TWO-MODE COHERENT AND SQUEEZED STATES OF ONE OF THE JAYNES—CUMMINGS' NONLINEAR MODELS

E.M. VERLAN, T.S. SIDENKO

Taras Shevchenko Kyiv National University, Faculty of Physics
(6, Academician Glushkov Prosp., Kyiv 03127, e-mail: teamoptp@ups.kiev.ua)

UDC 535.33:538.6

The interaction of a two-level atom having the level energies $E_1 < E_2$ in states $|1\rangle$ and $|2\rangle$, respectively, with two modes of the electromagnetic field having frequencies ω_1 (pumping) and ω_2 (Stokes component) is considered. The transition of $|1\rangle \to |2\rangle$ is allowed in the quadrupole approximation. We assume the difference $\Delta = \omega_1 - \omega_2 - \omega_0 (\Delta/\omega_i << 1, \quad i = 0, 1, 2)$ to be small in studies of the energy spectrum and stationary states of the atom+field system. It is shown that the spectrum of energies $E_\nu(N_1', N_2')$ consists of three groups conditioned by the eigenvalues of the operators of excitation numbers $N_i' (i = 1, 2)$, being the integrals of motion. The interaction between the atom and the field modes appears only for those steady states where $N_i' \geq 1$. Being a superposition of the atom states $|1\rangle, |2\rangle$ and quantum states of the field modes, they are referred to as dressed states. The states do not mix up by interaction if one of the numbers N_i' equals zero. In this case, a value of the level energy does not depend on the interaction nature. Using the unitary transformation in the basis of "dressed" states, we define the Bose operators of creation $\hat{A}_i^+$ of excitations and the spin operators $\hat{\Sigma}_z, \hat{\Sigma}_\pm$, in terms of which all basic operators are presented, namely, the number of photons in the modes $\hat{n}_i$, the difference of atomic level populations $\hat{\sigma}_z$, and the Hamiltonian operator $\hat{H}$ of the model under consideration. We represent the operator $\hat{H}$ as a sum of Hamiltonians of two interacting anharmonic oscillators and study statistical properties of the two-mode coherent and squeezed states of a nonlinear Jaynes—Cummings' (JC) model of the Raman type constructed by using the operators $\hat{A}_i^+$, $\hat{A}_i$ and the squeeze operator $\hat{S}\left(\hat{A}_i^+, \hat{A}_i, r\right)$. It is shown that the interaction between the atom and the field mode results in a time dependence of average values of all investigated physical quantities except for the integrals of motion. By its nature, this dependence is collapse and revival of the Rabi oscillations and reaches its maximum at the resonance ($\Delta = 0$). The impact of spontaneous and stimulated Raman scattering on the time dependence of the average value of a difference of atom-level populations $\langle \tilde{\sigma}_z(t) \rangle$ is discussed.

Introduction

A multilevel atom interacting with the multimode electromagnetic field is of considerable interest as an object of study in quantum optics. Its most simple prototype is the standard JC model in which a two-level atom interacts with one mode of the field being resonant to a transition in an atom [1]. In the approximation of a rotating wave, this model possesses an exact solution and describes the quantum nature of the dynamics of an atom and a field interacting with each other. In particular, it describes such phenomena as collapse and revival of the Rabi oscillations of the atomic inversion and polarization, and also a non-classical behavior of the field which is in a coherent state under the interaction absence. Subsequently, numerous generalizations of this model were described and studied, including many-photon versions [2—4] and nonlinear modifications with the interaction constant depending on intensity [5]. As was shown, the beating of Rabi oscillations is typical of all nonlinear modifications of the above-mentioned JC model. In the last years, much attention was paid to the research of quantum systems with three (and more) levels interacting with two or several resonance modes of the field [6—8]. In this case, fields "mix up" the stationary states of an isolated atom. In this case, peculiar interference effects, which substantially change the spectrum of spontaneous and induced radiation of atoms, arise [9]. To one of such interference effects, a phenomenon of quantum capture of populations [10] can be referred. On its basis, the development of lasers for the amplification and generation of light beams without creation of the population inversion becomes possible [11—13].

Mixed states (also named dressed) of the atom+field system can arise in the most simple case of a two-level atom interacting with the quantum mode of the field (the standard JC model). They are stationary states of this model, being a superposition of states of an isolated atom and a field mode. New superposition states, in which an electromagnetic field is similar in its properties to the coherent and squeezed states of an isolated mode, were constructed in our previous works [14] in the basis of dressed states.

In this work, we construct two-mode coherent and squeezed states for a more complicated nonlinear model of the Raman type and study the statistical properties of a field in these states, by using the method of construction of similar states described in [14].

1. Description of the Model, its Stationary States, and the Energy Spectrum

Let us consider a three-level atom with energies $E_1 < E_2 < E_3$ (λ − configuration) in states $|1\rangle, |2\rangle, |3\rangle$, interacting with both the pumping ω_1 and the Stokes mode ω_2 of the electromagnetic field. The Hamiltonian $\hat{H}$ of this system is

$$\hat{H} = \sum_{i=1}^{3} E_i \hat{\sigma}_{ii} + \hbar\omega_1\, \hat{a}_1^+ \hat{a}_1 + \hbar\omega_2\, \hat{a}_2^+ \hat{a}_2 +$$

$$+\hbar\, g_1 \left[\hat{a}_1 \hat{\sigma}_{31} + \hat{a}_1^+ \hat{\sigma}_{13}\right] + \hbar\, g_2 \left[\hat{a}_2 \hat{\sigma}_{32} + \hat{a}_2^+ \hat{\sigma}_{23}\right]. \quad (1)$$

Here, $\hat{a}_i (i = 1, 2)$ are the field operators of annihilation of excitation modes 1 and 2, $\hat{\sigma}_{ii} = |i\rangle\langle i|$, $\hat{\sigma}_{ij} = |i\rangle\langle j|$ $(i \neq j)$ are the atomic operators of level populations and the transition from level j to level i, respectively. Transitions $|1\rangle(|2\rangle) \to |3\rangle$ are allowed in the electric dipole approximation (g_1, g_2 are the atom-field coupling constants), while $|1\rangle \to |2\rangle$ are allowed in the quadrupole one.

As shown in [15], such a three-level system can be exactly transformed to a two-level one with the following Hamiltonian operator (see Appendix 1):

$$\hat{H}_{\text{eff}} = \hbar\omega_1\, \hat{a}_1^+ \hat{a}_1 + \hbar\omega_2\, \hat{a}_2^+ \hat{a}_2 +$$

$$+(1/2)\, \hbar\omega_0 \hat{\sigma}_z + \hbar\, k \left[\hat{a}_1^+ \hat{a}_2 \hat{\sigma}_- + \hat{a}_2^+ \hat{a}_1 \hat{\sigma}_+\right]. \quad (2)$$

Here, $\hat{\sigma}_z = (|2\rangle\langle 2| - |1\rangle\langle 1|)$, $\hat{\sigma}_+ = |2\rangle\langle 1|$, $\hat{\sigma}_- = |1\rangle\langle 2|$ are the operators of "energy spin". They satisfy the Pauli commutation relations for spin operators; $\omega_0 = (E_2 - E_1)/\hbar$, and k is the coupling constant. We assume that the detuning $\Delta = \omega_1 - \omega_2 - \omega_0$ is small in comparison with the frequencies $\omega_i (i = 0, 1, 2)$, that is, $\Delta/\omega_i \ll 1$. Below, we discuss the properties of model (2) and construct its two-mode coherent and squeezed states.

