INTERPRETATION OF THE NUCLEAR STATES OF EVEN ISOTOPES OF SELENIUM IN TERMS OF A PHENOMENOLOGICAL COLLECTIVE MODEL

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A classification of excited nucleus states by energy level bands is considered for the $^{72-82}$Se nuclei using various collective models. The dependence of the state identification on the choice of a particular model as well as the dependence of parameters of a nucleus shape on the choice of the system of base levels is established.

Introduction

Collective phenomenological models of deformed soft nuclei are attractive by their comparative simplicity, the clearness of calculation procedures, and a possibility for their further developing on the base of new experimental data with the use of values of the parameters that have been obtained before. This was observed, for instance, under the developing of versions of the model of interacting bosons (MIB) and the models built on the basis of A. Bohr's Hamiltonian [1]. As a result, a generalized model of nuclei that takes into account the coexistence of collective and one-particle modes in a nucleus was developed. However, the description of the spectra of soft deformed nuclei remains problematic in many cases in the region of low-lying excited states of a nucleus. A consistent microscopic description of the structure of such nuclei needs introducing additional parameters, which complicates the numerical calculations of the spectroscopic characteristics of nuclei.

In the models with A. Bohr's Hamiltonian, the identification of excited states of a nucleus by the collections of quantum numbers characterizing a certain mode and the corresponding symmetry of the states of a nucleus has a great importance [2]. Particularly, a consideration of the collective states with negative parity demands to account the octupole deformation of the nuclear surface along with the quadrupole-type deformation. Softness of the nucleus in any direction of the axes of its symmetry initiates the appearance of additional energy bands, the comparison of which with real observable levels needs exact identification of the latter. The consideration of this problem with reference to even nuclei of selenium within the framework of a phenomenological model of soft nuclei is the subject of the present paper.

The choice of the mentioned nuclei was caused by several reasons. First, those nuclei possess the characteristic feature such as the existence of low-lying excited states of the level with odd parity and spin $I = 3$. Secondly, by the number of neutrons in those nuclei, they are situated between their magic values ($N = 28$ and 50), which can cause the appearance of many collective and non-collective modes resulting in the instability of their shapes.

The type of the Hamiltonian of the collective model influences not only the quality of description of the energy structure of a nucleus, but also the description of inelastic nucleon-nucleus scattering which directly depends on the treating of the nature of excited states of a nucleus and on their belonging to certain energy bands. For example, 2_2^+- and 0_2^+-states were interpreted in the rotational-vibrational models as the beginnings of β- and γ-vibrational bands, correspondingly [3, 4], while they are regarded within the framework of the Davydov—Chaban model (DChM) [5] as the beginnings of the first anomalous and rotational-vibrational bands at fixed values of the independent parameters, namely of the longitudinal quadrupole softness μ_β and non-axiality of the nucleus in its ground state ($\gamma_0 \neq 0$). Such an ambiguity of the interpretation of the states of nuclei (particularly, of a nucleus ^{76}Se [6]) is the consequence of the presence (among them) of the fundamental vibrational band of states with spins $I^\pi = 0_2^+, 2_2^+, 4_1^+$ which can be considered as members of the vibrational quadrupole two-phonon triplet built on the state $|0_1^+\rangle$. In the cases where the softness of a nucleus is accounted for not only by the β (longitudinal), but also by the γ (transversal) quadrupole deformations (M$\beta\gamma$D) [7], the existence of additional (in comparison with DChM) energy bands is predicted and the collective parameters of a nucleus are refined along with the possible introduction of the octupole and/or hexadecapole types of deformation.

1. Basic Principles of the Model of Soft Nucleus

The Hamilton operator in the quadrupole approximation describing the excitation of even-even nuclei in the proper coordinate frame of a nucleus (PCFN), can be written as [5]

$$\hat{H} = \frac{\hbar^2}{2B}\left\{\hat{T}_\beta + \frac{1}{\beta^2}\hat{T}_\gamma + \frac{1}{\beta^2}\hat{T}_{\rm rot}\right\} + V(\beta,\gamma), \qquad (1)$$

where B is the mass parameter of a nucleus under quadrupole deformation. The operators of the kinetic energy of rotation $\hat{T}_{\rm rot}$, and β-, γ-vibrations of the surface of a nucleus $\widehat{T}_\beta$ and $\hat{T}_\gamma$, respectively, are defined by the known expressions [5—7]. Moreover, $\hat{T}_{\rm rot}$ contains the components of the operator of angular momentum $\hat{L}'_k$ with respect to the axes of PCFN.

The eigenfunctions of Hamiltonian (1) depend on the internal variables $0 \leq \beta < \propto$ and $0 \leq \gamma \leq \pi/3$ which describe the total deformation of a nucleus and its asymmetry (non-axiality), respectively, as well as on the Euler angles determining the orientation of a nucleus with regard to the axes of the laboratory coordinate frame (LCF). Operators (1) act in a five-dimensional phase space of variables β, γ, θ_1, θ_2, θ_3 with a volume element $d\tau = 2B^{5/2}\beta^4|\sin 3\gamma|\sin\theta_2 d\beta d\gamma d\theta_1 d\theta_2 d\theta_3$.

At not large excitation energies of a nucleus, its potential energy may be represented by a sum of functions of the variables β and γ in enough general form, i.e. $V(\beta) + V(\gamma) + V(\beta,\gamma)$ [5], where the third term characterizes the connection between β- and γ-vibrations of the surface; at small excitation energies of a nucleus, this connection can be considered as a perturbation in the operator of potential energy (the influence of the γ-vibrations on the β-vibration is taken into account by means of the operator of kinetic energy $\hat{T}_{\rm rot}(\gamma,\hat{\theta})$ [7]).

