EFFECT OF ELASTO-OPTIC CONTRIBUTION ON SELF-BENDING OF SPECKLED LIGHT BEAM IN $BaTiO_3$[1]

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We consider an influence of the additional elasto-optic contribution to the dielectric permittivity change on the self-bending effect for a speckled extraordinary light beam in photorefractive $BaTiO_3$. The real trajectory of the speckled beam is calculated.

Introduction

Self-bending of a laser beam in photorefractive crystals [1—9] occurs in the presence of a gradient-type component of the response [6—9]. This phenomenon consists in the propagation of a light beam along a curved path inside the crystal. In a crystal of the $BaTiO_3$ type with the strong diffusion mechanism of the photorefractive nonlinearity, a radius of the trajectory curvature of several centimeters is typically observed with a beam diameter of $\sim$ 1mm. An asymmetric beam fanning accompanies the light propagation in this situation. It was found [10] that a reduction of the spot size of the beam weakens the fanning. For the narrow Gaussian beam with the waist of 15 μm at the entrance face of a $BaTiO_3$ crystal, Segev et al. [10] have only observed a wave front tilting of about 0.2° with a small change of the beam shape after the propagation distance of $\sim$ 5 mm.

The various theoretical models have been applied to discussions of these phenomena. In 1982, Feinberg [4] explained the asymmetric beam fanning by using the diffraction integral for a TEM_{00} Gaussian laser beam which itself deflects by creation of a transverse gradient of the refractive index. The evolution problem of one-dimensional Gaussian beams propagating in photorefractive crystals was considered in the paraxial approximation [5, 8]. Any noise fanning effects have been neglected in these works. On addition of the noise to the system, it is possible to describe the fanning effect both for linear [10, 11] and nonlinear models [12] of material response in the case of propagation a wide-size beam. In 1994, Lyubomudrov and Shkunov [6] introduced a theoretical model of self-bending with the assumption of a speckled structure for an input beam. This model describes the beam with the conserving Gaussian shape of the angular spectrum. The photorefractive interaction between angular components with random phases causes energy redistribution and the bending of the beam trajectory in a crystal. In such an approach, the trajectories of the whole beam with a strong curvature (without the frame of paraxial approximation) can be considered.

The strong anisotropy of a photorefractive response in $BaTiO_3$ influences both the quantitative parameters and the qualitative peculiarities of the self-bending phenomenon [4—8, 10, 11]. However, the simple electro-optic mechanism of photorefraction has been taken into account [4—8] and [10—12] in the analysis of this effect. It is believed that the spatial variations in dielectric permittivity are primarily due to the electro-optic contribution of the space-charge field which is formed in a crystal.

Nevertheless, the additional elasto-optic contribution to the dielectric permittivity changes of the photorefractive grating was introduced into account [13, 14]. More recently Gunter and Zgonik [15] considered the roto-optic contribution that is evident in the crystal with strong linear birefringence only. Both the elasto-optic and roto-optic contributions are associated with the inhomogeneous elastic field induced by the piezoelectric effect in a non-centrosymmetric crystal during photorefractive grating formation. In addition, a nonuniform piezoelectric polarization of the medium must be included into consideration of the effective static dielectric constant [14—16] that influences the condition of trap saturation. The influence of the elasto-optic effect on Bragg diffraction, two-wave mixing, and

[1]This article is dedicated to Professor Marat Soskin on the occasion of his 75th birthday.

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beam fanning has been experimentally verified in various photorefractive crystals such as $LiNbO_3$ [13], $Bi_{12}GeO_{20}$ [17], $Bi_{12}SiO_{20}$ [18—20], $Bi_{12}TiO_{20}$ [14, 21—24], GaAs [25], $KNbO_3$ [26], and $BaTiO_3$ [26—28].

Following the simple model of Lyubomudrov and Shkunov [6], we investigate the trajectories for a self-bent extraordinary speckle-beam in $BaTiO_3$, considering the elasto-optic contribution to the changes of dielectric permittivity for the photorefractive response. We neglect the roto-optic contribution because of the weak linear birefringence for $BaTiO_3$.