The quantum properties of the described model are fully determined by the operator $\hat{H}_{\text{eff}}$. It commutes with the operators of excitation numbers $N_1' = \hat{n}_1 + 1/2(1 + \hat{\sigma}_z)$, $N_2' = \hat{n}_2 + 1/2(1 - \hat{\sigma}_z)$ and that of total number of photons $\hat{n}_{\text{ph}} = \hat{n}_1 + \hat{n}_2 = \hat{N}_1' + \hat{N}_2' - 1$ $(\hat{n}_i = \hat{a}_i^+ \hat{a}_i)$. They together with $\hat{H}_{\text{eff}}$ are the motion integrals of the system. Steady states of the system are eigenvectors of three independent integrals $\hat{N}_i' (i = 1, 2)$ and $\hat{H}_{\text{eff}}$. In these states, eigenvalues of the operatorsN_i' equal $\hat{N}_i' = 0, 1, 2..., \infty$, and the energy can be presented as $E_\nu(N_1', N_2')$, where the index ν numbers the different values of energy in states with fixed N_i'.

In the energy spectrum $E_\nu(N_1', N_2')$, it is possible to select three groups conditioned by numbers N_1', N_2'. One of them corresponds to the case where both numbers are nonzero and satisfy the inequality $\hat{N}_i' \geq 1$. In the two other cases, one of the numbers N_i' equals zero.

Let us consider, for example, the states

$$|\varphi_1\rangle = |n_1 + 1,\ n_2; -1\rangle \quad \text{and} \quad |\varphi_2\rangle = |n_1,\ n_2 + 1; +1\rangle. \quad (3)$$

Here, $|m_1, m_2 \pm 1\rangle \equiv |m_1\rangle |m_2\rangle | \pm 1\rangle$ is a product of the Fock states of field modes with numbers of photons m_1 and m_2 in the mode and the atom states $| \pm 1\rangle$. Vectors $|\varphi_1\rangle$ and $|\varphi_2\rangle$ are the eigenvectors of operators $\hat{N}_i'$, and their values are $N_i' = (n_i + 1)$, $n_i = 0, 1, 2, ..., \infty$ $(i = 1, 2)$. In the absence of interaction ($k = 0$), the energies of these states $E_i = \langle\varphi_i|\hat{H}_{\text{eff}}|\varphi_i\rangle$ are close, and their difference $E_2 - E_1 = \hbar(\omega_1 - \omega_2 - \omega_0) = \hbar\Delta$ is small as a result of the assumption, $\Delta/\omega_1 << 1$, made above.

The interaction of field modes with the atom mixes states $|\varphi_1\rangle$ and $|\varphi_2\rangle$. To find eigenvalues and eigenstates of the operator $\hat{H}_{\text{eff}}$, it is worth using the perturbation theory for two degenerate or close levels [16]. Two values of energy in this case are:

$$\text{E}_\pm\,(N_1',\, N_2') = \left(\hat{H}_{11} + \hat{H}_{22}\right)\Big/ 2 \pm$$

$$\pm \sqrt{\left[\left(\hat{H}_{11} - \hat{H}_{22}\right)\Big/ 2\right]^2 + |\hat{H}_{12}|^2}\quad, \quad (4)$$

where $\hat{H}_{ik} = \langle\varphi_i|\hat{H}_{\text{eff}}|\varphi_k\rangle (i, k = 1, 2)$ are matrix elements, for example, $H_{12} = \hbar k\sqrt{(n_1 + 1)(n_2 + 1)} \equiv \hbar k\sqrt{N_1' N_2'}$. Substituting H_{ik} in (4), we find

$$\text{E}_\pm\,(N_1',\, N_2') = -\hbar(\omega_1 + \omega_2)/2 + \hbar\omega_1 N_1' +$$

$$+\hbar\omega_2 N_2' \pm \hbar\sqrt{(\Delta/2)^2 + k^2 N_1' N_2'} \equiv$$

$$\equiv \hbar\omega_1(n_1 + 1/2) + \hbar\omega_2(n_2 + 1/2) \pm$$

$$\pm\hbar\sqrt{(\Delta/2)^2 + k^2(n_1 + 1)(n_2 + 1)}. \quad (5)$$

The following stationary (dressed) states of the atom+field system correspond to energies $E_\pm$:

$$|\varphi_+(n_1n_2)\rangle = \cos\theta_{n_1n_2}|n_1+1,\, n_2;-1\rangle +$$

$$+\sin\theta_{n_1n_2}|n_1,\ n_2+1;+1\rangle,$$

$$|\varphi_-(n_1n_2)\rangle = -\sin\theta_{n_1n_2}|n_1+1,\, n_2;-1\rangle +$$

$$+\cos\theta_{n_1n_2}|n_1,\, n_2+1;+1\rangle, \tag{6}$$

where $n_1, n_2 = 0, 1, 2, ..., \infty$. In formulas (6), the following designations are used:

$$\cos\theta_{n_1n_2} = \left(\frac{\lambda_{n_1n_2}+\Delta/2}{2\lambda_{n_1n_2}}\right)^{1/2},$$

$$\sin\theta_{n_1n_2} = \left(\frac{\lambda_{n_1n_2}-\Delta/2}{2\lambda_{n_1n_2}}\right)^{1/2},$$

$$\lambda_{n_1n_2} = \sqrt{(\Delta/2)^2 + k^2\,(n_1+1)\,(n_2+1)}. \tag{7}$$

It follows from the expression $H_{ik} = \hbar k\sqrt{N_i',\, N_k'}$ that if any one of the numbers N_i' and N_k' equals zero in one of the states i, k, they do not mix with each other while interacting. For example, such states are

$$|\psi_1\rangle = |n_1 = 0,\ n_2;-1\rangle, \quad N_1' = 0,\ \ N_2' = n_2+1,$$

$$|\psi_2\rangle = |n_1,\ n_2 = 0;+1\rangle, \quad N_1' = n_1+1,\ \ N_2' = 0. \tag{8}$$

They correspond to such groups in the energy spectrum

$$E_1\,(n_1 = 0,\ n_2;-1) = \hbar\omega_2 n_2 - \hbar\omega_0/2,$$

$$n_2 = 0,\ 1,\ 2,\ \ldots,\infty,$$

$$E_2\,(n_1,\ n_2 = 0;+1) = \hbar\omega_1 n_1 + \hbar\omega_0/2,$$

$$n_1 = 0,\ 1,\ 2,\ \ldots,\ \infty. \tag{9}$$

It is worth noting that this energy spectrum can be also obtained from the first equality (5), substituting $N_1'\cdot N_2' = 0$ and $\Delta = \omega_1 - \omega_2 - \omega_0$.

Below, we are interested in the interaction between an atom and field modes affecting the statistical properties of the model (the average numbers of photons $\langle\hat n_i(t)\rangle$, products $\langle\hat n_1(t)\hat n_2(t)\rangle$, atomic level population $\langle\hat\sigma_z(t)\rangle$, etc.). Therefore, we construct initial states of the atom+field system, in particular two-mode coherent and squeezed states, as the superpositions of dressed states (6), in which this interaction manifests itself.