Thus, we consider the Schrödinger equation [6, 7]

$$\left\{\frac{\hbar^2}{2B}\left[\hat{T}_\beta + \frac{1}{\beta^2}\hat{T}_\gamma + \frac{1}{\beta^2}\hat{T}_{\rm rot}\right] + V(\beta) + \right.$$

$$\left. + \frac{\beta_0^4}{\beta^2}V(\gamma) - E_{I\tau n_\beta n_\gamma}\right\}\Psi_{IM\tau n_\beta n_\gamma}(\beta,\gamma,\hat{\theta}) = 0, \qquad (2)$$

where I and M — the angular momentum of the state of a nucleus and its projection on the z axis in LCF, respectively; τ — the ordinal number of the eigenvalue of the operator of a rigid rotator for the given value of spin I; quantum numbers n_β and n_γ vary over $0, 1, 2, \ldots$ and characterize longitudinal and transversal vibrations of the surface of a nucleus, respectively; $\hat{\theta} \equiv \{\theta_1,\theta_2,\theta_3\}$ – Euler angles determining the orientation of the PCFN with regard to the axes of the LCF.

We seek the wave function of Eq. (2) in the form

$$\Psi_{IM\tau n_\beta n_\gamma}(\beta,\gamma,\hat{\theta}) = \frac{1}{\beta^2}\varphi_{I\tau n_\beta n_\gamma}(\beta)\Phi_{IM\tau n_\gamma}(\gamma,\hat{\theta}), \qquad (3)$$

which allows us to carry out the separation of variables in Eq. (2) with a separation constant $(-\frac{\hbar^2}{2B}\varepsilon_{I\tau n_\gamma})$ for the functions $\varphi_{I\tau n_\beta n_\gamma}(\beta)$ and $\Phi_{IM\tau n_\gamma}(\gamma,\hat{\theta})$.

The equation for the function $\varphi_{I\tau n_\beta n_\gamma}(\beta)$ will be reduced to the known equation for the parabolic cylinder function [8] with parameters C_β and β_0 which determine elasticity and deformation of a nucleus in its ground state ($I = I_0 = 0$), respectively. At $I \neq 0$, the parameter of equilibrium deformation β_0 shifts to $\beta_{I\tau n_\gamma}$, and the parameter C_β does to $\bar{C}_\beta(n_\gamma)$ (the index n_γ at quantities describing β-vibrations means their dependence on the separation constant of the variables in Eq. (2), i.e. on $\varepsilon_{I\tau n_\gamma}$). After the introduction of the parameters

$$\mu_\beta = \frac{1}{\beta}\left(\frac{\hbar^2}{BC_\beta}\right)^{1/4}, \quad p_{I\tau n_\gamma} = \frac{1}{\beta_0}\beta_{I\tau n_\gamma} \geq 1,$$

$$\omega_{0\beta} = \sqrt{C_\beta/B}, \qquad (4)$$

where μ_β is the parameter of non-adiabaticity (softness) with regard to β-vibrations of the surface of a nucleus, the equation for the function $\varphi_{I\tau n_\beta n_\gamma}$ transforms to that for a parabolic cylinder function $D_\nu(\varsigma)$ with the argument

$$\varsigma = \sqrt{2}(\mu_\beta \beta_0)^{-1}(4 - 3/p_{I\tau n_\gamma})^{1/4}(\beta - \beta_{I\tau n_\gamma}). \quad (5)$$

Eigenvalues ν of the function $D_\nu(z)$ were found by the technique described in paper [8].

The influence of γ-vibrations of the surface of a nucleus on its β-vibrations is taken into account by means of the parameters $p_{I\tau n_\gamma}$, μ_β as well as solutions of the equation for the function $\Phi_{IM\tau n_\gamma}(\gamma, \hat{\theta})$ under the assumption of existence of energy bands of the levels by the quantum number n_γ. This allows us to find the quantities $\varepsilon_{I\tau n_\gamma}$ and to write down the energy spectrum of excited states of a nucleus [6]:

$$\frac{E_{I\tau n_\beta n_\gamma}}{\hbar\omega_0} = \left\{ \left(\nu_{I\tau n_\beta n_\gamma} + \frac{1}{2}\right)\sqrt{4 - \frac{3}{p_{I\tau n_\gamma}}} + \right.$$

$$\left. + \frac{1}{2}\left(\frac{\mu_\beta}{p_{I\tau n_\gamma}}\right)^2 \varepsilon_{I\tau n_\gamma}\left[1 + \left(\frac{\mu_\beta}{p_{I\tau n_\gamma}}\right)^4 \varepsilon_{I\tau n_\gamma}\right]\right\}. \quad (6)$$

As the operator $\hat{T}_{\rm rot}$ acts on the variables $\hat{\theta}$ and γ, a finding of the function $\Phi_{IM\tau n_\gamma}(\gamma, \hat{\theta})$ is very complicated and some simplifications are required. In particular, we suppose that $\hat{T}_{\rm rot}$ does not depend on the variable γ, and only contains the parameter γ_0 (zero-order approximation). In addition, the dependence of $\hat{T}_{\rm rot}$ on γ as on a variable is taken into account within the framework of perturbation theory [6, 7]. With this purpose, the operator $\hat{T}_\gamma$ is transformed in the appropriate way, and the operator $\hat{T}_{\rm rot}$ is represented as the expansion in a Taylor series in powers of the value $(\gamma - \gamma_0)$ with the subsequent expression through the components of the momentum $\hat{L}'_k$ with respect to the axes of PCFN [9].

