COMPARISON OF APPROXIMATE TO EXACT NEXT-TO-NEXT-TO LEADING ORDER CORRECTIONS FOR HIGGS AND PSEUDOSCALAR HIGGS BOSON PRODUCTION

A.P. CONTOGOURIS,[1,2] P.K. PAPACHRISTOU[2]

[1]**Department of Physics, McGill University**
(Montreal, Quebec, H3A 2T8, Canada; e-mail: apcont@physics.mcgill.ca, acontog@cc.uoa.gr),

[2]**Nuclear and Particle Physics, University of Athens**
(Panepistimiopolis, Athens 15771, Greece; e-mail: ppapachr@cc.uoa.gr)

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Recently obtained NNLO exact corrections for Higgs and pseudoscalar Higgs boson production in hadron colliders are compared with approximate ones. As shown before, it is found that there is a range of a proper variable where these corrections differ little.

Some time ago, it was argued that for processes involving structure functions and/or fragmentation functions, over a range of a proper kinematic variable w, there is a part that dominates the next-to leading order (NLO) correction and that this part contains the distributions $\delta(1-w)$ and $[\ln^n(1-w)/1-w)]_+$ $n=0,1$ [1]. Subsequently this argument was extended to the then existing next-to-next-to leading order (NNLO) calculations, namely Drell—Yan (DY) production of lepton pairs ($q+\overline{q}\rightarrow\gamma^*$) and deep inelastic structure (DIS) functions ($q+\gamma^*\rightarrow q$) [2]*.

In the meantime, two more processes have been calculated in NNLO: Higgs boson production in hadron-hadron collisions ($g+g\rightarrow H$) [3] and neutral pseudoscalar Higgs boson production in hadron-hadron collisions ($g+g\rightarrow A$) [4]. Clearly, it would be important to see whether the procedures developed in [2] apply also to Higgs and pseudoscalar Higgs boson production as well.

In the calculation of Higgs boson production in hadron- hadron collisions to leading order (LO) [5] and to NLO [6], no approximations of the Higgs two gluon vertex are necessary†. This vertex is dominated by the top quark which is known to have a mass m_t much greater than that of the other quarks. However, the NNLO calculation was possible only in the limit where the Higgs boson mass

$$m_H << 2m_t. \tag{1}$$

In this limit, the top-quark loops are replaced by point-like vertices, and the corresponding effective Lagrangian [6–8] is known to provide a satisfactory description of the cross section for a Higgs boson at NLO [6].

In the calculation of the pseudoscalar Higgs boson production, the situation is more complicated. The Higgs boson sector of the Minimal Supersymmetric Standard Model consists of two complex Higgs doublets. Thus, apart from the mass of the neutral pseudoscalar Higgs boson m_A, the ratio of the vacum expectation values of the two Higgs doublets $\upsilon_1/\upsilon_2\equiv\tan\beta$ also enters. The calculation of [4] is valid for small and moderate values of $\tan\beta$; only then the $gg\rightarrow A$ is dominated by a top-quark loop. Then, for

$$m_A << 2m_t, \tag{2}$$

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†To NLO, this is due to an accidental cancellation of the dependence on the top quark mass between real and virtual corrections. See the last paper of Ref. [6].

the interaction of the pseudoscalar Higgs boson can be described by an effective Lagrangian [9].

In our approach [2], the proper variable is proportional to $\tau = m_H^2/S$ (or $\tau = m_A^2/S$), where $\sqrt{S}$ is the total c.m. energy of the initial hadrons. Our approach requires the inclusion of the region of τ large (inclusion of τ near 1). Since experiment excludes values of m_H (or m_A) ≤ 100 GeV, we have to consider $\sqrt{S}$ well exceeding this value. On the other hand, the inclusion of $\sqrt{S} \geq 2$ TeV would require m_H (or m_A) well exceeding 1 TeV, which would render questionable the field-theoretic approach. We then have considered a nominal energy of $\sqrt{S} = 520$ GeV. Clearly, for m_H (or m_A) ≥ 200 GeV, inequality (1) (or (2)) is violated and the whole results of [3] and [4] we use should be considered as just providing a mathematical model, where the approach of [2] can be tested.

