ISOTOPE SEPARATION IN A SYSTEM WITH CUSP-GEOMETRY MAGNETIC FIELD

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Motion of particles in a system with cusp-geometry magnetic field is studied. It is shown that, with a sufficiently high radial velocity component of a particle, the latter can be reflected, after passing the zero-magnetic-field plane, by the growing barrier field even if the conditions are met for passing particles with $\dot{r}_0 = 0$ through the barrier. In the central part of the system, the radial size of the particles' flux gets increased if their radial velocities differ from zero. An example of separation of the carbon isotopes ^{12}C and ^{13}C is adduced. Possibilities for isotopes' taking out are discussed.

Need in the separation of isotopes of different elements is urgent in a number of branches of science and technology. First of all, this relates to the branches of nuclear power engineering, ecology, and medicine. In this connection, the development of isotope separation techniques is under way. In the past decades, a number of so-called plasma techniques have been suggested and now are studied [1—4]. These studies are targeted at the development of technologies which allow extraction of highly pure isotopes in needed quantities with being more efficient than the existing ones. One of such techniques is the isotope separation in a system with magnetic field of the cusp geometry. This system is relatively simple as compared with some other ones. The method of ion-cyclotron resonance (ICR) [2] demands the presence, besides the magnetic field, of an RF energy source and multistage (to enhance the isotopes purification) plasma systems making the particles to move in the crossed E and H fields.

The isotope separation ability is connected with the peculiarity of particles' motion in the cusp geometry magnetic field, considered in work [5] in which a classification of the particles' trajectories is presented along with an experimental confirmation of efficient separation of isotopes ^{6}Li and ^{7}Li.

In a theoretical consideration of particles' motion in the cusp geometry magnetic field, it was supposed that the particles start their motion with a velocity vector directed along force lines of the magnetic field. The presence of transversal velocity components of the injected particles makes the picture more complicated. The particles' motion in a presence of non-zero transversal velocity components is considered in works [6, 7]. Our paper is a further development of these studies. As is done in work [7], we consider a system of two coils with oppositely directed currents forming the magnetic field with zero strength at the central point. The rhs-adjacent system of coils keeps the magnetic field strength to be equal to that at the rhs plug located in a definite section outside the coils. The scheme of the setup is presented in Fig. 1. The particles are injected from the region of maximum magnetic field. The particles' source is shifted to the lift by distance r_0 from the field axis.

For simplification, it is supposed that the magnetic field in the inter-coil region is

$$H_z = -H_0 \sin(kz),\ H_y = H_0 (kr/2) \cos(kz),$$

where $k = \pi/(2L)$ with L being a distance between the two plugs [8]. H_0 is a maximal strength of the magnetic field on the axis. The same field strength is kept at a certain section outside the coil.

The system of equations written in the cylindrical coordinates (r, φ, z) takes a form

$$\ddot{r} - r\dot{\varphi}^2 = \frac{e}{Mc} r\dot{\varphi} B_z, \quad \ddot{z} = -\frac{e}{Mc} r\dot{\varphi} B_r,$$