At a fixed collection of the quantum numbers (I, τ) and with the parameter of nonaxiality γ_0, the eigenvalues $\varepsilon^{(0)}_{I\tau}$ and wavefunctions of a rigid non-axial rotator [5], which are used in perturbation theory as the initial ones in solving the problem on eigenvalues and eigenfunctions $\Phi_{IM\tau n_\gamma}(\gamma, \hat{\theta})$, are found.

The description of γ-vibrations of the surface of a nucleus is directly connected to the solution of the equation (see Eq. (27) in [7])

$$\left\{ d^2/d\gamma^2 + \left[-\Im_{I\tau n_\gamma}(\gamma) + \left(\varepsilon^{(0)}_{I\tau n_\gamma} - \varepsilon^{(0)}_{I\tau}\right)\right]\right\} u_{I\tau n_\gamma}(\gamma) = 0, \quad (7)$$

in which the model function $\Im_{I\tau n_\gamma}(\gamma)$ with $V(\gamma) = \frac{1}{2}C_\gamma(\gamma - \gamma_0)^2$ looks like

$$\Im_{I\tau n_\gamma}(\gamma) = \frac{BC_\gamma}{\hbar^2}\beta_0^4(\gamma - \gamma_0)^2 - \frac{9}{4}(1 + \sin^{-2} 3\gamma), \quad (8)$$

where C_γ is the constant of elasticity, and γ_0 is the deformation of a non-axial nucleus in its ground state.

Function (8) (in contrast to the one in [10]) allows identifying the states of a nucleus more adequately. At the excitation of a nucleus, the second term in the r.h.s. of (8) shifts the parameter of equilibrium non-axiality from γ_0 for the ground state ($I = I_0 = 0$) to the value $\gamma = \gamma_{I\tau n_\gamma}$, which is determined by the condition $d\Im_{I\tau n_\gamma}(\gamma)/d\gamma = 0$.

For solving Eq. (7) with potential (8), the additional parameters

$$q_{I\tau n_\gamma} \equiv (\gamma_{I\tau n_\gamma}/\gamma_0) \geq 1,$$

$$\bar{\mu}_\gamma = \frac{1}{\beta_0\gamma_0}\left(\frac{\hbar^2}{BC_\gamma}\right)^{1/4} = \frac{1}{\beta_0}\mu_\gamma \quad (9)$$

are introduced. Here, $\bar{\mu}_\gamma$ (or μ_γ) is the parameter of softness of a nucleus with regard to γ-vibrations. The quantity $q_{I\tau n_\gamma}$ at fixed values of γ_0 and $\bar{\mu}_\gamma$ is determined from solving the nonlinear equation [6]

$$q_{I\tau n_\gamma} \equiv q_\gamma = 1 + \frac{27}{4}\gamma_0^3\bar{\mu}_\gamma^4\left|\frac{\cos(3\gamma_0 q_\gamma)}{\sin^3(3\gamma_0 q_\gamma)}\right| > 1. \quad (10)$$

Equation (7) is reduced by the appropriate transformations to the equation that has parabolic cylinder functions as a solution [7], and the eigenvalues of these functions are used in the determination of energies of the excited states of a nucleus.

2. Calculation of the Parameters of the Model

The parameters for each of the considered models were determined by the minimum of a root-mean-square deviation of the experimental quantities (energy levels, spins, parity) from the corresponding theoretical ones for the base states. The results of calculation for even nuclei of selenium are presented in Tables 1—5 (similar results on a nucleus ^{76}Se are given in Table 3 in [6]), where the asterisk (*) marks the experimental basic levels, on which the determination of the model parameters was made.

A set of varied parameters includes the non-axiality γ_0 of a nucleus in the ground state under its quadrupole deformation as well as the deformability (softness) under longitudinal μ_β and transversal μ_γ quadrupole deformations. With regard for higher multipole deformations (octupole ($\lambda = 3$), hexadecapole ($\lambda = 4$), etc.), the additional varied parameters of softness of a nucleus and the parameters of non-axiality are introduced at the corresponding multipolarities [11]. The energy factor $\hbar\omega_0$ is determined from the condition of the minimum of a root-mean-square deviation of theoretical quantities from experimental ones by the totality of base states, which gives

$$\hbar\omega_0 = \left\{ \sum_{I,\tau,n_i} \frac{(\hbar\omega_0)^{-1}\Delta E_{I\tau n_i}}{E^{\text{exp}}_{I\tau n_i}} \right\} \times$$

$$\times \left\{ \sum_{I,\tau,n_i} \left[\frac{(\hbar\omega_0)^{-1}\Delta E_{I\tau n_i}}{E^{\text{exp}}_{I\tau n_i}} \right]^2 \right\}^{-1}. \qquad (11)$$

Here, the summation is carried over all the basic states (indices $n_i \equiv \{n_\beta, n_\gamma, n_\xi\}$ characterize the types of energy bands by the directions of deformability of a nucleus).

The quantities included in relation (11), i.e. $\Delta E_{I\tau n_i}$, determine the excitation energy of a state $|I\tau n_i\rangle$ in the frame of the considered model as the difference between the energies of this state and the ground state of a nucleus $|I = 0, \tau = 1, n_i = 0\rangle$ (both these quantities are calculated according to relation (6)); $E^{\text{exp}}_{I\tau n_i}$ are experimental values of the excitation energies of the base levels.