We begin with $pp \to H + X$ (or $p\overline{p} \to H + X$) mediated via the subprocess $gg \to H^*$ and consider the cross-section

$$\sigma_{h_1+h_2\to H+X}(m_H^2, S) =$$

$$= \int_0^1 dx_1 dx_2 \overline{f}_{g/p}(x_1)\overline{f}_{g/p}(x_2)\sigma_{gg\to H}(m_H^2, x_1x_2S), \quad (3)$$

where h_1, h_2 denote p, p (or $p, \overline{p}$) and $\overline{f}_{g/p}(x)$ is the standard distribution of gluons inside p (or $\overline{p}$). Using dimensional analysis, we write the partonic cross-section in terms of the dimensionless variable

$$z = \frac{m_H^2}{x_1x_2S} = \frac{\tau}{x_1x_2}. \quad (4)$$

After factoring the collinear singularities (usually in the $\overline{MS}$ scheme), we end up with the expression [3]

$$\sigma_{h_1+h_2\to H+X}(\tau, S) = \tau f_{g/p} \otimes f_{g/p} \otimes (\sigma_{gg}(z)/z)(\tau), \quad (5)$$

where $\otimes$ denotes the standard convolution defined as

$$[f_1 \otimes f_2](\tau) = \int_0^1 dx_1 dx_2 f_1(x_1) f_2(x_2) \delta(\tau - x_1x_2). \quad (6)$$

The partonic cross-section $\sigma_{gg}(z)$ is given by the perturbation expansion

$$\sigma_{gg}(z) = \sigma_0 \left[\eta_{gg}^{(0)}(z) + \frac{\alpha_s}{\pi}\eta_{gg}^{(1)}(z) + \right.$$

$$\left. + \left(\frac{\alpha_s}{\pi}\right)^2 \eta_{gg}^{(2)}(z) + O(\alpha_s^3)\right], \quad (7)$$

where the functions $\eta_{gg}^{(k)}$, $k = 0, 1, 2$, are given in Eqs. (44), (45), (47), (48), and (49) of [3] and

$$\sigma_0 = \frac{\pi}{576v^2}\left(\frac{\alpha_s}{\pi}\right)^2 \quad (8)$$

with $v \approx 246$ GeV the Higgs vacuum expectation value.

Subsequently, we proceed as in [2]†. We write, for simplicity, $\sigma_{h_1+h_2\to H+X}(\tau, S) \equiv \sigma_H(\tau, S)$ and denote by $\sigma_H^{(k)}(\tau, S)$, $k = 0, 1, 2$, the $O(\alpha_s^k)$ part of $\sigma_H(\tau, S)$, by $\sigma_{Hs}^{(k)}$ the part of $\sigma_H^{(k)}$ arising from the distributions $\delta(1-z)$ and $[\ln^n(1-z)/1-z]_+$, $n = 0, 1, 2, 3$ (virtual, collinear and soft gluons), and by $\sigma_{Hh}^{(k)}$ the rest. We also define

$$L_H^{(k)}(\tau, S) = \frac{\sigma_{Hh}^{(k)}(\tau, S)}{\sigma_H^{(k)}(\tau, S)}. \quad (9)$$

In the subsequent calculations, we use $n_f = 5$ flavors and fix the renormalization and factorization scales at $\mu = M = m_H$. For the gluon distributions, we use the updated $\overline{MS}$ CTEQ5M1 set of [10].

Fig. 1, upper part, shows $L_H^{(k)}$, $k = 1, 2$, as functions of $\sqrt{\tau}$. For $L_H^{(1)}$, while for relatively small $\sqrt{\tau}$ is significant, for $\sqrt{\tau} \geq .63$ is below 30%. As for $L_H^{(2)}$, for $\sqrt{\tau} \geq .43$ is smaller than 20%. Moreover, both $L_H^{(k)}$ decrease fast as $\sqrt{\tau}$ increases towards 1.