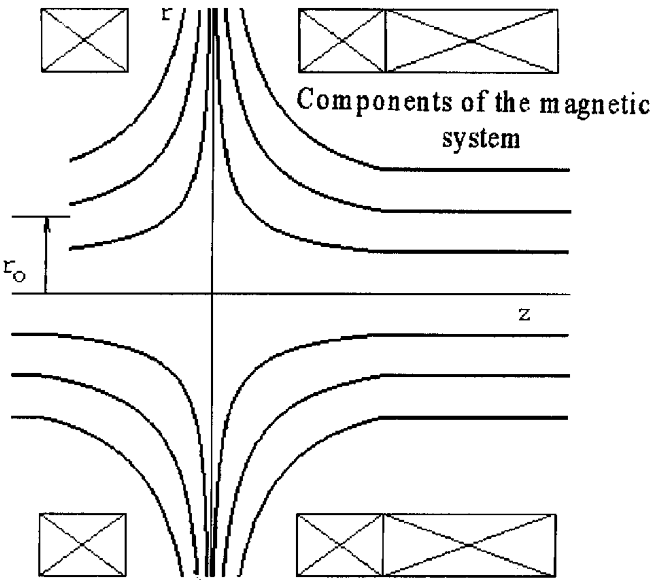

Fig.1. Schematic drawing of the isotopes' separation setup

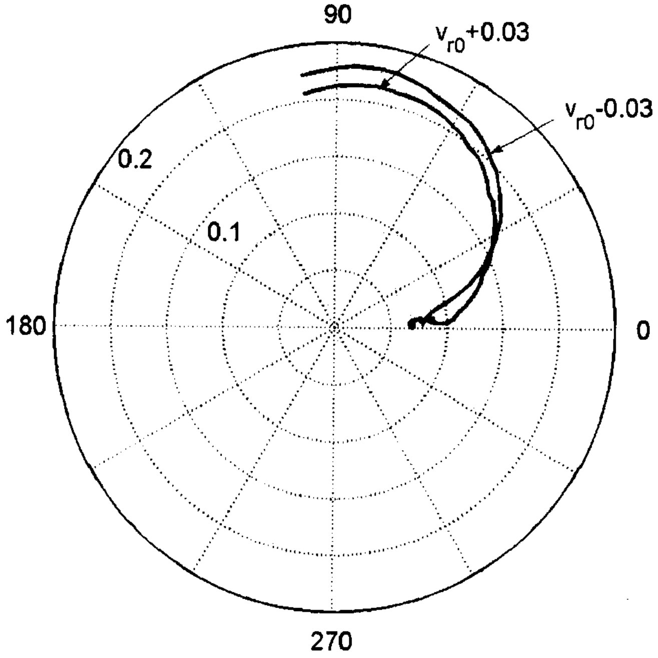

Fig. 2. Projection of partial trajectories of the particles possessing initial radial velocities $\nu_{r0} = \pm 0.03$ at $kz = (-1.57) - 0.5$

$$\dot{\varphi} = \frac{eH_0}{2Mc}\left(\sin kz + \frac{r_0^2}{r^2}\right).$$

The equations have been resolved under initial conditions $t = 0$, $z = -L$, $\dot{z} = v_0$, $r = r_0$, $\dot{r} = v_{r0}$.

To accomplish a numerical calculation, the system was reduced to a non-dimensional form. As characteristic values, the following ones were taken: H_0— maximal on-axis magnetic field value; ν_0 — initial velocity of a particle; the dimensionless variables of coordinates and time are written in the form kr, kz, $\tau \to (eH_0/2Mc)t$. The dimensionless parameters of the system of equations are $\eta = (eH_0 r_0/Mc\nu_0)$ and $kr_0 = \pi r_0/(2L)$, with e being the charge of a particle, c — speed of light, and M — ion mass.

As was shown previously, the particles, which obey the condition $\eta < \eta_{cr} < 1$ and start from the r_0 radius of the left boundary of the system, pass through it successfully. Calculations set forth in [7] show that particles obeying the condition $\eta < \eta_{cr}$ in the right-plunge-adjacent region are moving along a helix shifted towards the axis after being caught in the almost homogeneous magnetic field (for which $kz \approx 1.4\ldots1.6$). In this case, the maximal $r_{\max}$ and minimal $r_{\min}$ deviations from the axis satisfy the relationship $r_{\max}\cdot r_{\min} = r_0^2$. In this region, trajectories of particles of different masses, which start from location r_0, are lying on a cylinder surface whose diameter is the same for any mass value. The trajectories differ from one another by their step and the angle of helix inclination relative to the system's axis. At certain values $kz = \text{const}$, in a plane perpendicular to the system axis, particles of mass M_1 experiencing a spread of the initial radial velocities, are grouped within a certain range of azimuth angles around a particle of $\nu_{r0} = 0$. Particles of mass M_2 with $\nu_{r0} \neq 0$ are grouped closely to the M_2-mass particle with $\nu_{r0} = 0$ within the region of angles shifted relative to that occupied by the M_1-mass particles. One can take a section of $kz = \text{const}$ for which the particles-occupied angle ranges are not mutually overlapping up to a certain critical velocity ν_{r0cr}. As calculations show, a satisfactory separation of the components can be obtained if the initial radial velocities are not exceed $0.015\ \nu_0$. Particles moving with velocities of $\nu_{r0} > \nu_{r0cr}$ will prevent the complete mass separation among the particles.