2. Bose Operators of Creation $\hat{\mathbf{A}}^+$ and Annihilation $\hat{\mathbf{A}}$ Excitations in the Basis of Dressed States

As in our previous works [14], the mentioned operators are obtained as a result of the unitary transformation of operators $\hat a_i^+$, $\hat a_i$ of the Fock basis. The operator $\hat R(\hat R^+\cdot\hat R = 1)$, which initiates this transformation, is given in Appendix. It links the Fock basis $|n_1, n_2;\pm1\rangle$ with the dressed states $|\varphi_\pm(n_1, n_2)\rangle$ by the following relation:

$$\hat R^+|n_1,\, n_2;\,\mp1\rangle = |\varphi_\pm(n_1,\, n_2)\rangle. \tag{10}$$

Such a transformation of the representation bases generates the transformation of operators:

$$\{\hat x_i\} = \left\{\hat a_i^+, \hat a_i,\, \hat\sigma_z,\, \hat\sigma_\pm\right\} \to$$

$$\to \left\{\hat X_i\right\} = \left\{\hat A_i^+,\, \hat A_i,\, \hat\Sigma_z,\, \hat\Sigma_\pm\right\} = \hat R^+\{\hat x_i\}\,\hat R.$$

Their action in basis $|\varphi_\pm(n_1, n_2)\rangle$ is determined by the following equalities:

$$\hat A_i^+|\varphi_\pm(\ldots,\ n_i,\ \ldots)\rangle =$$

$$= \sqrt{n_i+1}\,|\varphi_\pm(\ldots,\ n_i+1,\ldots)\rangle, \quad i = 1,\, 2;$$

$$\hat A_i|\varphi_\pm(\ldots,\ n_i,\ldots)\rangle = \sqrt{n_i}\,|\varphi_\pm(\ldots,\ n_i-1,\ldots)\rangle,$$

$$\hat N_i|\varphi_\pm(\ldots,\ n_i,\ldots)\rangle =$$

$$= n_i\,|\varphi_\pm(\ldots,\ n_i,\ldots)\rangle, \quad \hat N_i = \hat A_i^+\hat A_i,$$

$$\hat\Sigma_z|\varphi_\pm(n_1,\ n_2)\rangle = \mp\,|\varphi_\pm(n_1,\ n_2)\rangle,$$

$$\hat\Sigma_\pm|\varphi_\pm(n_1,\ n_2)\rangle = |\varphi_\mp(n_1,\ n_2)\rangle,$$

$$\hat\Sigma_\pm|\varphi_\mp(n_1,\ n_2)\rangle = 0. \tag{11}$$

As an illustration, we present the expressions $\hat{\Sigma}_z$, $\hat{\Sigma}_{x(y)}$, ($\hat{\Sigma}_{\pm} = (1/2)(\hat{\Sigma}_x \pm i\hat{\Sigma}_y)$), written in terms of the operators $\{\hat{x}_i\}$:

$$\hat{\Sigma}_z = (1/\hat{\lambda}_{N_1' N_2'}) \{(\Delta/2)\,\hat{\sigma}_z - k(\hat{a}_1^+\,\hat{a}_2\,\hat{\sigma}_- + \hat{a}_1\,\hat{a}_2^+\,\hat{\sigma}_+)\},$$

$$\hat{\Sigma}_y = (i/\hat{\lambda}_{N_1' N_2'})\,(\hat{a}_1^+\,\hat{a}_2\,\hat{\sigma}_- - \hat{a}_1\,\hat{a}_2^+\,\hat{\sigma}_+)\,, \hat{\Sigma}_x = -i\,\hat{\Sigma}_y \cdot \hat{\Sigma}_z,$$

$$\hat{\lambda}_{N_1' N_2'} = \sqrt{(\Delta/2)^2 + k^2 \hat{N}_1' \hat{N}_2'}. \tag{12}$$

It is not difficult to make sure that states (8) are also eigenvectors of the operator $\hat{\Sigma}_z$,

$$\hat{\Sigma}_z|\psi_1\rangle = -|\psi_1\rangle, \qquad \hat{\Sigma}_z|\psi_2\rangle = |\psi_2\rangle. \tag{13}$$

Substituting $\{\hat{x}_i\} = \hat{R}\{\hat{X}_i\}\hat{R}^+$ into the expressions for $\hat{N}_i'$ and $\hat{H}_{\rm eff}$, it is possible to write them in the basis of dressed states $\varphi_{\pm}(n_1, n_2)\rangle$:

$$\hat{H}_{\rm eff} = -\hbar(\omega_1 + \omega_2)/2 + \hbar\omega_1\,\hat{N}_1' +$$

$$+\hbar\omega_2 \hat{N}_2' - \hbar\,\hat{\lambda}_{N_1', N_2'} \cdot \hat{\Sigma}_z,\ \ \hat{N}_1' = \hat{N}_1 + 1,$$

$$\hat{N}_2' = \hat{N}_2 + 1,\ \ \hat{N}_i = \hat{A}_i^+\,\hat{A}_i\ \ (i = 1,\ 2). \tag{14}$$

Actually, this expression for $\hat{H}_{\rm eff}$ is an operator form for the first equality (5). As was already pointed, the operator $\hat{H}_{\rm eff}$ (14) describes the entire energy spectrum of the model, including the levels $\mathrm{E}_{\pm}(N_1', N_2')$ of dressed states and the totality of levels (9), for which one of the numbers N_i' equals zero. If we limit our consideration only by the dressed states (6), substituting $\hat{N}_i' = \hat{N}_i + 1$ in (14), we get the following expression for $\hat{H}_{\rm eff}$:

$$\hat{H}_{\rm eff} = \hbar\omega_1(\hat{N}_1 + 1/2) + \hbar\omega_2\,(\hat{N}_2 + 1/2) - \hbar\,\hat{\lambda}_{N_1, N_2}\,\hat{\Sigma}_z,$$

$$\hat{N}_i = \hat{A}_i^+\,\hat{A}_i\ (i = 1,\ 2),$$

$$\hat{\lambda}_{N_1, N_2} = \sqrt{(\Delta/2)^2 + k^2(\hat{N}_1 + 1)(\hat{N}_2 + 1)}. \tag{15}$$

One can see that, in the basis of dressed states, operator (15) of a nonlinear JC model of the Raman type can be written as the sum of the Hamiltonians of interacting anharmonic oscillators. In other words, the atom interaction with the field modes at the frequencies $\omega_1,\ \omega_2(\omega_1 - \omega_2 \approx \omega_0)$ causes the energy exchange between modes. It follows from the commutation relations $[\hat{A}_i^+, \hat{A}_i] = 1$ that the eigennumbers of the operators $\hat{N}_i = \hat{A}_i^+, \hat{A}_i$ take values $n_i = 0, 1, 2, ..., \infty$ $(i = 1, 2)$, and the basis of dressed states $|\varphi_{\pm}(n_1, n_2)\rangle$ in relation to the operators $\{\hat{A}_i^+, \hat{A}_i, \hat{\Sigma}_{\pm}, \hat{\Sigma}_z\}$, defined in it, is complete. The lowest (vacuum) states in this basis are the states $|\varphi_{\pm}(n_1 = 0, n_2 = 0)\rangle$. For them, we have $\hat{A}_i|\varphi_{\pm}(0,0)\rangle = 0, \quad \hat{N}_i|\varphi_{\pm}(0,0)\rangle = 0.$