Basic Equations

Fig. 1 shows the curved path geometry for pump beam propagation in $BaTiO_3$, where α_0 and α are the entrance and local angles of the beam inside the crystal. In order to extend the configuration from [6] and [7], we consider an arbitrary orientation of the optical axis **C** in the (010) crystallographic plane. In the diffusion regime without trap saturation, the trajectory of the whole pump beam can be calculated from the following set of equations [6]:

$$\frac{d\alpha}{dx} = \frac{1}{\cos\alpha}\frac{d\alpha}{d\varsigma} = k\Delta\theta_f^2\frac{\gamma(\beta)}{\cos\alpha}, \qquad (1)$$

$$z(x) = \int_0^x \mathrm{tg}\alpha(x_1)dx_1, \qquad (2)$$

where $k = 2\pi/\lambda$ is the magnitude of the light wave vector in vacuum, $\Delta\theta_f$ is the angular half-width divergence of the speckled beam, λ is the wavelength, $\beta = \alpha + \theta_0$, and θ_0 is the angle between the axes **C** and **z** (Fig. 1). In a general way, the nonlinearity coefficient $\gamma(\beta)$ for a beam with the extraordinary polarization can be represented as

$$\gamma(\beta) = -\frac{1}{2}k\left(\frac{k_{\rm B}T}{e}\right)\left(\Delta\varepsilon_{11}(\beta)\sin^2\beta + \right.$$

$$\left. +\Delta\varepsilon_{33}(\beta)\cos^2\beta + 2\Delta\varepsilon_{13}(\beta)\sin\beta\cos\beta\right)\frac{1}{\varepsilon_o}, \qquad (3)$$

where $k_{\rm B}T$ is the thermal energy, e is the charge of the mobile charge carriers, and ε_0 is the vacuum permittivity. The change in the dielectric permittivity tensor, $\Delta\varepsilon_{ij}$, for the photorefractive grating with the unit amplitude of space-charge field in a non-centrosymmetric crystal, can be written as the sum of the electro-optic and elasto-optic contributions [13, 14]:

$$\Delta\varepsilon_{ij} = -\frac{\varepsilon_{im}\varepsilon_{jn}}{\varepsilon_0}\left(r^S_{mnp}m_p + \right.$$

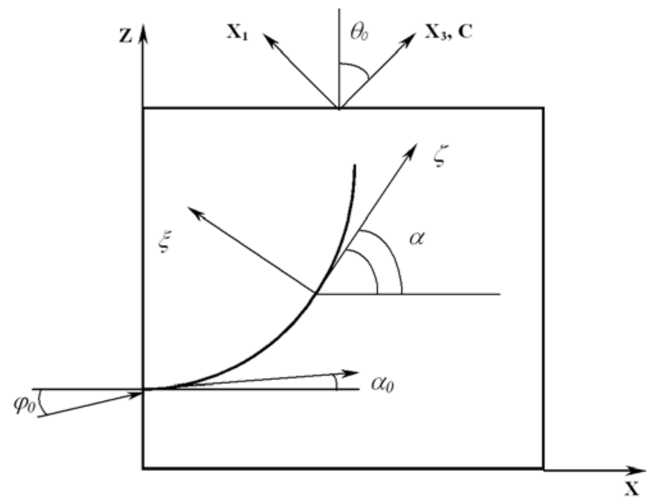

Fig. 1. Geometric configuration for the self-bending of a laser beam in the crystal with an arbitrary orientation of the optical axis **C** in the (010) crystallographic plane

$$\left. +p^E_{mnkl}m_lG_{ks}e_{psr}m_pm_r\right), \qquad (4)$$

where ε_{im} is the dielectric permittivity tensor at optical frequencies in the absence of space-charge field; r^S_{mnp} and p^E_{mnkl}, respectively, are the electro-optic tensor in a clamped crystal and the elasto-optic tensor for a constant electric field; G_{ks} are the components of the tensor $G = \Gamma^{-1}$ inverse to the Christoffel's tensor Γ with the components $\Gamma_{sk} = C^E_{sqkt}m_qm_t$; C^E_{sqkt} and e_{psr}, respectively, are the tensor of elastic constants for a constant electric field and the piezoelectric stress tensor; and m_p are the components of the unit vector $\boldsymbol{m}$ which is perpendicular to the local beam direction inside a crystal. Using the approach described in [16] and [29], we derive the following expressions for $\Delta\varepsilon_{11}$, $\Delta\varepsilon_{33}$ and $\Delta\varepsilon_{13}$ in $BaTiO_3$ crystal:

$$\Delta\varepsilon_{11} = n_0^4\left(r^S_{13}\cos\beta + \right.$$

$$\left. +\frac{p^E_{11}\left(\Gamma_{33}e'_1-\Gamma_{13}e'_3\right)\sin\beta+p^E_{13}\left(\Gamma_{11}e'_3-\Gamma_{13}e'_1\right)\cos\beta}{\Gamma_{11}\Gamma_{33}-\Gamma^2_{13}}\right),$$

$$\Delta\varepsilon_{33} = n_e^4\left(r^S_{33}\cos\beta + \right.$$

$$\left. +\frac{p^E_{31}\left(\Gamma_{33}e'_1-\Gamma_{13}e'_3\right)\sin\beta+p^E_{33}\left(\Gamma_{11}e'_3-\Gamma_{13}e'_1\right)\cos\beta}{\Gamma_{11}\Gamma_{33}-\Gamma^2_{13}}\right),$$

