CLASSIFICATION OF THE EQUILIBRIUM STATES OF A QUANTUM FLUID WITH TENSOR ORDER PARAMETER

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A classification of the equilibrium states of a quantum fluid with d-pairing is carried out on the basis of the concept of quasiaverages. It is shown that the set of such equilibrium states can be classified in terms of a quantum number relevant to the projection of the orbital momentum of a Cooper pair on the anisotropy direction. The explicit form of three admissible generators of the residual symmetry and the corresponding equilibrium values of the order parameter are found.

The classification of the equilibrium states of condensed media based on the phenomenological Ginzburg—Landau approach requires to represent the free energy as a function of the order parameter and depends essentially on the form of a model under consideration. The other group-theoretic approach is based on the representation of the residual symmetry of a degenerate equilibrium state as a subgroup of the symmetry of the normal phase. In this approach, significant are the corresponding transformational properties of the order parameter upon transformations of the Hamiltonian symmetry. This consideration is free from any model assumptions on the form of free energy. The classification of homogeneous states in the frames of both indicated approaches was performed for superfluid ^{3}He [1–3] which is described with a tensor order parameter. The question on possible inhomogeneous superfluid states is studied in [4]. Other important example of degenerate condensed media is a superfluid in the state of d-pairing which is also described by the tensor order parameter [5]. Such a state describes the nuclear matter of neutron stars [6—9]. By virtue of the strong spin-orbital coupling, there occurs $^3P_2$ pairing of neutrons ($l = 1, S = 1; J = 2$). The pairing of the same type is possible for ^{3}He with $l = 2$ [10] and for a number of high-temperature superconductors [11—13]. The order parameter describing the states with $J = 2$ is a symmetric traceless tensor. A classification of possible states for this type of pairing was carried out on the basis of a phenomenological approach in [5, 14]. In the present work, we present a microscopic approach to the classification of homogeneous equilibrium states which is based on the concept of quasiaverages [14—18, 4]. We found the admissible properties of a symmetry of the equilibrium state of a quantum liquid and the corresponding structures of the order parameter from the conditions of an residual symmetry for nonzero values of the order parameter.

The theoretical basis of the statistical physics of condensed media is the concept of quasiaverages advanced by N. N. Bogolyubov [15] which generalizes the Gibbs distribution to degenerate condensed media. According to [15], the quasiaverage of a physical quantity in the state of statistical equilibrium with broken symmetry is defined by the formula

$$\langle \hat{a}(x) \rangle \equiv \lim_{\nu \to 0} \lim_{V \to \infty} \mathrm{Sp} \hat{w}_\nu \hat{a}(x),$$

$$\hat{w}_\nu \equiv \exp\left(\Omega_\nu - Y_a \hat{\gamma}_a - \nu \hat{F}\right). \quad (1)$$

Here, $\hat{\gamma}_a$ are additive integral of motion ($\hat{H}$ is the Hamiltonian, $\hat{P}_k$ is the momentum operator, $\hat{N}$ is the particle number operator, $\hat{S}_\alpha$ is the spin operator), and ($Y_a \equiv Y_0, Y_k, Y_4, Y_\alpha$) are the relevant thermodynamic forces. For the convenience of the further presentation, we assume that macroscopic flows are absent in the lab system of coordinates, i.e., $Y_k = 0$ and the internal magnetic field is equal to zero. The thermodynamic potential Ω_ν is defined by the normalization condition $\mathrm{Sp}\hat{w}_\nu = 1$. The operator $\hat{F}$ possesses the symmetry of the studied phase of a condensed medium and is a linear functional of the order parameter operator $\hat{\Delta}_a(x)$

$$\hat{F} \equiv \int d^3x (f_\alpha(x) \hat{\Delta}_\alpha(x) + h.c.). \quad (2)$$

Here, $f_a(x)$ is a certain function of coordinates conjugated to the order parameter operator that sets its equilibrium values in the sense of quasiaverages $\Delta_a(x) = \left\langle \hat{\Delta}_a(x) \right\rangle$. The structure of functions $f_a(x)$ is defined by properties of the symmetry of the studied states of a quantum liquid. This gives us the possibility to introduce additional thermodynamic parameters in the frame of a microscopic theory to the Gibbs distribution.

The description of condensed media with spontaneously broken symmetry leans essentially on the idea of order parameter. Within the scheme of secondary quantization, the order parameter operators $\hat{\Delta}_x$ are constructed from the field operators of creation and annihilation. Below, we state transformational properties of the order parameter operators. The condition of translational invariance has the form

$$i[\hat{P}_k, \hat{\Delta}_\alpha(x)] = -\nabla_k \Delta_\alpha(x). \tag{3}$$

A generator of the group of phase transformations is the particle number operator $\hat{N}$. The order parameter operators $\hat{\Delta}_a(x)$ are transformed according to the relations

$$\left[\hat{N}, \hat{\Delta}_a(x)\right] = -g_a \hat{\Delta}_a(x). \tag{4}$$

The constants g_a depend on the tensor dimension of the order parameter operator.