At the comparison of the theoretical and experimental values of energy levels in the cases where spin and parity of the given state are not determined experimentally, an ambiguity in the interrelationship of the given level to a certain band arises. In this case, it is necessary to take into account the experimental data on the probabilities of electromagnetic transitions with participation of the researched state. The agreement of the model calculations with the experimental characteristics of the lowest excited states with spin $I = 2$ is of great importance, because the cross-sections of nucleon-nucleus collisions with their excitation are most precisely measured and we have a rather extensive information about their properties.

3. Identification of the States of a Nucleus

Within the framework of various versions of collective models, we calculated the energy spectra of the low-excited states of even nuclei $^{72-82}$Se. The analysis was based on the identification of the states of a nucleus by the collections of values of the quantum numbers n_β, n_γ, n_ξ, τ, which are given below:

$|g\rangle \equiv (n_\beta = n_\gamma = n_\xi = 0, \tau = 1)$ — the first rotational band;
$|\beta\rangle \equiv (n_\beta = 1, n_\gamma = n_\xi = 0, \tau = 1)$ — the first β-vibrational band;
$|\gamma\rangle \equiv (n_\beta = n_\gamma = n_\xi = 0, \tau = 1$ or $2)$ — the first anomalous rotational band;
$|\varepsilon\rangle \equiv (n_\beta = 0, n_\gamma = 1, n_\xi = 0, \tau = 0)$ — the first γ-vibrational band;
$|\xi\rangle \equiv (n_\beta = n_\gamma = 0; n_\xi = 0, 1; \tau = 1)$ — the octupole bands;
$|\delta\rangle \equiv (n_\beta = 2, n_\gamma = n_\xi = 0, \tau = 1)$ — the second β-vibrational band.

On the basis of this list of energy bands, their belonging to a certain band for each level of a nucleus is specified in Tables 1—5 within the framework of the considered models. This concerns only those experimental states that presumably have the collective nature and may be described in the framework of A.Bohr's Hamiltonian. As initial parameters in the M$\beta\gamma$D, the optimal parameters of DChM are taken. In such a way on the basis of M$\beta\gamma$D, the states of even parity were described with taking into account only the quadrupole deformation (λ=2). If the octupole deformation (λ=3) is also taken into account, the lowest states with negative parity can be described. However, the calculated characteristics of a certain level may be different depending on the used version of a model. It will be mentioned below in the discussion of the experimental information in view of the performed model calculations.

4. Discussion

At the correct choice of the initial values of parameters of a model, their optimal theoretical values for DChM and M$\beta\gamma$D appear close to each other. Otherwise (see Table 3 in [6] on the structure of a nucleus ^{76}Se), essential divergences in the values of $\hbar\omega_0$, μ_β, and μ_γ may arise between these models. The reason for this may be not only the incorrectness of a choice of the initial values of parameters of a model, but also the model distinctions in the account of a transversal deformability of the surface of a nucleus (see [7] and [11]). Moreover, the value $\mu_\beta > 1$ obtained in [11] is an evidence for the inapplicability of such an approach to the given nucleus.

The choice of the system of basic levels of a nucleus for the definition of values of the parameters of a collective Hamiltonian plays an essential role even within the framework of the same model. If, among the lowest excited states of a nucleus, several levels with spins $I^\pi = 2^+$ are observed, their identification by energy bands becomes complicated. In this case, for the definition of

T a b l e 1. Characteristics of excited states of a nucleus ^{72}Se

$N_{\rm lev}$	Experiment [12]		Calculation by the model								
			DChM				MβγD (with account of $\lambda = 3$)				
	$E_{\rm lev}^{\rm exp}$, MeV	$I_{\rm lev}^{\pi}$	$E_{\rm lev}^{\rm theor}$, MeV	$I_{\rm lev}^{\pi}$	τ	n_β	$E_{\rm lev}^{\rm theor}$, MeV	$I_{\rm lev}^{\pi}$	τ	n_β	n_γ
1	0.0	0^+	0.0	0^+	1	0	0.0	0^+	1	0	0
2	0.86208 *	2^+	0.8377	2^+	1	0	0.8265	2^+	1	0	0
3	0.93722	0^+					0.9372	0^+	1	0	1
4	1.31668 *	2^+	1.3599	2^+	2	0	1.3485	2^+	2	0	0
5	1.63686 *	4^+	1.6511	4^+	1	0	1.6360	4^+	1	0	0
6	1.87620	(2, 4)	1.7854	3^+	1	0	1.7732	3^+	1	0	0
7	2.40573	3^-									
8	2.43376	3^-					2.4156	3^-	1	0	0
9	2.46677 *	6^+	2.4862	6^+	1	0	2.4704	6^+	1	0	0
10	2.58642	(3)	2.5492	2^+	1	1	2.5261	2^+	1	1	0
11	2.843	5^-									
12	2.965						2.9909	5^-	1	0	0
13	3.4248 *	8^+	3.3335	8^+	1	0	3.3216	8^+	1	0	0
14	4.5043	10^+	4.1991	10^+	1	0	4.1943	10^+	1	0	0
15	5.7097	12^+	5.0734	12^+	1	0	5.0741	12^+	1	0	0
16	7.0381	14^+	5.9582	14^+	1	0	5.9688	14^+	1	0	0
Parameters	$\hbar\omega_0$, MeV		1.0663				1.0663				
	μ_β		0.9852				0.9852				
	γ_0, rad/deg.		0.4341/24.9°				0.4341/24.9°				
	β_0		—				0.236				
	$\mu_\gamma^{\rm DChM} = \beta_0 \cdot \mu_\gamma^{\rm M\beta\gamma D}$		—				0.2098				

N o t e. Here and further, the asterisk denotes the levels, on which parameters of the collective model were determined.

parameters of a model, the state with spin $I^\pi = 3^+$ of the γ-vibrational band is chosen as one of the basic levels, because this state is understood unambiguous by the model interpretation. After the description of the levels of the basic rotational band, we identify the other states (including those with spin 2^+). In such an approach, the advantage of MβγD is revealed in comparison with the technique in [11].

