DETERMINATION OF AVERAGE RESONANCE PARAMETERS FROM ELASTIC SCATTERING CROSS-SECTIONS OF LOW-ENERGY NEUTRONS BY EVEN-EVEN NUCLEI

M.M. PRAVDIVY, I.O. KORZH, M.T. SKLYAR

Institute for Nuclear Researches, Nat. Acad. Sci. of Ukraine
(47, Nauky Prosp., Kyiv 03028, Ukraine)

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A new method for the determination of average resonance parameters by analyzing the differential cross-sections of keV-neutrons elastically scattered by even-even nuclei has been developed on the basis of the relations established between the coefficients ω_1 and ω_2 of the expansion of the differential cross-sections in Legendre polynomials and the diagonal elements η_0 and η_1 of the averaged S-matrix. For illustration, a complete set of average resonance parameters S_0, S_1, R'_0, R'_1, $S_{1,1/2}$, and $S_{1,3/2}$ for a ^{116}Sn nucleus has been obtained and their analysis has been performed.

Introduction

The strength functions S_l and radii of potential scattering R'_l determine average cross-sections in the region of unresolved resonances [1]. Values amplitudes can be obtained by making use of two main methods: by analyzing resonances in an energy region where they are resolved, and by analyzing average cross-sections in a region of unresolved resonances. The first method is straightforward. But it has substantial shortcomings: the number of resolved resonances is small (several tens or less) for most nuclei, resonance parameters are often unknown, and weak resonances can be missed in experiment. The second method is not straightforward. It requires knowing experimental cross-sections with a high accuracy. Moreover, the procedure of theoretical description of cross-sections becomes often problematic if a considerable intermediate structure exists. As a result, parameters defined by different methods are often inconsistent. Therefore, although a great body of experimental data concerning resonance parameters is now available, the recommended values [2—4] are of low reliability and, with the appearance of new experimental data, can change their magnitudes by a factor of 2—3. Hence, the development of new methods for determination of average resonance parameters from experimental data remains an actual problem until now.

A new method for the determination of average resonance parameters from the experimental differential cross-sections of low-energy neutron elastic scattering was developed and applied in [5]. In the framework of the single-level R-matrix approximation, expressions connecting the differential cross-section parameters $\sigma_{\rm el}$, ω_1, and ω_2 with strength functions and potential scattering phases for orbital momenta $l = 0$ and 1, have been derived. By fitting those parameters to their experimental values, the complete sets of average resonance parameters S_0, S_1, R'_0, R'_1, $S_{1,1/2}$, and $S_{1,3/2}$ have been obtained for many nuclei. Nevertheless, the method is very complicated and cumbersome, and the obtained resonance parameters deviate in many cases from recommended ones and hence require an additional verification.

In this work, we have shown that the average resonance parameters can be determined from the differential cross-sections of neutron elastic scattering and in the framework of the nuclear optical model.

1. Procedure for the Determination of Average Resonance Parameters

The optical model allows one to estimate the diagonal elements $\eta_l = \eta_{l\rm Re} - i\eta_{l\rm Im}$ of the scattering matrix which determine the total cross-sections as well as the cross-sections of potential and compound scatterings [1]. For many nuclei, mainly the orbital momenta $l = 0$ and 1 make contributions to the neutron interaction in the investigated energy range, while the neutron radiative capture cross-sections can be neglected ($\sigma_t \approx \sigma_{\rm el}$). In this case, the cross-sections in the optical model can be expressed as follows:

$$\sigma_{s0} = \pi\lambda\!\!\!^{-2}(2 - 2\eta_{0\rm Re} - T_0), \quad (1)$$

$$\sigma_{s1} = 3\pi\lambda\!\!\!^{-2}(2 - 2\eta_{1\rm Re} - T_1), \quad (2)$$

$$\sigma_c = \sigma_{c0} + \sigma_{c1} = \pi \lambda\!\!\!^{-2} T_0 + 3\pi \lambda\!\!\!^{-2} T_1, \quad (3)$$

$$\sigma_{\rm el} = 2\pi \lambda\!\!\!^{-2} (4 - \eta_{0\rm Re} - 3\eta_{1\rm Re}), \quad (4)$$