3. Two-mode Coherent and Squeezed States

For two isolated modes of the field, the states indicated by $|\alpha_1, \alpha_2, r\rangle$ can be defined in such a way [17]:

$$|\alpha_1,\,\alpha_2,\,r\rangle = \hat{D}\,(\alpha_1)\,\hat{D}\,(\alpha_2)\,\hat{S}\,(r)\,|0\rangle_{a_1 a_2}. \tag{16}$$

Here, $\hat{D}(\alpha) = \exp(\alpha_i \hat{\alpha}_i^+ - \alpha_* \hat{\alpha}_i)$ is the displacement operator of the mode i, $\hat{S}(r) = \exp(r[\hat{a}_1 \hat{a}_2 - \hat{a}_1^+ \hat{a}_2^+])$ is the two-mode squeezed operator, $|0\rangle_{\alpha_1\alpha_2} = |0\rangle_{\alpha_1}|0\rangle_{\alpha_2}$ are the states of vacuum of modes. At $r = 0$, they are coherent states of two modes. The state$|\alpha_1 = \alpha_2 = 0, r\rangle$ is named the two-mode squeezed vacuum. In the Fock basis $|n_1, n_2\rangle = |n_1\rangle|n_2\rangle$, the state $|\alpha_1, \alpha_2, r\rangle$ looks as:

$$|\alpha_1,\,\alpha_2,\,r\rangle = \sum_{n_1, n_2 = 0}^{\infty} C\,(n_1,\,n_2,\,\alpha_1, \alpha_2,\,r)\,|n_1,\,n_2\rangle. \tag{17}$$

The quantity $W_{n_1 n_2}(\alpha_1, \alpha_2, r) = |C(n_1, n_2, \alpha_1, \alpha_2, r)|^2$ is the probability of that the numbers of photons which are measured in the state $|\alpha_1, \alpha_2, r\rangle$ be n_1, n_2. It is given by the expression [18]

$$W_{n_1 n_2}(\alpha_1,\ \alpha_2,\ r) = P_{n_1}(\bar{n}_1)\,P_{n_2}(\bar{n}_2) \times$$

$$\times \left| \sum_{n=0}^{\infty} n!\,\frac{L_n^{n_1 - n}\,(\bar{n}_1)\,L_n^{n_2 - n}\,(\bar{n}_2)}{\mathrm{ch\,r}}\,\left(\frac{z}{\alpha_1 \alpha_2}\right)^n \right|^2, \tag{18}$$

where $\bar{n}_i = |\alpha_i|^2$, $z = -\mathrm{th}r, P_{n_i}(\bar{n}_i) = e^{-n_i}\frac{(\bar{n}_i)\,n_i}{n_i!}$ is the Poisson distribution, L_n^{α} are generalized Laguerre polynomials. At $r = 0$ $W_{n_1 n_2}(\alpha_1,\ \alpha_2,\ r) = P_{n_1}(\bar{n}_1)\,P_{n_2}(\bar{n}_2)$, and, for the “squeezed vacuum”, the probability distribution is:

$$W_{n_1 n_2}\,(r) = \frac{(\mathrm{th}\,r)^{2n}}{\mathrm{ch}^2 r} \delta_{n_1, n} \delta_{n_2, n}. \tag{19}$$

In the absence of interaction between an atom and field modes ($k = 0$), it is possible to construct two two-mode squeezed states $|\alpha_1, \alpha_2, r\rangle| + 1\rangle$ and $|\alpha_1, \alpha_2, r\rangle| - 1\rangle$ with the atom in an excited or the ground state

$$|\alpha_1,\,\alpha_2; r; \pm 1\rangle =$$

$$= \sum_{n_1 n_2=0}^{\infty} C\left(n_1, n_2, \alpha_1, \alpha_2; r\right)|n_1, n_2\rangle|\pm 1\rangle. \tag{20}$$

Let us act from the left by the operator $\hat{R}^{+}$ on this expression. Then, taking into account equality (10), we come to the following two-mode states of the atom and the field interacting with each other:

$$|\psi_{\pm}(\alpha_1, \alpha_2; r)\rangle =$$

$$= \sum_{n_1 n_2=0}^{\infty} C\left(n_1, n_2, \alpha_1, \alpha_2; r\right) \hat{R}^{+}|n_1, n_2; \pm 1\rangle =$$

$$= \sum_{n_1 n_2=0}^{\infty} C\left(n_1, n_2, \alpha_1, \alpha_2; r\right)|\varphi_{\mp}(n_1, n_2)\rangle. \tag{21}$$

Further , as an initial state ($t = 0$) we choose $|\Psi(0)\rangle$ which is a superposition of states (21):

$$|\psi\left(0\right)\rangle = \sum_{n_1 n_2=0}^{\infty} C\left(n_1, n_2, \alpha_1, \alpha_2; r\right) \times$$

$$\times\{\cos(\vartheta/2)|\varphi_{-}(n_1, n_2)\rangle + \sin(\vartheta/2)|\varphi_{+}(n_1, n_2)\rangle\}. \tag{22}$$

It can be also presented in the form similar to expression (16)

$$|\psi(0)\rangle = \hat{D}(\alpha_1, \hat{A}_1, \hat{A}_1^{+})\ \hat{D}(\alpha_2, \hat{A}_2, \hat{A}_2^{+}) \times$$

$$\times\hat{S}(r,\ A_i^{+}, A^{+}, A_i, \hat{A}_i)\{\cos(\vartheta/2)|\varphi_{-}(n_1 = n_2 = 0)\rangle +$$

$$+\sin(\vartheta/2)|\varphi_{+}(n_1 = n_2 = 0)\rangle\}, \tag{23}$$

where the new operators of displacement $\hat{D}(\alpha_i, \hat{A}_i, \hat{A}_i^{+})$ and squeezing $\hat{S}(r, \hat{A}_i, \hat{A}_i^{+})$ can be obtained from ones defined above by replacement $\hat{a}_i \to \hat{A}_i, \hat{a}_i^{+} \to \hat{A}_i^{+}$. States (23) are the two-mode squeezed states of a nonlinear JC model of the Raman type.

In work [19], the state $|\alpha_1, \alpha_2, r = 0; -1\rangle$ was considered as the initial one. In the case where $\Delta = 0$, it is

$$|\alpha_1,\ \alpha_2,\ r = 0;\ -1\rangle =$$

$$= \sum_{n_1 n_2} C\left(n_1, n_2, \alpha_1, \alpha_2\right)\, |n_1, n_2; -1\rangle =$$

$$= \sum_{n_1 n_2} C\left(n_1, n_2, \alpha_1, \alpha_2\right) \times$$

$$\times\{|\varphi_{+}\left(n_1 - 1, n_2\right)\rangle - |\varphi_{-}\left(n_1 - 1, n_2\right)\rangle\}. \tag{24}$$

Here, $C(n_1, n_2, \alpha_1,\ \alpha_2) = \exp[(|\alpha_1|^2 + |\alpha_2|^2)/2] \times \times(\alpha_1^{n_1}/\sqrt{n_1!})(\alpha_2^{n_2}/\sqrt{n_2!})$ In this case, both modes of the field are in coherent states, while the atom is in its ground state ($|-1\rangle$).