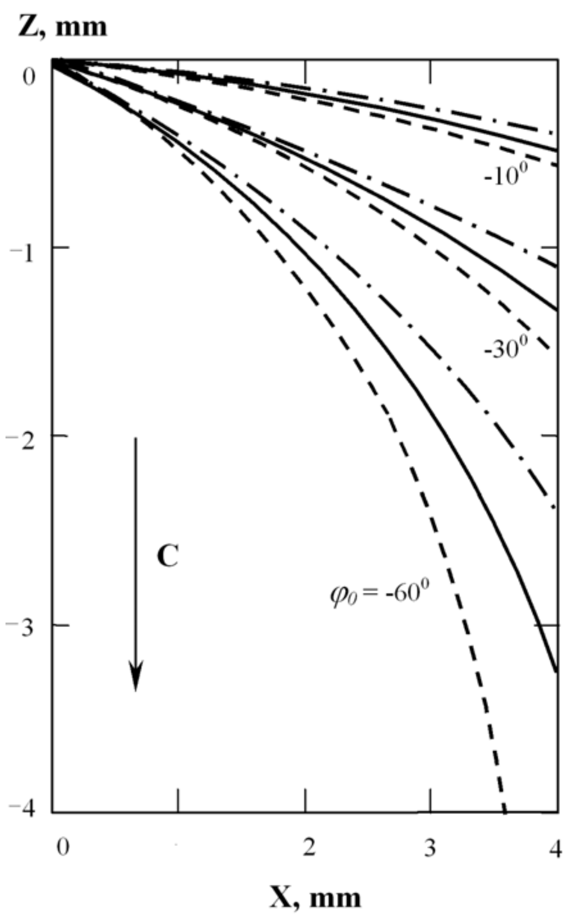

Fig. 2. Trajectories of the self-bent beam for various values of the incident angle φ_0 in the 0°-cut $BaTiO_3$ crystal at the angular half-width divergence of the speckled beam $\Delta\theta_f$=0.076 and for the angle between the axes **C** and **z** θ_0=180°. The solid curves correspond to the calculation with elasto-optic contribution. The dash-dotted and the dashed curves indicate results calculated without elasto-optic contribution (p^E_{mn}=0) by using electro-optic coefficients for clamped and unclamped crystals, respectively

$$\Delta\varepsilon_{13} = n_0^2 n_e^2 \left(r^S_{42} \sin\beta + \right.$$
$$\left. + \frac{p^E_{44}\left(\Gamma_{33}e'_1 - \Gamma_{13}e'_3\right)\cos\beta + p^E_{33}\left(\Gamma_{11}e'_3 - \Gamma_{13}e'_1\right)\sin\beta}{\Gamma_{11}\Gamma_{33} - \Gamma^2_{13}} \right), \tag{5}$$

where n_o and n_e are the ordinary and extraordinary indices of refraction. The relevant components of the Christoffel's tensor Γ_{sk} and some vector e'_i are given by [16]:

$$\Gamma_{11} = C^E_{11}\sin^2\beta + C^E_{44}\cos^2\beta,$$
$$\Gamma_{33} = C^E_{44}\sin^2\beta + C^E_{33}\cos^2\beta,$$
$$\Gamma_{13} = \left(C^E_{13} + C^E_{44}\right)\sin\beta\cos\beta, \tag{6}$$
$$e'_1 = (e_{15} + e_{31})\sin\beta\cos\beta,$$
$$e'_3 = e_{15}\sin^2\beta + e_{33}\cos^2\beta, \tag{7}$$

where r^S_{mn} are the clamped electro-optic constants, p^E_{mn} are the elasto-optic constants, C^E_{mn} are the elastic constants, and e_{mn} are the piezoelectric stress constants in the matrix contracted notations.