where $T_l = 1 - |\eta_l|^2$. Differential cross-sections in the optical model are

$$\sigma_{\rm sel}(\mu) = \frac{\lambda\!\!\!^{-2}}{4} \left| \sum_l (2l+1)(1-\eta_l) P_l(\mu) \right|^2, \quad (5)$$

$$\sigma_{\rm cel}(\mu) = \frac{\lambda\!\!\!^{-2}}{2} \sum_{L=0,2} B_L P_L(\mu), \quad (6)$$

where the coefficients B_L for even-even nuclei and $l = 0, 1$ have the forms

$$B_0 = \frac{1}{2}(T_0 + 3T_1) = \frac{\sigma_{\rm cel}}{2\pi \lambda\!\!\!^{-2}}, B_2 = T_{1,3/2}. \quad (7)$$

If we expand the experimental differential neutron elastic scattering cross-sections in Legendre polynomials as

$$\sigma_{\rm el}(\mu) = \frac{\sigma_{\rm el}}{4\pi} \left\{ 1 + \omega_1 P_1(\mu) + \omega_2 P_2(\mu) \right\}, \quad (8)$$

the expansion coefficients ω_1 and ω_2 are determined by the averaged cosines $\overline{\mu}$ of the elastic scattering angle and by averaged squares of cosines $\overline{\mu^2}$ [6], which, in turn, are expressed through the matrix elements η_l [7]. It made us possible, using Eqs. (5)—(7), to obtain the following expressions for the coefficients:

$$\omega_1 = \frac{6\pi \lambda\!\!\!^{-2}}{\sigma_{\rm el}} \left(1 - \eta_{0\rm Re} - \eta_{1\rm Re} + \eta_{0\rm Re}\eta_{1\rm Re} + \eta_{0\rm Im}\eta_{1\rm Im} \right), \quad (9)$$

$$\omega_2 = \frac{2}{\sigma_{\rm el}} \left(\sigma_{s1} + \pi \lambda\!\!\!^{-2} T_{1,3/2} \right). \quad (10)$$

In resonance theory, average cross-sections are determined by average resonance parameters S_0, S_1, R'_0, and R'_1. For narrow resonances, the average cross-sections of potential and resonance scatterings coincide with those of potential and compound scatterings in the optical model. Taking into consideration that penetration factors depend on the quoted neutron strength functions through the relation

$$T_l = 1 - \exp\left(-2\pi S_l v_l \sqrt{E} \right), \quad (11)$$

where $v_0 = 1$, $v_1 = \frac{(kR)^2}{1+(kR)^2}$, and that average matrix elements are determined by strength functions and potential scattering phases, the expansion coefficients of differential cross-sections $\sigma_{\rm el}$, ω_1, and ω_2 can be expressed through the resonance parameters S_0, S_1, R'_0, R'_1, and $S_{1,3/2}$, which can be determined from the fitting to experimental values of $\sigma_{\rm el}$, ω_1, and ω_2. The average cross-sections of the potential scattering were taken in the fitting procedure as [1]

$$\sigma_{s0} = 4\pi \lambda\!\!\!^{-2} \sin^2 \delta_0 (1 - 0.5T_0),$$

$$\sigma_{s1} = 12\pi \lambda\!\!\!^{-2} \sin^2 \delta_1 (1 - 0.5T_1), \quad (12)$$

where $\delta_0 = \arcsin(\rho R_0^\infty) - \rho$, $\delta_1 = \arcsin(\rho v_1 R_1^\infty) - \rho + \arctan\rho$, $\rho = kR$, $R = 1.23\sqrt[3]{A} + 0.8$, $k = \lambda\!\!\!^{-}{}^{-1} = 0.21968\frac{A}{A+1}\sqrt{E}$, $R'_l = R\,[1 - (2l+1)R_l^\infty]$. Thus, a complete set of average resonance parameters S_0, R'_0, R'_1, $S_{1,1/2}$, $S_{1,3/2}$, and $S_1 = (S_{1,1/2} + 2S_{1,3/2})/3$ can be determined from a fitting to experimental values of $\sigma_{\rm el}$, ω_1, and ω_2.