4. Statistical Properties of the JC Model in Two-mode Coherent and Squeezed States

Statistical properties of an arbitrary system are determined by the average values of physical quantities in the states under investigation. While finding the average value of a quantity $\hat{O}$ in state (23),

$$\left\langle \hat{O}\left(t\right)\right\rangle = \left\langle \psi\left(0\right) \left|\hat{O}\left(t\right)\right|\psi\left(0\right)\right\rangle,$$

we use the Heisenberg representation for the operator

$$\hat{O}\left(t\right) = \hat{U}^{+}\left(t\right)\ \hat{O}\ \hat{U}\left(t\right)\ ,$$

where $\hat{O}$ is an operator in the Schrödinger representation, and $\hat{U}(t) = \exp\{-i\hat{H}t/\hbar\}$ is the operator of evolution with Hamiltonian $\hat{H}$ (15). An explicit form of operators $\hat{O}(t)$ is shown in Appendix 2.

For average values, such expressions are obtained

$$\langle\hat{n}_i\left(t\right)\rangle = [(\bar{n}_1 + (1/2)) - 1/2\ \cos\vartheta\langle\cos\ 2\theta_{n_1 n_2}\rangle]\ \delta_{i,1} +$$

$$+[(\bar{n}_2 + (1/2)) + 1/2\ \cos\vartheta\langle\cos\ 2\theta_{n_1 n_2}\rangle]\ \delta_{i,2} +$$

$$+(1/2)\ \sin\vartheta\langle\sin\ 2\theta_{n_1 n_2}\cos\ 2\lambda_{n_1 n_2}t\rangle\ (\delta_{i,2} - \delta_{i,1}), \tag{25}$$

where $\delta_{i,1}$, $\delta_{i,2}$ are the Kronecker deltas,

$$\langle\hat{\sigma}_z\left(t\right)\rangle = \cos\vartheta\langle\cos\ 2\theta_{n_1 n_2}\rangle +$$

$$+\sin\vartheta\langle\sin\ 2\theta_{n_1 n_2}\cos\ 2\lambda_{n_1 n_2}\ t\rangle, \tag{26}$$

$$\langle\hat{n}_i^2\left(t\right)\rangle = \overline{n_i^2} + (n_i + 1/2) -$$

$$-\cos\theta\langle(n_i + 1/2)\ \cos 2\vartheta_{n_1 n_2}\rangle\,(\delta_{i,1} - \delta_{i,2}) -$$

$$-\sin\vartheta\,\langle(n_i+1/2)\ \sin 2\theta_{n_1n_2}\ \cos 2\lambda_{n_1n_2}t\rangle\times$$

$$\times\left(\delta_{i,1}-\delta_{i,2}\right), \tag{27}$$

$$\langle\hat{n}_1\left(t\right)\ \hat{n}_2\left(t\right)\rangle=\overline{n_1n_2}+$$

$$+(1/2)\left(\bar{n}_1+\bar{n}_2\right)+(1/2)\cos\vartheta\langle(n_1-n_2)\cos 2\theta_{n_1n_2}\rangle+$$

$$+(1/2)\sin\vartheta\langle(n_1-n_2)\sin 2\theta_{n_1n_2}\cos 2\lambda_{n_1n_2}t\rangle, \tag{28}$$

In formulas (26) – (28), following designations are used:

$$\cos 2\theta_{n_1n_2}=\frac{\Delta/2}{\lambda_{n_1n_2}},$$

$$\sin 2\theta_{n_1n_2}=\frac{k\sqrt{\left(n_1+1\right)\left(n_2+1\right)}}{\lambda_{n_1n_2}}$$

$$\lambda_{n_1n_2}=\sqrt{\left(\Delta/2\right)^2+k^2\left(n_1+1\right)\left(n_2+1\right)},$$

$$\bar{n}_i=|\alpha_i|^2+\mathrm{sh}^2\mathrm{r},$$

$$\overline{n_i^2}=|\alpha_i|^4+|\alpha_i|^2+4|\alpha_i|^2\mathrm{sh}^2\mathrm{r}+\mathrm{sh}^2\mathrm{r}+2\,\mathrm{sh}^4\mathrm{r},$$

$$\overline{n_1n_2}=|\alpha_1|^2\mathrm{sh}^2\mathrm{r}+|\alpha_2|^2\mathrm{sh}^2\mathrm{r}+|\alpha_1|^2|\alpha_2|^2-$$

$$-2\mathrm{Re}\left(\alpha_1\alpha_2\right)\,\mathrm{sh\,r\,ch\,r}+\left(\mathrm{sh}^2\,\mathrm{r}+\ \mathrm{ch}^2\mathrm{r}\right)\ \mathrm{sh}^2\,\mathrm{r}. \tag{29}$$

In these expressions, broken brackets$\langle\,\rangle$ designate average values. Moreover, the averaging is carried out with the weight function $W_{n_1n_2}(\alpha_1,\alpha_2,r)$ (18), for example

$$\langle\cos 2\theta_{n_1n_2}\rangle=\sum_{n_1n_2}^{\infty}\cos 2\theta_{n_1n_2}W_{n_1n_2}\left(\alpha_1,\alpha_2,r\right).$$

Average values $\bar{n}_i$, $\bar{n_i^2}$, $\overline{n_1n_2}$ can be found in the same way. In the squeezed vacuum ($\alpha_1=\alpha_2=0$) $W_{n_1n_2}(r)$ is defined by expression (19).

5. Discussion of Results

As one can see from expressions (25) – (4.), the statistical properties of the model system being in the two-mode states (23) differ always from those which take place in states (17) of isolated modes. The distinctions are conditioned by the parameters k and Δ of the JC model, and also by the parameter ϑ in superposition (23). It is substantial that if the interaction is absent ($k=0$, $\sin 2\theta_{n_1n_2}=0$, $\cos 2\theta_{n_1n_2}=1$), the time-dependent elements disappear in all expressions. The inclusion of the interaction between an atom and the field modes causes the time dependence of the average values. This dependence has the form of collapse and revival. It manifests itself maximally at a resonance, where $\Delta=0$, $\cos 2\theta_{n_1n_2}=0$, $\sin 2\theta_{n_1n_2}=1$, $\lambda_{n_1n_2}=2k\sqrt{(n_1+1)\,(n_2+1)}$ and for $\vartheta=\pi/2$. In this case, the states $|\varphi_\pm(n_1,\,n_2)\rangle$ in superposition (22) have the identical weight $1/\sqrt{2}$. The collapse and revival are absent if $\vartheta=0,\ \pi$, and the two-mode state is constructed only from one of the doublet components $|\varphi_\pm(n_1,\,n_2)\rangle$. In the case where $r=0$, $\vartheta=\pi/2$, $\Delta=0$, the average value of the atomic inversion equals

$$\langle\sigma_z\left(t\right)\rangle=\left\langle\cos\,2kt\,\sqrt{\left(n_1+1\right)\left(n_2+1\right)}\right\rangle=$$

$$=\sum_{n_1n_2}P_{n_1}\left(\bar{n}_1\right)\ P_{n_2}\left(\bar{n}_2\right)\times$$