A more detailed consideration of the motion of particles with high radial velocities shows that in the case of $\nu_{r0} < 0$, the bigger the ν_{r0} value the stronger the retardation of a particle and, under a certain value $\nu_{r0} = \nu_{r0refl}$, the particle experiences the reflection in the growing magnetic field of the second coil even if the condition $\eta < \eta_{cr}$ is met for this particle. The higher the initial radial velocity, the earlier the particle reflects. Therefore, if the selection of components is done near the system's outlet, then the particles of both the masses, for which inequality $\nu_{r0} > \nu_{r0refl}$ is true, do not enter this region and, hence, will not introduce an impurity into

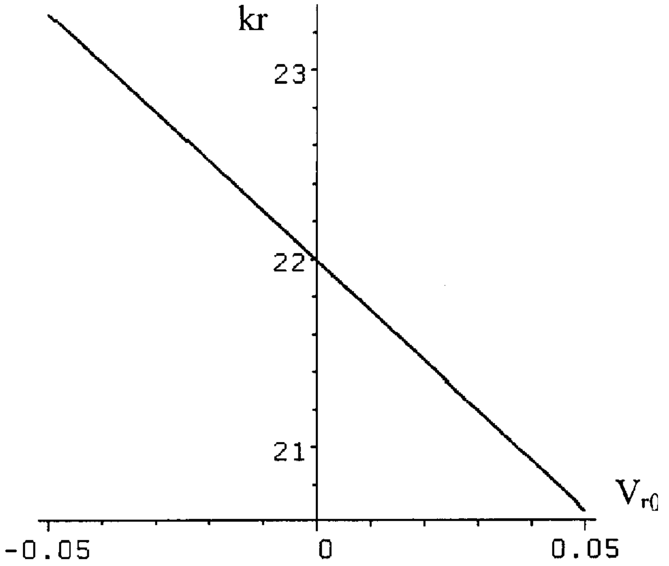

Fig. 3. Radial shift of the particle's trajectory vs its initial radial velocity ($kz = 0.49$)

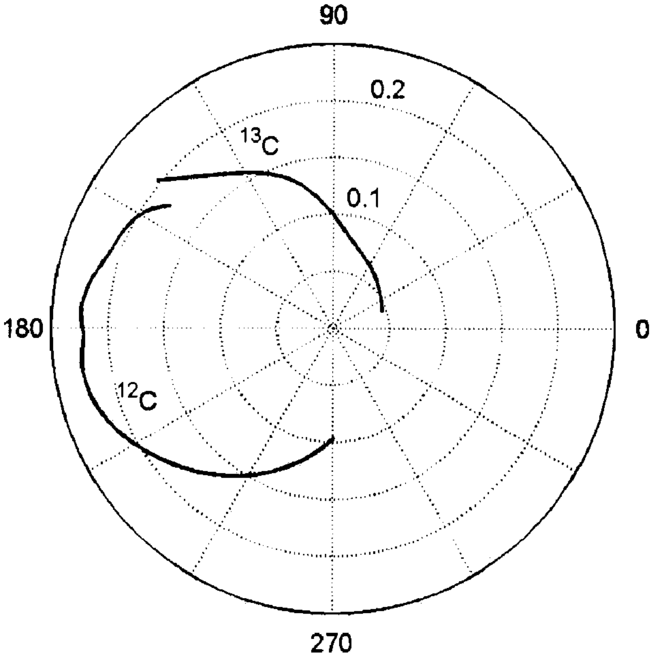

Fig. 4. Ranges of azimuth angles occupied by the carbon isotopes (^{12}C and ^{13}C) in the section plane $kz = 1.54$

the corresponding component. But if the initial radial velocity ν_{r0} is positive, then its augmentation causes a less deceleration of a particle as compared with a particle for which $\nu_{r0} = 0$, and leaves the system being faster than a particle with $\nu_{r0} < 0$.

The inclination angle of a trajectory (helical line) for particles with sufficiently large value $\nu_{r0} > 0$ will be visibly different from that of the main massif of particles [7] and such particles may be excluded from those hitting upon a properly chosen and aligned receiver. The mixing of particles with different masses will be predominantly determined by particles obeying the inequality $\nu_{r0cr} \leq \nu_{r0} \leq \nu_{r0refl}$. For particles with $\eta = 0.7$, the reflection begins from the value $\nu_{r0} > |0.065\nu_0|$ at $kz = 1.48$.