As in [2], it is of interest to see the percentage of $\sigma_{Hh}^{(k)}$ of the total cross section determined up to $O(\alpha_s^k)$. Fig.1, upper part, also shows the ratios $\sigma_{Hh}^{(1)}/\left(\sigma_H^{(0)} + \sigma_H^{(1)}\right)$ and $\sigma_{Hh}^{(2)}/\left(\sigma_H^{(0)} + \sigma_H^{(1)} + \sigma_H^{(2)}\right)$. The former is below 31% and the latter below 15% for all $\sqrt{\tau}$‡. Again, both ratios decrease fast as $\sqrt{\tau}$ increase.

Now we turn to the calculation of the pseudoscalar Higgs boson production and consider $pp \to A + X$ (or $p\overline{p} \to A + X$) mediated via the subprocess $gg \to A$. As before, the partonic cross-sections $\sigma_{gg}(z)$ have an expansion similar to (7),

$$\sigma_{gg}(z) = \sigma_0 \left[\phi_{gg}^{(0)}(z) + \frac{\alpha_s}{\pi}\phi_{gg}^{(1)}(z) + \right.$$

$$\left. + \left(\frac{\alpha_s}{\pi}\right)^2 \phi_{gg}^{(2)}(z) + O(\alpha_s^3)\right], \quad (10)$$

*We recall that, in our approach [1,2], the various perturbation orders (LO, NLO, NNLO) should refer to the same subprocess.
†Although known since long ago (see [1]), as in [2], we present also results for $k = 1$.
‡This can also be seen in the first paper of [11].

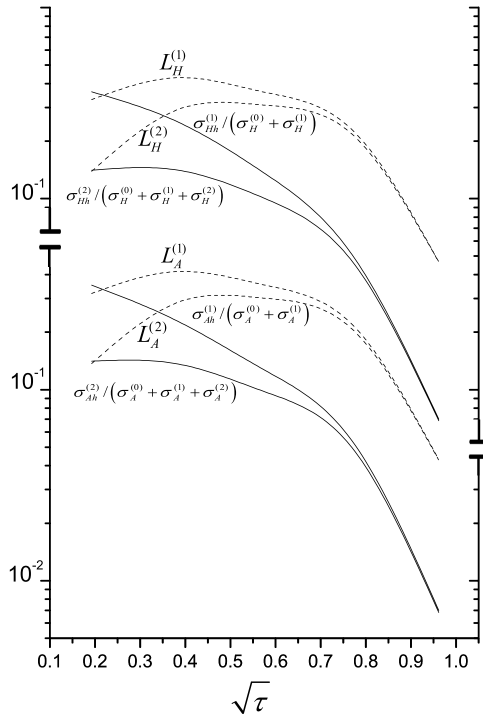

Fig.1. Upper part: The ratios $L_H^{(1)}$ and $\sigma_{Hh}^{(1)}/\left(\sigma_H^{(0)}+\sigma_H^{(1)}\right)$ (dashed lines) and the ratios $L_H^{(2)}$ and $\sigma_{Hh}^{(2)}/\left(\sigma_H^{(0)}+\sigma_H^{(1)}+\sigma_H^{(2)}\right)$ (solid lines) versus $\sqrt{\tau}=m_H/\sqrt{S}$. Lower part: The quantities $L_A^{(1)}$ and $\sigma_{Ah}^{(1)}/\left(\sigma_A^{(0)}+\sigma_A^{(1)}\right)$ (dashed) and the quantities $L_A^{(2)}$ and $\sigma_{Ah}^{(2)}/\left(\sigma_A^{(0)}+\sigma_A^{(1)}+\sigma_A^{(2)}\right)$ (solid) versus $\sqrt{\tau}=m_A/\sqrt{S}$