Results and Discussion

Fig. 2 shows the calculated curved trajectory followed by the speckled beam (λ=633 nm, $\Delta\theta_f$=0,076) for various values of the incident angle φ_0 specified at the entrance face outside the 0°-cut crystal. The solid curve in Fig. 2 corresponds to the calculation on the basis of exact equations (3) and (5)—(7). For comparison, we plot these curved trajectories without considering the additional elasto-optic contribution (p^E_{mn}=0) by using clamped (dash-dotted curve) and unclamped (dashed curve) electro-optic constants of $BaTiO_3$ crystal. We have used the following material parameters of $BaTiO_3$ crystal for λ= 633 nm from [30]: n_o = 2.412, n_e = 2.360, r^S_{13} = 10.2 pm/V, r^S_{33} = 40.6 pm/V, r^S_{42} = 730 pm/V, p^E_{11} = 0.50, p^E_{13} = 0.20, p^E_{31} = 0.07, p^E_{33} = 0.77, $e_{31} = -0.7$ C/m^2, e_{33} = 6.7 C/m^2, e_{15} = 34.2 C/m^2, $C^E_{11} = 2.22\cdot10^{11}$ N/m^2, $C^E_{13} = 1.11\cdot10^{11}$ N/m^2, $C^E_{33} = 1.51\cdot10^{11}$ N/m^2, $C^E_{44} = 0.61\cdot10^{11}$ N/m^2, r^T_{13} = 8 pm/V, r^T_{33} = 105 pm/V, and r^T_{42} = 1300 pm/V.

It may be seen from Fig. 2 that the real trajectory of the speckled beam lies between two curves that are commonly calculated on the base of the clamped or unclamped electro-optic coefficients. The form of a beam trajectory is dictated by the angular dependence of the nonlinear coefficient $\gamma(\beta)$ and the initial angle $\alpha_0 + \theta_0$ at the entrance face of a crystal.

In Fig. 3, we show the dependence of the real nonlinear coefficient with the elasto-optic contribution γ on β and, as a comparison, the nonlinear coefficients γ^S and γ^T which are calculated for p^E_{mn} = 0 by using clamped and unclamped electro-optic constants. It is clear that $\gamma^T(\beta)$ and $\gamma^S(\beta)$, respectively, indicate the overestimated and underestimated magnitudes of the nonlinear response relative to $\gamma(\beta)$.

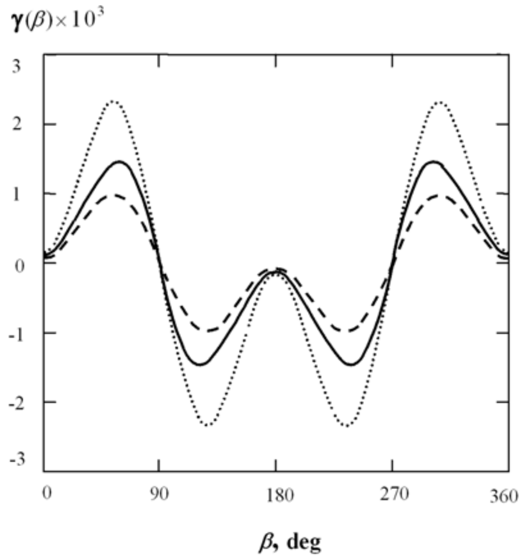

Fig. 3. Orientation dependence of the nonlinear coefficient $\gamma(\beta)$ in $BaTiO_3$ with elasto-optic contribution (solid curve) and without one for the calculation by clamped (dashed curve) and unclamped (dotted curve) electro-optic coefficients

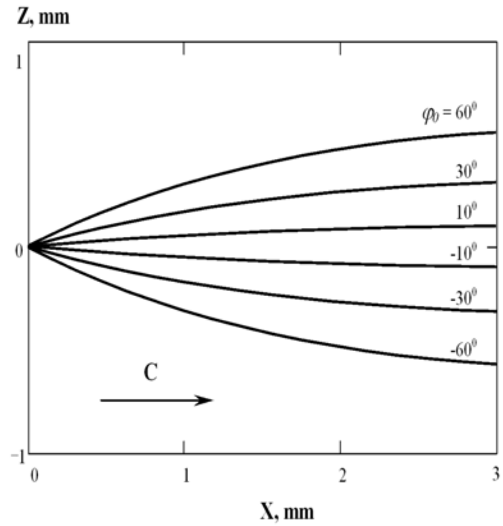

Fig. 5. Trajectories of the self-bent beam for various values of the incident angle φ_0 in $BaTiO_3$ crystal at $\Delta\theta_f$=0.076 and θ_0=90°. The curves calculated with elasto-optic contribution