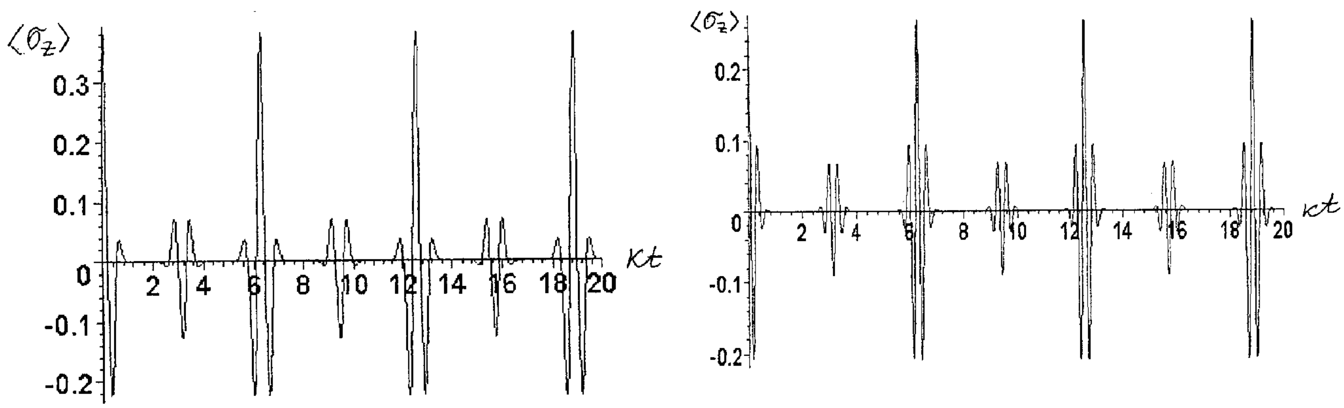
$$\times\cos\,2kt\,\sqrt{\left(n_1+1\right)\left(n_2+1\right)}. \tag{30}$$

In state (24), it is equal to [19]

$$\langle\sigma_z\left(t\right)\rangle=-\sum_{n_1n_2}P_{n_1}\left(\bar{n}_1\right)\ P_{n_2}\left(\bar{n}_2\right)\times$$

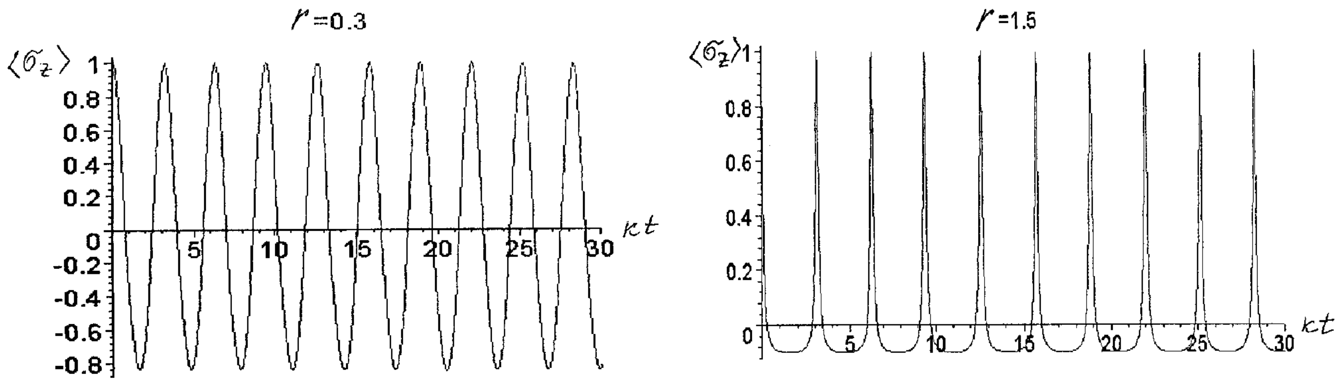
$$\times\cos\,2kt\,\sqrt{n_1\left(n_2+1\right)}. \tag{31}$$

Both these expressions are close in the case of large values of $\bar{n}_1$, but substantially differ at $\bar{n}_1=0$. In this case, $P_{n_1}\left(\bar{n}_1\right)=\delta_{n_1,0}$ and expression (31) gives $\langle\sigma_z\left(t\right)\rangle=-1$. This is consistent with the condition that an atom in state (24) is in its ground state $|-1\rangle$. In expression (22), this condition is absent, and, at the resonance $\left(\Delta=0,\ \cos 2\theta_{n_1n_2}=\sin 2\theta_{n_1n_2}=1/\sqrt{2}\right)$ in the states $|\varphi_\pm(n_1,\,n_2)\rangle$, the atom with equal probability can be both in the ground state $|-1\rangle$ and in the excited one $|+1\rangle$. In this case, the processes of spontaneous radiation of photons with frequency ω_1 (if $\bar{n}_1=0$) or



a **b**

Fig. 1. Time dependence of the atomic inversion $\langle\hat{\sigma}_z(t)\rangle$ in the coherent states for ($\vartheta=\pi/2, \varphi=0, r=0, \overline{n_1}=\overline{n_2}=5(a); 10(b)$)



a **b**

Fig. 2. Time dependence of the atomic inversion $\langle\hat{\sigma}_z(t)\rangle$ in the squeezed vacuum for $r=0.3$ (a); 1.5 (b) $\Delta=0$, $\overline{n_1}=\overline{n_2}=0$

with frequency ω_2 (if $\bar{n}_2=0$) take place. Both these processes cause a change of the atomic inversion $\langle\sigma_z(t)\rangle$ as a function of time. At $\overline{n_2}=0$, it has a form:

$$\langle\sigma_z(t)\rangle=\sum_{n_1=0}^{\infty}P_{n_1}(\bar{n}_1)\ \cos 2k\sqrt{(n_1+1)}\,t,$$

that coincides with analogous expressions derived in [14] in the case where $\Delta=0$, $\vartheta=\pi/2$. There in Fig. 1,*a,b*, the time dependence of this quantity is shown. The similar dependence takes place also in our case (30) if $\overline{n_1}=0$. The processes of spontaneous and induced combination scattering also condition a correlation in numbers of photons of different modes.

The time dependences $\langle\sigma_z(t)\rangle$ in the coherent state (22) ($\vartheta=\pi/2, \varphi=0, r=0, \overline{n_1}=\overline{n_2}=5; 10$) and in the squeezed vacuum ($\Delta=0, \overline{n_1}=\overline{n_2}=0, r=0.3; 1.5$) are presented in Fig. 1,*a* and Fig. 2,*a*, respectively. In the average $\langle\hat{n}_i(t)\rangle$, the collaps and revival have a nature typical of those presented in Fig. 1,*a,b*, but it occurs relative to the quantities $(\overline{n_i}+1/2)$. Averages for the integrals of motion $\langle\hat{N}_i\rangle$ $\langle\hat{n}_1(t)\rangle+\langle\hat{n}_2(t)\rangle$ are constant and do not depend on time.