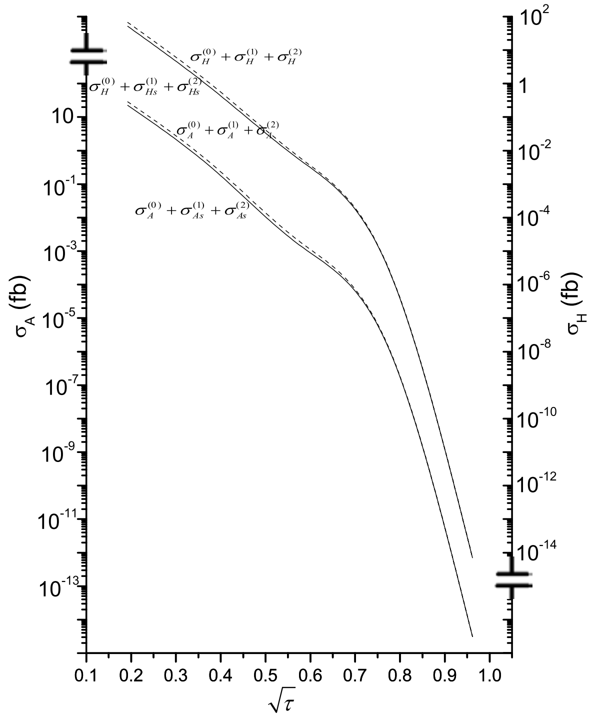

Fig.2. Upper part: The cross-sections $\sigma_H^{(0)}+\sigma_H^{(1)}+\sigma_H^{(2)}$ (dashed line) and $\sigma_H^{(0)}+\sigma_{Hs}^{(1)}+\sigma_{Hs}^{(2)}$ (solid line) versus $\sqrt{\tau}=m_H/\sqrt{S}$. Lower part: The quantities $\sigma_A^{(0)}+\sigma_A^{(1)}+\sigma_A^{(2)}$ (dashed) and $\sigma_A^{(0)}+\sigma_{As}^{(1)}+\sigma_{As}^{(2)}$ (solid) versus $\sqrt{\tau}=m_A/\sqrt{S}$

where z is given by (4) with $\tau=m_A^2/S$, $\phi_{gg}^{(k)}(z)$ are given in Eqs. (8)-(11) of [4] (together with the expressions of $\eta_{gg}^{(k)}(z)$), and here

$$\sigma_0=\frac{\pi}{256v^2\tan^2\beta}\left(\frac{\alpha_s}{\pi}\right)^2. \tag{11}$$

Now we write $\tan^2\beta\sigma_{h_1+h_2\to A+X}\equiv\sigma_A(\tau,S)$ and, as before, denote by $\sigma_A^{(k)}(\tau,S)$ the $O(\alpha_s^k)$ part of $\sigma_A(\tau,S)$, by $\sigma_{As}^{(k)}$ the part of $\sigma_A^{(k)}$ arising from distributions, and by $\sigma_{Ah}^{(k)}$ the rest. We define

$$L_A^{(k)}(\tau,S)=\frac{\sigma_{Ah}^{(k)}(\tau,S)}{\sigma_A(\tau,S)} \tag{12}$$

and fix the renormalization and factorization scales at $\mu=M=m_A$. Again, for the gluon distributions, we use the CTEQ5M1 set of [10].

Fig. 1, lower part, shows $L_A^{(k)}$, $k=1,2$, as functions of $\sqrt{\tau}$. All the results are similar as for $L_H^{(k)}$. Similar are also the results for the ratios $\sigma_{Ah}^{(1)}/\left(\sigma_A^{(0)}+\sigma_A^{(1)}\right)$ and $\sigma_{Ah}^{(2)}/\left(\sigma_A^{(0)}+\sigma_A^{(1)}+\sigma_A^{(2)}\right)$.