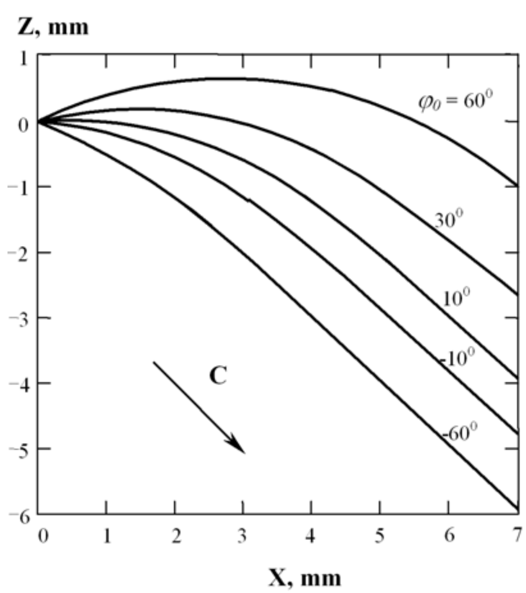

Fig. 4. Trajectories of the self-bent beam for various values of the incident angle φ_0 in the 45°-cut $BaTiO_3$ crystal at $\Delta\theta_f$=0.076 and θ_0=135°. The curves calculated with elasto-optic contribution

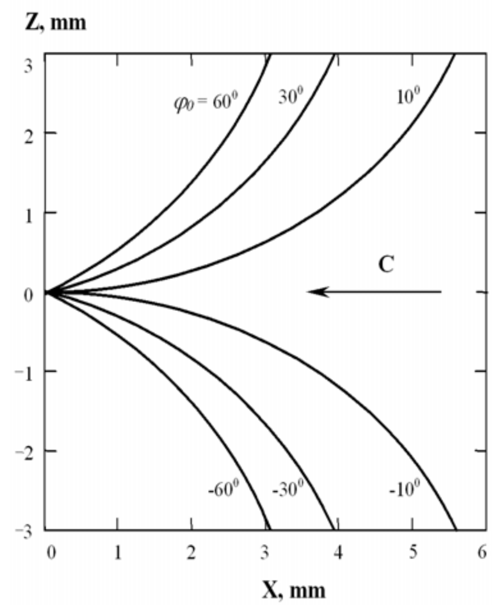

Fig. 6. Same as in Fig. 5, but for $\theta_0 = -90°$

Fig. 4 shows the trajectory of the self-curving beam in the 45°-cut crystal. In that case, a curvature of an initial part of the beam trajectories is far in excess of one for the corresponding incident angles in the 0°-cut sample (Fig. 2). This distinction is due to the dissimilar initial values of nonlinear coefficient $\gamma(\alpha_0 + \theta_0)$ (see Figs. 1 and 3). Note that calculated trajectory tends to be aligned along the direction of the optical axis C [6] in this approach.

The view of self-bent beams in a sample with the $(00\bar{1})$ input face is shown in Fig. 5. The negative or positive sign of the nonlinear coefficient $\gamma(90° + \theta_0)$ at $\alpha_0 > 0$ or $\alpha_0 < 0$ leads to the convex upwards or convex downwards trajectories of the beam.

The behavior of the beams in such a nonlinear medium closely resembles that in a positive gradient lens. However, the pattern of the beams in the crystal with (001) entrance face (Fig. 6) indicates that the properties of negative lens for a speckled beam is exhibited in that case.

Conclusion

In summary, we have considered the self-bending effect for a speckled extraordinary light beam in $BaTiO_3$ on the base of a model that takes into account the additional elasto-optic contribution. The analytical expressions for the nonlinearity coefficient have been derived. The substantial influence of the elasto-optic effect on the beam self-bending was demonstrated. It was shown that $BaTiO_3$ crystal exhibits the property of a positive or negative gradient lens depending on whether the specled laser beam propagates along the optical axis **C** or in the opposite direction.

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ЕФЕКТ ЕЛАСТООПТИЧНОГО ВНЕСКУ ПРИ САМООГИНАННІ СВІТЛОВОГО ПУЧКА ЗІ СПЕКЛ-СТРУКТУРОЮ У $BaTiO_3$

С.М. Шандаров, М.І. Бурімов, О.А. Кашин, В.В. Дацюк

Р е з ю м е

Розглянуто вплив додаткового еластооптичного внеску до зміни діелектричної проникності на ефект самоогинання для незвичайної світлової хвилі зі спекл-структурою у фоторефрактивному $BaTiO_3$. Розраховано реальну траєкторію хвилі зі спекл-структурою.