Specific results of the analysis of each nucleus for even isotopes of selenium (the structure of a nucleus ^{76}Se is considered separately in [6]) are submitted below.

4.1. ^{72}Se

This nucleus raised more questions than answers have been given as to the applicability of collective models to the description of its energy band structure (see Table 1). Besides the ground state with spin 0^+, there is only one state $N_{\rm lev} = 3$ with spin 0^+ among the lowest excited states (levels of nuclei in Tables 1—5 are numerated by numbers of the first column, $N_{\rm lev}$) which is considered within the framework of MβγD as the beginning of the band of quadrupole *transverse* vibrations (without presence of the other states of this band). Because of the absence of the state on which the band of quadrupole *longitudinal* vibrations could be built, a value of the parameter μ_β cannot be unambiguously established. First, the use of $N_{\rm lev} = 10$ as the basic state leads to the unjustified overestimate of μ_β. Secondly, the model calculations give $I^\pi = 2^+$ for this state. The experimental value of the spin in the state $N_{\rm lev} = 6$ is predicted as possible of the values (2, 4), however the calculations within the frameworks of both DChM and MβγD determine it as $I^\pi = 3^+$.

As a basic system, we choose those experimental levels (in Tables 1—5, they are marked by the asterisk) which are identified by spin I and τ, by numbers of the bands $n_\beta$in DChM, and by the additional number of the band n_γ in the case of MβγD. On the basis of the system of basic levels, the parameters of models as well as the numerical values of energies of the excited states with their identification by bands were obtained for the nucleus ^{72}Se.

The account of symmetric octupole deformations has allowed us to describe the lowest experimental levels with negative parity ($N_{\rm lev} = 8$ and 12), however there is a difficulty in the identification of the experimental states $N_{\rm lev} = 7$ or 8, as well as 11 or 12.

The parameter of transversal softness μ_γ in [7] contains the value of full deformation β_0 and thus differs from the similar parameter $\mu_\gamma^{\rm DChM}$ in [5], i.e. $\mu_\gamma^{\rm DChM} = \beta_0\mu_\gamma^{\rm M\beta\gamma D}$. The parameter $\mu_\gamma^{\rm DChM}$ in MβγD was defined for ^{72}Se by the energy of the level $N_{\rm lev} = 3$, representing the beginning of the first γ-vibration band which

T a b l e 2. Characteristics of excited states of a nucleus ^{74}Se

N_{lev}	Experiment [13, 14]		Calculation by the model									
			DChM				MβγD					
	$E_{\text{lev}}^{\text{exp}}$, MeV	I_{lev}^{π}	$E_{\text{lev}}^{\text{theor}}$, MeV	I_{lev}^{π}	τ	n_β	$E_{\text{lev}}^{\text{theor}}$, MeV	I_{lev}^{π}	τ	n_β	n_γ	n_ξ
1	0.0	0^+	0.0	0^+	1	0	0.0	0^+	1	0	0	0
2	0.635	2^+	0.622*	2^+	1	0	0.598 *	2^+	1	0	0	0
3	0.854	0^+					0.8538	0^+	1	0	1	0
4	1.269	2^+	1.258*	2^+	2	0	1.243	2^+	2	0	0	0
5	1.363	4^+	1.417*	4^+	1	0	1.397 *	4^+	1	0	0	0
6	1.6574	0^+	1.581	0^+	1	0	1.630 *	0^+	1	1	0	0
7	1.838	2^+										
8	1.884	3^+	1.629	3^+	1	0	1.613	3^+	1	0	0	0
9	2.108	4^+	2.299	4^+	2	0	2.279	4^+	2	0	0	0
10	2.2314	6^+	2.251*	6^+	1	0	2.246 *	6^+	1	0	0	0
11	2.314	(2^+)	2.346	2^+	1	1	2.356	2^+	1	1	0	0
12	2.349	3^-					2.166	3^-	1	0	0	1
13	2.379	$(1, 2^+)$					2.356	2^+	1	1	0	0
14	2.661	5^+	2.560	5^+	1	0	2.552	5^+	1	0	0	0
15	2.842	5^-					2.787	5^-	1	0	0	1
16	3.198	8^+	3.130*	8^+	1	0	3.122	8^+	1	0	0	0
17	3.253	$(2\div6)$					3.272	4^+	1	1	0	0
18	3.516	7^-					3.512	7^-	1	0	0	1
19	3.525	7^+	3.552	7^+	1	0	3.565	7^+	1	0	0	0
20	3.980	(6^+)					4.222	6^+	1	1	0	0
21	4.256	(10^+)	4.040	10^+	1	0	4.059	10^+	1	0	0	0
22	4.403	(9^-)					4.253	9^-	1	0	0	1
23	4.449	9^+	4.558	9^+	1	0	4.599	9^+	1	0	0	0
24	5.492	11^+					5.107	11^-	1	0	0	1
25	5.443	12^+	4.9807	12^+	1	0	5.023	12^+	1	0	0	0
Parameters	$\hbar\omega_0$, MeV		1.3592				1.4301					
	μ_β		0.5756				0.5499					
	γ_0, rad/deg.		0.4341/24.9°				0.4341/24.9°					
	β_0		—				0.236					
	$\mu_\gamma^{\text{DChM}} = \beta_0\mu_\gamma^{\text{M}\beta\gamma\text{D}}$		—				0.2606					

initiated by transverse deformations of the surface of a nucleus.