To illustrate the application of the method proposed, the results of calculations of resonance parameters are shown below for a nucleus ^{116}Sn.

2. Results of Calculations of Resonance Parameters and Their Analysis

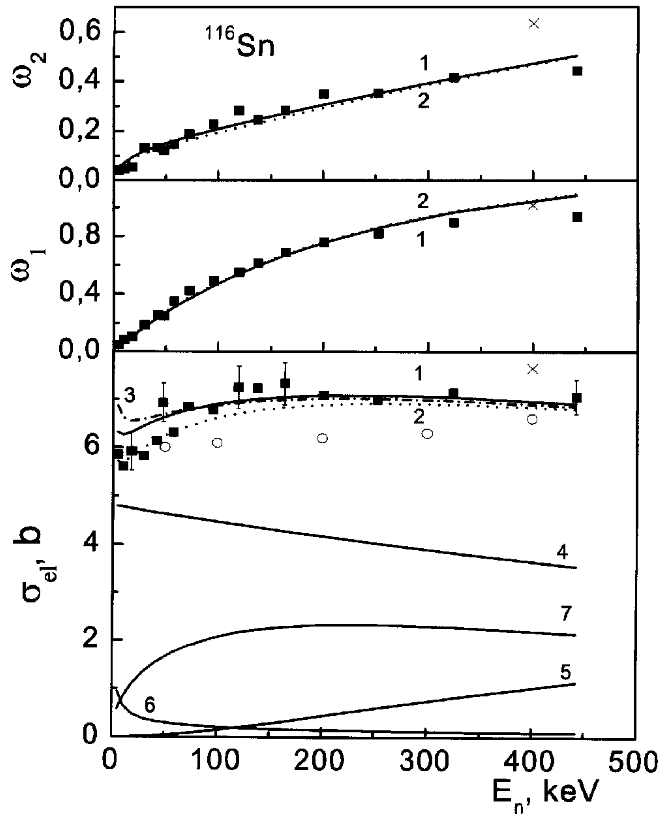
In [5], the experimental data on $\sigma_{\rm el}$, ω_1, and ω_2 for a number of nuclei, including ^{116}Sn, were published. The measurements were carried out by the time-of-flight method for various energies of an interval of 1—442 keV. The obtained resonance parameter values were published in [9]. Since the energy resolution of the experiment at the beginning of the interval was much less than the average distance between s-resonances of ^{116}Sn ($D_0 \approx 0.6$ keV [2,4]), we carried out an extra averaging of experimental data over energies.

The recommended values for various resonance parameters, calculated by different methods on the basis of different experimental data and already passed an earlier analysis, have been published in the literature for many nuclei [2–4]. They are worth to be preserved while carrying out a fitting procedure for the determination of other resonance parameters. This will allow one to test the recommended values for their agreement to experimental data on $\sigma_{\rm el}$, ω_1, and ω_2 and, moreover, to diminish, to some extent, a negative influence of correlations, existing between resonance parameters, on reliability of the results obtained.

For ^{116}Sn nuclei, the recommended resonance parameters are $S_0 = 0.26(5)$ [4] or $0.40(25)$ [5], and $R'_0 = 6.2(1)$ [3] (the errors are indicated in parentheses;

hereafter, all the parameters S_i are measured in 10^{-4} units, and R'_i in fm). At first, the fitting to the experimental values of $\sigma_{\rm el}$, ω_1, and ω_2 was fulfilled at the fixed $S_0 = 0.26$ and $R'_0 = 6.2$. The results obtained are shown by curve *3* in the figure, where the cross-section components $\sigma_{\rm el} = \sigma_{c0} + \sigma_{c1} + \sigma_{s0} + \sigma_{s1}$ are also depicted. It is seen that, for an energy decrease below of ~ 10 keV, the cross-sections σ_{c0} drastically grow (the value of $\pi\lambda\!\!\!^{-2}$ has a 10-fold rise as the energy decreases from 10 to 1 keV). It is also seen from the figure that the use of the value $S_0 = 0.26$ results in a notable increasing of the cross-sections $\sigma_{\rm el}$ at the beginning of the energy interval, which evidences for an overestimated value of that parameter. An optimal fitting was obtained for $S_0 = 0.16$, which is consistent within the error limits with a recommended value in [4] and, at the same time, makes it possible to describe well the cross-sections $\sigma_{\rm el}$ (curve *1*). The relevant obtained values for resonance parameters are quoted in the table (the components of the cross-sections $\sigma_{\rm el}$, which are shown in the figure, were calculated using those quoted values). The results of calculations with the values of resonance parameters taken from [9] are also depicted in the figure (curves *2*) and, in general, agree well with the results of our fitting, excluding the cross-sections $\sigma_{\rm el}$, especially at the beginning of the energy interval. Those inconsistencies arise from the preservation of the recommended value $R'_0 = 6.2$ in our calculations.