The authors express the sincere gratitude to Professor V. I. Sugakov for the fruitful debate and valuable remarks while discussing the work results.

APPENDIX 1

Reducing a three-level atomic system, which interacts with two modes of the electromagnetic field, to an effective two-level one, we carry out the canonical transformation of the operator $\hat{H}$ (1) (see [15]). It is initiated by the unitary operator $e^{\hat{S}}$ ($\hat{S}^{+}=-\hat{S}$) and looks as

$$\hat{H}'=e^{\hat{S}}\ \hat{H}\,e^{-\hat{S}}. \tag{1.1}$$

In this case, the operator

$$\hat{S} = \alpha\,(\hat{a}_1\,\hat{\sigma}_{31} - \hat{a}_1^+\,\hat{\sigma}_{13}) + \beta\,(\hat{a}_2\,\hat{\sigma}_{32} - \hat{a}_2^+\,\hat{\sigma}_{23}) \qquad (1.2)$$

where α and β are transformation parameters.

The operator $\hat{H}'$ is similar to the initial operator $\hat{H}$, but with the renormalized constants of interaction

$$\hat{H}' = E_0 + \hbar\omega_1\hat{N}_1 + \hbar\omega_2\hat{N}_2 + \hbar\lambda\,(\hat{a}_1^+\hat{a}_2\hat{\sigma}_{12} + \hat{a}_2^+\hat{a}_1\hat{\sigma}_{21}) +$$

$$+\hbar\omega\,(\hat{\sigma}_{22} - \hat{\sigma}_{11}) + (1/2)\hbar\eta\hat{\sigma}_{33} + \hbar\gamma_1(\hat{a}_1\hat{\sigma}_{31} +$$

$$+\hat{a}_1^+\hat{\sigma}_{13}) + \hbar\gamma_2(\hat{a}_2\hat{\sigma}_{32} + \hat{a}_2^+\hat{\sigma}_{23}). \qquad (1.3)$$

Here, $\hat{N}_1$, $\hat{N}_2$ are the operators of excitation numbers (integrals of motion), and new constants ω, η, γ_1, γ_2 are functions of the parameters α, β, eigennumbers N_i in the steady states, and deviations of the frequencies of modes ω_i from those of resonance transitions$\Delta_i = (E_3 - E_i)/\hbar - \omega_i$ $(i = 1, 2)$. The parameters α, β are selected to make the operator $\hat{H}'$ have the simplest form. This takes place under $\gamma_1 = \gamma_2 = 0$. In this case, as is evident from (1.3), the level E_3 separates from levels E_1, E_2, and the field modes ω_1, ω_2 interact effectively only with two first levels.

APPENDIX 2

Below, we present an explicit form of the operator $\hat{R}^+$ in the Fock basis $|n_1,\, n_2;\, \pm 1\rangle$ and the basis of the dressed states $|\varphi_\pm(n_1,\, n_2)\rangle$.

In the Fock basis,

$$\hat{R}^+ = \begin{vmatrix} \hat{a}_2^+\,\frac{\hat{\cos}\theta_{n_1 n_2}}{\sqrt{\hat{n}_2+1}} & \hat{a}_2^+\,\frac{\hat{\sin}\theta_{n_1 n_2}}{\sqrt{\hat{n}_2+1}} \\ -\hat{a}_1^+\,\frac{\hat{\sin}\theta_{n_1 n_2}}{\sqrt{\hat{n}_1+1}} & \hat{a}_1^+\,\frac{\hat{\cos}\theta_{n_1 n_2}}{\sqrt{\hat{n}_1+1}} \end{vmatrix}. \qquad (2.1)$$

In the basis $|\varphi_\pm(n_1,\, n_2)\rangle$,

$$\hat{R}^+ = \begin{vmatrix} \hat{A}_1^+\,\frac{\hat{\mathrm{Cos}}\theta_{N_1 N_2}}{\sqrt{\hat{N}_1+1}} & -\hat{A}_1^+\,\frac{\hat{\mathrm{Sin}}\theta_{N_1 N_2}}{\sqrt{\hat{N}_1+1}} \\ \hat{A}_2^+\,\frac{\hat{\mathrm{Sin}}\theta_{N_1 N_2}}{\sqrt{\hat{N}_2+1}} & \hat{A}_2^+\,\frac{\hat{\mathrm{Cos}}\theta_{N_1 N_2}}{\sqrt{\hat{N}_2+1}} \end{vmatrix} \qquad (2.2)$$

Here, the operators $\hat{\sin}\theta_{n_1 n_2}$, $\hat{\cos}\theta_{n_1 n_2}$, $(\hat{\mathrm{Sin}}\theta_{N_1 N_2}$, $\hat{\mathrm{Cos}}\theta_{N_1 N_2})$ are found from equalities (7) by replacement of numbers n_1, n_2 with operators $\hat{n}_1$, $\hat{n}_2$ (or operators $\hat{N}_i = \hat{A}_i^+\hat{A}_i$ $(i = 1, 2)$). Below, we give the operators $\{\hat{x}_i\} = \{\hat{n}_i,\, \hat{\sigma}_z,\, \ldots\}$ found by using the operator $\hat{R}^+$ (2.2) and the relation $\langle\hat{x}_i\rangle = \hat{R}\Big\{\hat{X}_i\Big\}\hat{R}^+$ in the basis $|\varphi_\pm(n_1,\, n_2)\rangle$ (the Schrödinger representation):

$$\hat{n}_1 = \Big(\hat{N}_1 + 1/2\Big)\cdot\hat{1} - (1/2)\,\hat{\mathrm{Cos}}2\theta_{N_1 N_2}\cdot\hat{\Sigma}_z - (1/2)\,\hat{\mathrm{Sin}}2\theta_{N_1 N_2}\cdot\hat{\Sigma}_x,$$

$$\hat{n}_2 = \Big(\hat{N}_2 + (1/2)\Big)\cdot\hat{1} + 1/2\,\hat{\cos}2\theta_{N_1 N_2}\cdot\hat{\Sigma}_z + (1/2)\,\hat{\mathrm{Sin}}2\theta_{N_1 N_2}\cdot\hat{\Sigma}_x,$$

$$\hat{\sigma}_z = \hat{\mathrm{Cos}}2\theta_{N_1 N_2}\cdot\hat{\Sigma}_z + \hat{\mathrm{Sin}}2\theta_{N_1 N_2}\hat{\Sigma}_x, \quad \hat{\Sigma}_x = \hat{\Sigma}_+ + \hat{\Sigma}_-. \qquad (2.3)$$

In the Heisenberg representation, operator $\hat{\sigma}_z\,(t)$ equals

$$\hat{\sigma}_z(t) = e^{i\hat{H}t\lambda}\hat{\sigma}_z e^{-i\hat{H}t\lambda} = \hat{\mathrm{Cos}}2\theta_{N_1 N_2}\hat{\Sigma}_z +$$

$$+\hat{\mathrm{Sin}}2\theta_{N_1 N_2}\,e^{-i\hat{\lambda}_{N_1 N_2}\hat{\Sigma}_z}\hat{\Sigma}_x e^{i\hat{\lambda}_{N_1 N_2}\hat{\Sigma}_z} =$$

$$= \hat{\mathrm{Cos}}2\theta_{N_1 N_2}\hat{\Sigma}_z + \hat{\mathrm{Sin}}2\theta_{N_1 N_2}\times$$

$$\times\{e^{-i2\lambda_{N_1 N_2}t}\hat{\Sigma}_+ + e^{i2\hat{\lambda}_{N_1 N_2}t}\hat{\Sigma}_-\}. \qquad (2.4)$$