4.2. ^{74}Se

The energy band structure of a nucleus ^{74}Se (see Table 2) both in theoretical and in experimental aspects still isn't studied enough, especially for high-spin states. Between papers [13] and [14], some information divergences are observed. In particular, in [14], not only quantum mechanical characteristics of this nucleus are specified, but also the unambiguous conclusion about the deformed shape of the nucleus ^{74}Se with its considerable softness along the three spatial axes was made on the basis of Hartree—Fock—Bogolyubov approach. Our calculations of the energy spectrum of this nucleus within the framework of DChM and M$\beta\gamma$D models also confirm a considerable multidirected deformation (of longitudinal and transversal softness) in the quadrupole approximation with the inclusion of deformations of higher multipolities.

The use of M$\beta\gamma$D has allowed us to describe the band of levels with negative parity (N_{lev}= 12, 15, 18, 22, and 24) on the basis of the use of representations of octupole deformation of a nucleus ($\lambda = 3$) with the quantum number n_ξ [11], as well as to specify values of spins and parities of the excited levels N_{lev} = 11, 13, 17, 20—22, and 24. The choice of various sets of basic levels has an influence not only on the numerical values of parameters of the model, but also on the quality of the agreement of the theoretical values of excitation energies of states with the experimentally measured ones.

4.3. ^{78}Se

In Table 3, the description of energy bands of ^{78}Se is presented on the basis of collective Hamiltonian with various collections of the basic levels (DChM$^{(1)}$ and DChM$^{(2)}$), as well as within the framework of M$\beta\gamma$D. For the estimation of the quality of model calculations of the energy structure of nuclei, experimental data on the corresponding levels of excitation of a nucleus [15]

T a b l e 3. Characteristics of excited states of a nucleus ^{78}Se

$N_{\rm lev}$	Experiment [15]		Calculation by the model													
			DChM$^{(1)}$ ($\lambda = 2$)				DChM$^{(2)}$ ($\lambda = 2$)				M$\beta\gamma$D ($\lambda = 2, 3$)				
	$E_{\rm lev}^{\rm exp}$, MeV	$I_{\rm lev}^{\pi}$	$E_{\rm lev}^{\rm theor}$, MeV	$I_{\rm lev}^{\pi}$	τ	n_β	$E_{\rm lev}^{\rm theor}$, MeV	$I_{\rm lev}^{\pi}$	τ	n_β	$E_{\rm lev}^{\rm theor}$, MeV	$I_{\rm lev}^{\pi}$	τ	n_β	n_γ
1	0.0	0^+	0.0	0^+	1	0	0.0	0^+	1	0	0.0	0^+	1	0	0
2	0.614	2^+	0.614 *	2^+	1	0	0.609 *	2^+	1	0	0.601	2^+	1	0	0
3	1.309	2^+	1.309 *	2^+	2	0	1.303 *	2^+	2	0	1.289	2^+	2	0	0
4	1.499	(0^+)									1.498	0^+	1	0	1
5	1.503	(4^+)	1.507 *	4^+	1	0	1.516 *	4^+	1	0	1.497	4^+	1	0	0
6	1.7587	$(0^+,1,2)$	1.7478	3^+	1	0	1.7570	3^+	1	0	1.7385	3^+	1	0	0
7	1.996	(2^+)									1.932	2^+	1	0	1
8	2.190		2.1481	0^+	1	1									
9	2.335	$(0^+,1,2)$					2.328 *	0^+	1	1	2.329	0^+	1	1	0
10	2.508	3^-									2.508	3^-	1	0	0
11	2.537	$(0^+,1,2)$	2.5748	4^+	2	0									
12	2.539	(6^+)	2.508 *	6^+	1	0	2.255 *	6^+	1	0	2.521	6^+	1	0	0
13	2.686	$(0^+,1,2)$					2.627	4^+	2	0	2.650	4^+	1	0	1
14	2.680	$(1,2)$									2.605	4^+	2	0	0
15	2.890		2.8891	5^+	1	0									
16	2.914										2.920	5^+	1	0	0
17	2.948						2.9475	5^+	1	0					
18	3.090						3.1033	2^+	1	1	3.088	2^+	1	1	0
19	3.255										3.272	5^-	1	0	0
20	3.574	(8^+)	3.605 *	8^+	1	0	3.6973	8^+	1	0	3.654	8^+	1	0	0
Parameters	$\hbar\omega_0$, MeV		2.0581				2.2631				2.2631				
	μ_β		0.4368				0.4125				0.4125				
	γ_0, rad/deg.		0.4383/25.1°				0.4415/25.3°				0.4415/25.3°				
	β_0		0.236				0.236				0.236				
	$\mu_\gamma^{\rm DChM} = \beta_0 \mu_\gamma^{\rm M\beta\gamma D}$		—				—				0.1966				

are also given in this table. Calculations in the quadrupole approximation with regard for octupole vibrations of a surface (by the technique in [11]) allow carrying out the identification of low-excited states with positive and negative parities more reliably.