It is seen from the figure that the experimental values of $\sigma_{\rm el}$, ω_1, and ω_2 obtained by various authors have notable dispersions that are especially large for the cross-sections $\sigma_{\rm el}$. They illustrate difficulties arising in the determination of resonance parameters from experimental data by fitting methods.



Energy dependences of the parameters $\sigma_{\rm el}$, ω_1, and ω_2 for a ^{116}Sn nucleus. Experimental data: [9] (squares), [10] (circles), [11] (crosses). Results of calculations: the fitting curves for the resonance parameters of the current work (*1*), of the work [9] (*2*), and for recommended resonance parameters (curve *3*); the components of the cross-sections $\sigma_{\rm el}$ obtained for the resonance parameters of the current work: σ_{s0} (*4*), σ_{s1} (*5*), σ_{c0} (*6*), and σ_{c1} (*7*)

Conclusions

In this work, a new method for the determination of average resonance parameters from the experimental differential cross-sections of elastic neutron scattering by even-even nuclei in the keV-energy region has been developed. The method is rather simple and allows one to verify the available values of resonance parameters upon their correspondence to experimental data and to diminish their errors, if needed. The method was illustrated by applying it to a ^{116}Sn nucleus, for which a complete set of average resonance parameters S_0, S_1, R'_0, R'_1, $S_{1,1/2}$, and $S_{1,3/2}$ has been obtained. The recommended values for the parameters S_0 and R'_0, within the error limits, were preserved in this set, and the other parameter values are in a good agreement with the available dependences of those parameters on A.

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Average resonance parameters of ^{116}Sn nucleus

S_0, $1\cdot10^{-4}$	S_1, $1\cdot10^{-4}$	$S_{1,1/2}$, $1\cdot10^{-4}$	$S_{1,3/2}$, $1\cdot10^{-4}$	R'_0, f	R'_1, f	Source
0.16(5)	3.26(31)	4.50(94)	2.64(22)	6.2(2)	10.4(4)	This work
0.16(4)	3.81(24)	7.0(1.0)	2.21(14)	5.7(2)	11.3(4)	[9]

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ВИЗНАЧЕННЯ СЕРЕДНІХ РЕЗОНАНСНИХ ПАРАМЕТРІВ ІЗ ПЕРЕРІЗІВ ПРУЖНОГО РОЗСІЯННЯ НЕЙТРОНІВ ПАРНО-ПАРНИМИ ЯДРАМИ В ОБЛАСТІ НИЗЬКИХ ЕНЕРГІЙ

М.М. Правдивий, І.О. Корж, М.Т. Скляр

Р е з ю м е

Розроблено новий метод визначення середніх резонансних параметрів з аналізу експериментальних диференціальних перерізів пружного розсіяння кілоелектронвольтних нейтронів парно-парними ядрами. Встановлено зв'язок коефіцієнтів розкладу ω_1 і ω_2 диференціальних перерізів за поліномами Лежандра з діагональними елементами η_0 і η_1 середньої S-матриці, що дає змогу визначати середні резонансні параметри. Як приклад для ядра ^{116}Sn із наявних експериментальних параметрів $\sigma_{\rm el}$, ω_1, ω_2 отримано повний набір середніх резонансних параметрів S_0, S_1, R'_0, R'_1, $S_{1,1/2}$, $S_{1,3/2}$ і проведено їх аналіз.