Here, the relation $e^{\hat{\xi}/2\cdot\hat{\Sigma}_z}\cdot\hat{\Sigma}_+ e^{-\hat{\xi}/2\cdot\hat{\Sigma}_z} = e^{\hat{\xi}}\cdot\hat{\Sigma}_+$, $\Big[\hat{\xi}, \hat{\Sigma}_\pm\Big] = 0$, (and the Hermitian conjugate one) and $\hat{H} = \hat{H}_{\mathrm{eff}}$ are taken into account (15). While finding $\langle\hat{\sigma}_z\,(t)\rangle = \langle\psi\,(0)\,|\hat{\sigma}_z\,(t)|\psi\,(0)\rangle$, it is worth taking into account equalities (11). The operators $\hat{n}_i\,(t)$, $\hat{n}_1\,(t)$ $\hat{n}_2\,(t)$, etc., and their average values $\langle\hat{n}_i\,(t)\rangle$, $\langle\hat{n}_1\,(t)\;\hat{n}_2\,(t)\rangle$ can be found in the same way.

1. *Jaynes E.T., Cummings F.W.* //Proc. IRE .— 1963.— **51**. – P.89.
2. *Mao-Fa Fang, Peng Zho* // J. Mod. Opt. — 1995. — **42**, N6. — P. 1199.
3. *Shumovsky A.S., Le Kien F., Aliskenderov E.I.* // Phys. Lett. A. — 1988 – **44**, N 9.— P. 351.
4. *Mubeen A. Mir, Razmi M.S.K.*// Phys. Rev. A.— 1991.— **44**, N 9. — P. 6071.
5. *Surendra Singh* // Ibid.— 1992. — 25, N 6. — P. 3206.
6. *Fu-li Li, Xiao-shen Li, Lin D.L., George T.F.* // Ibid. — 1992.— **45**, N 5. — P.3133.
7. *Karawajczyk A., Zakrzewsky J., Gawlik W.* // Ibid. — 1992. — **45**, N 1. — P.420.
8. *Chun-rong Fu, Yue-ming Zhang, Chang-de Gong* // Ibid. — 1992. — **45**, N 1. — P.505.
9. *Notkin G.E., Rautian S.G., Feoktistov A.A.* // Zh. Eksp. Teor. Fiz. – 1967. – **52**, N 6. – P.1673.
10. *Agap'ev B.D., Gorskii M.B., Matisov B.G., Rozhdestvenskii Yu.V.* // Uspekhi Fiz. Nauk. – 1993. – **163**, N 9. – P.1.
11. *Kocharovskaja O.A., Mandel P.* // Phys. Rev. A. - 1990. - **42**. — P. 542.
12. *Kocharovskaja O.A* // Kvant. Electr. – 1990. — **17**. – P.20.
13. *Guao Qin Ge, Xueli Luo, Ying Wu, Zajguang Li* // Phys. Rev. A. — 1990. — **42**. — P. 542.
14. *Verlan E.M., Razumova M.A.* // Opt. Spektr. - 2001. – **90**, N 6.— P.928. — **91**, N5 . — P.802 — 2002; **93**, N 4.– P.618; Ukr. Fiz. Zh.. — 2001. — **46**, N7. — C.755.
15. *Wu Y.* // Phys. Rev. A. — 1996. — **54**, N2. — P. 1586.
16. *Landau L.D., Lifshits E.M.* Quantum Mechanics. – Moscow: Nauka, 1974 (in Russian).
17. *Caves C.M., Chang Zhu* // Phys. Rev. A. — 1991. — **43**, N 7. — P.3854.
18. *Antoni M.A., Ortiz U.P., Birman J.L.* // Ibid. — 1991. — **43**, N7. — P. 3954.
19. *Gerry C.C., Eberli J.H.* // Ibid. — 1990. — **42**, N11. — P. 6805.

Received 12.05.03.
Translated from Ukrainian by T. Semenets

ДВОМОДОВІ КОГЕРЕНТНІ ТА СТИСНУТІ СТАНИ ОДНІЄЇ З НЕЛІНІЙНИХ МОДЕЛЕЙ ДЖЕЙНСА—КАММІНГСА

Е.М. Верлан, Т.С. Сіденко

Р е з ю м е

Розглянуто взаємодію дворівневого атома з енергіями рівнів $E_1 < E_2$ у станах $|1\rangle$ і $|2\rangle$ відповідно з двома модами електромагнітного поля з частотами ω_1 (накачування) і ω_2 (стоксова компонента). Перехід $|1\rangle \to |2\rangle$ дозволений у квадрупольному наближенні. Передбачається мале значення розстройки $\Delta = \omega_1 - \omega_2 - \omega_0 (\Delta/\omega_i << 1, i = 0,1,2)$. Вивчено спектр енергій і стаціонарні стани системи атом+поле. Показано, що спектр енергій $E_\nu(N_1', N_2')$ складається з трьох сукупностей, зумовлених власними значеннями операторів числа збуджень N_i' $(i = 1,2)$, які є інтегралами руху. Взаємодія між атомом і модами поля виявляється тільки в тих стаціонарних станах, у яких $N_i' \geq 1$. Вони є суперпозицією атомних станів $|1\rangle, |2\rangle$ та квантових станів мод поля і мають назву одягнених станів. Стани, у яких одне з чисел N_i' дорівнює нулю, взаємодією не змішуються. Значення енергії рівнів у цьому випадку не залежать від характеру взаємодії. У базисі одягнених станів унітарним перетворенням визначені бозе-оператори народження $\hat{A}_i^+$ і знищення $\hat{A}_i$ збуджень та спінові оператори $\hat{\Sigma}_z, \hat{\Sigma}_\pm$, через які представлені всі основні оператори — числа фотонів мод $\hat{n}_i$, різниці заселеностей атомних рівнів $\hat{\sigma}_z$ і оператор Гамільтона $\hat{H}$ — розглянутої моделі. Оператор $\hat{H}$ у цьому базисі має вигляд суми гамільтоніанів двох взаємодіючих ангармонічних осциляторів. За допомогою операторів $\hat{A}_i^+$, $\hat{A}_i$ і оператора стиску $\hat{S}(\hat{A}_i^+, \hat{A}_i, r)$ побудовані двомодові когерентні і стиснуті стани нелінійної моделі Джейнса—Каммінгса раманівського типу і вивчені їх статистичні властивості. Показано, що взаємодія між атомом і модами поля приводить до змін у часі середніх значень усіх вивчених фізичних величин за винятком інтегралів руху. Ці зміни носять характер биття осциляцій Рабі і максимально виявляються в резонансі $(\Delta = 0)$. Обговорюється роль спонтанного і вимушеного комбінаційного розсіяння у зміні з часом середнього значення різниці заселеностей атомних станів $\langle\tilde{\sigma}_z(t)\rangle$.