Having chosen the system of levels $N_{\rm lev}$=2, 3, 5, 12, and 20 (the case of DChM$^{(1)}$) as basic, we determined the parameters of the model with the account of the softness of a longitudinal deformation only and carried out the identification of the states of a nucleus ^{78}Se. In particular, the experimental level $N_{\rm lev}$= 8 is identified as the basis of the band of longitudinal quadrupole vibrations. Spins 3^+, 4^+ and $5^+$are accordingly attributed to the experimental levels $N_{\rm lev} = 6$, 11 (or 13), 15 (or 17); they are considered as fragments of the abnormal rotational band.

When the levels $N_{\rm lev} = 2$, 3, 5, 9, 12 (DChM$^{(2)}$) were used, we got different values of the parameters as a result. Thus, the choice of the basic system of levels plays an essential role not only in the determination of the parameters of a model, but also in the identification of states. This becomes evident from the comparison of the results of DChM$^{(1)}$ and DChM$^{(2)}$ (see Table 3).

The calculation of the band structure of a nucleus ^{78}Se within the frameworks of M$\beta\gamma$D is based on the use of parameters of DChM$^{(2)}$ with the subsequent choice of the parameter of transversal quadrupole softness $\mu_\gamma^{\rm DChM}$ [5]. At the same time, the experimental level $N_{\rm lev} = 4$ is supposed to be the beginning of the energy band of transversal quadrupole vibrations, into which the levels $N_{\rm lev} = 4$, 7, 13 (or 14) enter. States with negative parity (i.e., $N_{\rm lev} = 10$, 19) are also described within the framework of M$\beta\gamma$D at values of the quantum number $n_\xi = 0$ or 1 for octupole longitudinal vibrations of the surface.

4.4. ^{80}Se

A nucleus ^{80}Se differs from the nuclei considered above by the absence of the experimental data on the high-spin states in its energy spectrum [16, 17] (see Table 4). This complicates the construction of a basic rotational band for it and, therefore, the determination of parameters in the framework of DChM or M$\beta\gamma$D.

When experimental levels $N_{\rm lev} = 2$, 3, and 5, were used as basic, we got the parameters $\hbar\omega_0 = 2.9160$ MeV, $\mu_\beta = 0.3791$, $\gamma_0 = 0.4428$ on the basis of DChM. For the states with spins, $I > 4$ we were unable to orient

T a b l e 4. Characteristics of excited states of a nucleus ^{80}Se

N_{lev}	Experiment [16, 17]		Calculation by the model															
			DChM$^{(1)}$ ($\lambda = 2$)				MβγD ($\lambda = 2$)					MβγD ($\lambda = 2, 3$)					
	E_{lev}^{exp}, MeV	I_{lev}^{π}	E_{lev}^{theor}, MeV	I_{lev}^{π}	τ	n_β	E_{lev}^{theor}, MeV	I_{lev}^{π}	τ	n_β	n_γ	E_{lev}^{theor}, MeV	I_{lev}^{π}	τ	n_β	n_γ	n_ξ
1	0.0	0^+	0.0	0^+	1	0	0.0	0^+	1	0	0	0.0	0^+	1	0	0	0
2	0.666118 *	2^+	0.66180	2^+	1	0	0.66610	2^+	1	0	0	0.65628	2^+	1	0	0	0
3	1.44922	2^+	1.5472	2^+	2	0	1.5472	2^+	2	0	0	1.5256	2^+	2	0	0	0
4	1.4790	0^+					1.4793	0^+	1	0	1	1.4792	2^+	1	0	1	0
5	1.70122 *	4^+	1.7015	4^+	1	0	1.7015	4^+	1	0	0	1.6775	4^+	1	0	0	0
6	1.96005	2^+					1.9749	2^+	1	0	1	1.9680	2^+	1	0	1	0
7	2.1210	(≤ 4)	2.0375	3^+	1	0	2.0375	3^+	1	0	0	2.0116	3^+	1	0	0	0
8	2.6271 ∗	(0,1,2)	2.6272	0^+	1	1	2.6272	0^+	1	1	0	2.6272	0^+	1	1	0	0
9	2.7166	3^-										2.73323	3^-	1	0	0	1
10	2.8253	$(2 \div 6)$					2.8249	4^+	1	0	1	2.8058	4^+	1	0	1	0
11	2.9475	(≤ 4)	2.9197	4^+	2	0	2.9197	4^+	2	0	0	2.8906	4^+	2	0	0	0
12	3.314		3.3341	5^+	1	0	3.3341	5^+	1	0	0	3.2981	5^+	1	0	0	0
13	3.491		3.4761	2^+	1	1	3.4761	2^+	1	1	0	3.4570	2^+	1	1	0	0
14	3.567											3.6067	5^-	1	0	0	1
Parameters	$\hbar\omega_0$, MeV		2.5564				2.5564					2.5564					
	μ_β		0.4108				0.4108					0.4108					
	γ_0, rad/deg.		0.4189/24°				0.4189/24°					0.4189/24°					
	β_0		0.236				0.236					0.236					
	$\mu_\gamma^{DChM} = \beta_0 \mu_\gamma^{M\beta\gamma D}$		—				0.2466					0.2466					