We note the following*: suppose that, in $\sigma_{Hs}^{(k)}$ and $\sigma_{As}^{(k)}$, apart from the terms arising from the distributions $\delta(1-z)$ and $[\ln^n(1-z)/1-z)]_+$, we include also the terms $\ln^m(1-z)$, $m=1,2,3$. Defining as $\sigma_{Hs}^{(k)}$ and $\sigma_{Ah}^{(k)}$ the rest, we find that the ratios $\sigma_{Hh}^{(k)}/\sigma_H^{(k)}$ and $\sigma_{Ah}^{(k)}/\sigma_A^{(k)}$ decrease significantly in magnitude over the entire range of $\sqrt{\tau}$. Of course, the same holds for the ratios $\sigma_{Hh}^{(1)}/\left(\sigma_H^{(0)}+\sigma_H^{(1)}\right)$, $\sigma_{Hh}^{(2)}/\left(\sigma_H^{(0)}+\sigma_H^{(1)}+\sigma_H^{(2)}\right)$ and the corresponding ratios with H replaced by A.

Note also that, in [11], numerical results very similar to [3] and [4] have been obtained by expanding the phase-space integrals around the kinematic point $z=$

*A similar remark regarding resummations was first made by M. Kramer, E. Laenen and M. Spira, Nucl. Phys. B **511** (1998) 523.

$\tau/x_1x_2 = 1$, where $\tau = m_H^2/S$ or $\tau = m_A^2/S$, and keeping a number of terms. Although the first paper of [11] was published before [3], we prefer the methods of [3] as they avoid expansions.†

Finally, in Fig. 2, upper part, we present the total cross-sections $\sigma_H^{(0)} + \sigma_H^{(1)} + \sigma_H^{(2)}$ (dashed line) and $\sigma_H^{(0)} + \sigma_{Hs}^{(1)} + \sigma_{Hs}^{(2)}$ (solid line). What is important is that as $\sqrt{\tau}$ increases towards 1, both cross-sections approach each other and practically coincide for $\tau \geq 0.8$. The same is observed in Fig. 2, lower part, which shows the quantities $\sigma_A^{(0)} + \sigma_A^{(1)} + \sigma_A^{(2)}$ (dashed) and $\sigma_A^{(0)} + \sigma_{As}^{(1)} + \sigma_{As}^{(2)}$ (solid).

In conclusion, under the assumptions discussed at the beginning, we have shown that not only in DY production and DIS [2], but also in Higgs and pseudoscalar Higgs boson production ($gg \to H$ and $gg \to A$), there is a part containing the distributions $\delta(1-z)$ and $[\ln^n(1-z)/1-z)]_+$, $n = 0, 1, 2, 3$ (virtual, soft, and collinear part), that, for $\sqrt{\tau}(= m_H/\sqrt{S}$ or $m_A/\sqrt{S})$ not too small, dominates the NLO and NNLO correction. This part is determined much easier than the NLO and in particular the NNLO correction. Of course, as was stressed in [2], this part should not be restricted to too small a region near 1, because the threshold resummation [12] becomes very important.

After the completion of this article, the paper by V. Ravindran, J. Smith and W. van Neerven, hep-ph/0302335, appeared, confirming the results of [3], [4] and [11] by a different method.

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ПОРІВНЯННЯ НАБЛИЖЕНИХ І ТОЧНИХ NNLO-ПОПРАВОК ДЛЯ НАРОДЖЕННЯ ХІГГСОВОГО ТА ПСЕВДОСКАЛЯРНОГО ХІГГСОВОГО БОЗОНІВ

А.П. Контогуріс, П.К. Папахрісту

Р е з ю м е

Нещодавно отримані точні поправки в NNLO-наближенні для народження хіггсових та псевдоскалярних хіггсових бозонів на адронних прискорювачах порівнюються з попередніми наближеннями. Як було показано раніше, існує певний діапазон відповідної змінної, де ці поправки мало відрізняються.

†In the first of [11], an error was found in the calculation of T. Matsuura et al., Nucl. Phys. B **319** (1989) 570 on DY production. We have repeated the relevant calculations of [2] and found no significant change.