Particles starting from the r_0 position with a certain spread of radial velocities begin to move, after several radial oscillations, away from the system axis along the spatially decaying magnetic field. And then they begin to approach the axis in the increasing field of the second coil after passing through the zero-field plane where the particles achieve their maximum deviation from the axis. When reaching the maximal deviation, a particle does rotate by $\pi/2$ in the azimuth angle relative to the plane of particles' injection. Fig. 2 shows projections of the particles' trajectories onto the plane perpendicular to the axis. The trajectories are taken from the starting point to the plane of maximal deviation of particles with initial radial velocities of $\pm 0.03\nu_0$ ($kz = 0.49$). The initial divergence among the trajectories is determined by the radial oscillations of particles (shown partially in the graph). In the region of maximal off-axis particles' deviation, a rather considerable divergence of the trajectories is achieved. More detailed calculations show that particles with radial velocities $\nu_{r0} < |0.03\nu_0|$ fill in the region confined by trajectories with $\nu_{r0} = \pm 0.03\nu_0$. Fig. 3 shows the dependence of the radial deviation of a particle, in its maximal-shifted position, on the initial radial velocity. A distribution is established in which the particles with maximal radial velocity are the most distant from the system axis if $\nu_{r0} < 0$, and the nearest to it if $\nu_{r0} > 0$. Therefore, the flux of particles with non-zero initial radial velocities expands in the radial direction after passing through the middle of the system. Since the particles with maximal radial velocity are concentrated on the flux's circumference, we can run off the particles with sufficiently high initial radial velocities and exclude their penetration into the region of isotopes' separation, if we place a disk with a properly oriented slot into the region of maximal deviation.

As an example, some results of calculations for the separation of ^{12}C and ^{13}C isotopes are presented in Fig. 4. We can choose the system parameters so that the isotopes separation does take place at the system's outlet. In the case considered, parameter η, which corresponds to ^{12}C, equals $\eta_1 = 0.7$, and isotope ^{13}C is characterized by $\eta_2 = 0.674$. Injection of particles is executed from a region with a radial width of 0.002.

It is suggested that particles starting from the upper and lower boundaries of the flux possess the spread of radial velocities that does not exceed $|0.02|$. In this case, in a plane $kz = 1.54$ perpendicular to the system axis, the isotopes are distributed as follows. Particles characterized by parameter η_2, come to this section being circulated within the range of angles from 662° to 769° and pass through the IV and I quadrants of this plane. The η_1-characterized particles experience the turn from 779° to 970° (59° to 250° of the third round). Therefore, if one creates the conditions in which a main part of the particles possesses the radial velocity within $\nu_{r0} \leq |0.02|$, then a sufficiently high purity of the separated isotopes can be achieved. In this case, the withdrawal of the isotopes can be carried out in the same section. In the region of particles' off-axis maximal deviation (the first maximum takes place at $kz = 0.57$), the radial size of the flux reaches 0.022, i.e. it increases approximately by an order of magnitude for radial velocities < 0.02. In this region, the particles are mixed as yet, and we can eliminate those possessing rather high radial velocities because, in the opposite case, they will experience a reflection and do not hit the region of receivers. The azimuth spread in the middle of the trap equals about 10 degrees.

We can also use another possibility for the isotopes' separation if we take into account the fact of different twists for different masses. In this case, after choosing the characteristic location as coinciding with the point of maximal off-axis deviation of a particle, we find that η_1-characterized particles are located, within the region of the third deviation maximum, in the interval $\Delta(kz) = 0.1291$ (from 1.4654 to 1.5945), whereas the η_2-characterized ones — within the interval $\Delta(kz) = 0.141$ (from 1.599 to 1.740) if one takes into account the spread of radial velocities. These regions are not overlapped under the same conditions. To separate each of the isotopes, the receiver should be placed in different locations along the z axis. It should be noted also that the admissible radial velocity and radial size of the injection region under which a complete separation of isotopes takes place, depend on the chosen parameters of the setup, in particular, on the magnetic field distribution.

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РОЗДІЛЕННЯ ІЗОТОПІВ У СИСТЕМІ З ГОСТРОКУТНОЮ ГЕОМЕТРІЄЮ МАГНІТНОГО ПОЛЯ

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Резюме

Вивчено рух частинок в системі з гострокутною геометрією магнітного поля. Показано, якщо початкова радіальна компонента швидкості достатньо велика, то після проходження площини нульового магнітного поля частинка може відбиватися в збільшуючому полі бар'єра навіть тоді, коли задовольняються умови для проходження крізь бар'єр частинок з $\dot{r}_0 = 0$. В центральній частині системи радіальний розмір потоку частинок збільшується, якщо їх радіальні швидкості відрізняються від нуля. Наведено приклад розділення ізотопів вуглецю ^{12}C і ^{13}C. Обговорюються можливості виведення виділених ізотопів.