T a b l e 5. Characteristics of excited states of a nucleus ^{82}Se

N_{lev}	Experiment [18]		Calculation by the model								
			DChM ($\lambda = 2$)				MβγD ($\lambda = 2$)				
	E_{lev}^{exp}, MeV	I_{lev}^{π}	E_{lev}^{theor}, MeV	I_{lev}^{π}	τ	n_β	E_{lev}^{theor}, MeV	I_{lev}^{π}	τ	n_β	n_γ
1	2	3	4	5	6	7	8	9	10	11	12
1	0.0	0^+	0.0	0^+	1	0	0.0	0^+	1	0	0
2	0.6547 *	2^+	0.6547	2^+	1	0	0.6392	2^+	1	0	0
3	1.4099	0^+					1.4103	0^+	1	0	1
4	1.73113	2^+	1.5373	2^+	2	0	1.6053	2^+	2	0	0
5	1.7350 *	4^+	1.7350	4^+	1	0	1.7506	4^+	1	0	0
6	2.55009	$(2,4^+)$	2.0745	3^+	1	0	2.1431	3^+	1	0	0
7	3.105	(4^+)	3.0857	4^+	2	0	3.1671	4^+	2	0	0
8	3.449 *	0^+	3.4490	0^+	1	1	3.5094	0^+	1	1	0
9	3.624		3.5349	5^+	1	0	3.6417	5^+	1	0	0
10	4.134	2^+					4.1465	2^+	1	1	0
11	4.396	2^+	4.2747	2^+	1	1					
Parameters	$\hbar\omega_0$, MeV		3.4306				3.4327				
	μ_β		0.3498				0.4005				
	γ_0, rad/deg.		0.4262/24.42°				0.4376/25.07°				
	β_0		0.236				0.236				
	$\mu_\gamma^{DChM} = \beta_0 \mu_\gamma^{M\beta\gamma D}$		—				0.2446				

ourselves on the correspondence between the model values of energies of the states of a nucleus of the basic rotational band and experimental ones (because of their absence), and therefore experimental states $N_{lev.} = 2$, 5 and 8 were used as the basic ones. As a result, theoretical parameters in the case of DChM were obtained, which (without a variation) were applied then in the calculations for MβγD ($\lambda = 2$) and MβγD ($\lambda = 2, 3$).

Experimental states $N_{lev} = 7$, 11 and 12 are identified as levels of the abnormal rotational band with $n_\beta = 0$, but the experimental level $N_{lev} = 13$, together with $N_{lev.} = 8$, forms the second band ($n_\beta = 1$) of quadrupole longitudinal vibrations within the framework of DChM and MβγD. As to the level $N_{lev} = 10$ ($E_{lev.}^{exp} = 2.8253$ MeV ($I^\pi = 2 \div 6$)), it is identified as belonging to the band of transversal quadrupole vibrations with spin $I^\pi = 4^+$ at $n_\gamma = 1$.

The analysis of energy bands of a nucleus ^{80}Se within the frameworks of DChM or M$\beta\gamma$D indicates its longitudinal and transversal deformabilities. Having taken parameters of DChM as a basis, and by considering the experimental level $N_{\rm lev} = 4$ as the beginning of the band of transversal quadrupole deformations, we determined the transversal softness of a nucleus, μ_γ, which provides the description of states $N_{\rm lev} = 9$ and 14 with negative parity within the framework of M$\beta\gamma$D. In this case, there was enough to take into account only the symmetric octupole vibrations with numbers $n_\xi = 0$ and 1.

4.5. ^{82}Se

The energy spectrum of a nucleus ^{82}Se (Table 5) is similar to that of a nucleus ^{76}Se [6] in its bottom part by the sequence of arrangements of spins. However, there are no states with spins $I > 4$ among the experimentally measured levels. This complicates the model construction of energy bands with sufficient reliability on the basis of collective models. Experimental and model characteristics of the nucleus ^{82}Se are given in Table 5 with the indication of calculated model parameters.

At the analysis of a spectrum of levels of ^{82}Se, the ambiguities about their belonging to the certain band arise concerning several states. Their solution is possible only in a joint consideration of the whole spectrum. So, as a basis of the band of quadrupole longitudinal vibrations, the level $N_{\rm lev} = 8$ with spin 0^+ is chosen, but spin and parity $I^\pi = 5^+$ were attributed to the level $N_{\rm lev} = 9$, which was identified as that belonging to the abnormal rotational band both within the framework of DChM and M$\beta\gamma$D.

The level $N_{\rm lev} = 3$ is considered by us as the beginning of the band of transversal quadrupole vibrations, and the parameter of transversal softness of a nucleus $\mu_\gamma^{\rm DChM}$ was determined by it. The order of the arrangement of the bases of *longitudinal* and *transversal* quadrupole vibrations of a nucleus ^{82}Se turns out to be the same as that in a nucleus ^{76}Se, i.e. the beginning of the band of transversal vibrations ($N_{\rm lev} = 3$) is located lower than the bottom of the band of longitudinal vibrations ($N_{\rm lev} = 8$).

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ІНТЕРПРЕТАЦІЯ ЯДЕРНИХ СТАНІВ ПАРНИХ ІЗОТОПІВ СЕЛЕНУ В ТЕРМІНАХ ФЕНОМЕНОЛОГІЧНОЇ КОЛЕКТИВНОЇ МОДЕЛІ

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Р е з ю м е

Проведено розрахунки енергетичних спектрів парних ядер $^{72-82}$Se з використанням різних версій колективної моделі. На основі отриманих даних розглянуто питання класифікації збуджених станів ядра за полосами рівнів і показано, что віднесення певного рівня ядра до тої чи іншої полоси в багатьох випадках залежить від використаної моделі.