Upon the transformations related to the group of internal symmetries with generators $\hat{S}_\alpha$, $(\alpha = x, y, z)$, the operators $\hat{\Delta}_a(x)$ are transformed by representations of this group as

$$i\left[\hat{S}_\alpha, \hat{\Delta}_a(x)\right] = -g_{\alpha ab}\hat{\Delta}_b(x), \tag{5}$$

or, in the compact form,

$$i\left[\hat{S}_\alpha, \hat{\Delta}(x)\right] = -\hat{g}_\alpha \hat{\Delta}(x),$$

where $(\hat{g}_\alpha)_{ab} \equiv g_{\alpha ab}$ are some constants. The generators of the group of internal symmetry, $\hat{S}_\alpha$, satisfy the relations

$$i\left[\hat{S}_\alpha, \hat{S}_\beta\right] = -\varepsilon_{\alpha\beta\gamma}\hat{S}_\gamma, \tag{6}$$

where the antisymmetric tensor $\varepsilon_{\alpha\beta\gamma}$ has the sense of structural constants. Formulas (5),(6) and the Jacobi identity for the operators $\hat{T}$ and $\hat{\Delta}(x)$ yield the relation

$$[\hat{g}_\alpha, \hat{g}_\beta] = -\varepsilon_{\alpha\beta\gamma}\hat{g}_\gamma. \tag{7}$$

Upon transformations related to the group of spatial rotations with generators $\hat{L}_i \quad (i = 1, 2, 3)$, the order parameter operators $\hat{\Delta}(x)$ at the point 0 are transformed by representations of this group:

$$i\left[\hat{L}_i, \hat{\Delta}_a(0)\right] = -g_{iab}\hat{\Delta}_b(0).$$

Whence, in view of the formula $\left[\hat{L}_i, \hat{L}_j\right] = i\varepsilon_{ijk}\hat{L}_k$, we get the relations analogous to (7):

$$[\hat{g}_i, \hat{g}_j] = -\varepsilon_{ijk}\hat{g}_k. \tag{8}$$

Since $\hat{\Delta}_a(x) = e^{-i\hat{P}x}\hat{\Delta}_a(0)e^{i\hat{P}x}$, we get

$$i\left[\hat{L}_i, \hat{\Delta}_a(x)\right] = -g_{iab}\hat{\Delta}_b(x) - \varepsilon_{ijk}x_k\nabla_j\hat{\Delta}_a(x). \tag{9}$$

by virtue of (3).

It is known from the phenomenological theory that, in order to adequately describe the thermodynamics of irreversible processes occurring in condensed media with broken symmetry, it is necessary, generally saying, to introduce new thermodynamic parameters into theory which are not related to the preservation laws, but are conditioned by the physical nature of a thermodynamic phase. In the case of normal condensed media, thermodynamic parameters are defined only by the densities of additive integrals of motion. Below, we show how properties of a symmetry of the equilibrium state are formulated and the additional thermodynamic parameters for degenerate condensed media are introduced. Consider translation-invariant subgroups H of the full symmetry group G which are related to the residual symmetry. The translational invariance means that the equilibrium statistical operator satisfies the symmetry relation

$$\left[\hat{w}, \hat{P}_k\right] = 0. \tag{10}$$

We will analyze translation-invariant subgroups related to the residual symmetry of equilibrium states according to [17] by starting from the relation

$$\left[\hat{w}, \hat{T}\right] = 0, \tag{11}$$

where the residual symmetry generator $\hat{T}$ (a generator of the subgroup H) is a linear combination of the integrals of motion

$$\hat{T} = a_i\hat{L}_i + b_\alpha\hat{S}_\alpha + c\hat{N} \equiv \hat{T}(\xi) \tag{12}$$

with some real parameters $(a_i, b_\alpha, c \equiv \xi)$. The unitary transformations form a continuous subgroup

$U(\xi)\,U(\xi') = U(\xi''(\xi,\xi'))$ related to the residual symmetry of the equilibrium state. From the equalities

$$i\mathrm{Sp}\left[\hat{w},\hat{T}(\xi)\right]\hat{\Delta}_a(x)=0,\quad i\mathrm{Sp}\left[\hat{w},\hat{P}_k\right]\hat{\Delta}_a(x)=0,$$

the algebraic relations (3)—(5),(9), and definition (12), we deduce the equations

$$a_i g_{iab}\Delta_b + b_\alpha g_{\alpha ab}\Delta_b + igc\Delta_a = 0. \tag{13}$$

According to (12), we obtain

$$T_{ab}\Delta_b = 0,\quad T_{ab}\equiv a_i g_{iab} + b_\alpha g_{\alpha ab} + igc\delta_{ab}. \tag{14}$$

The condition for a nontrivial solution $\Delta_a \neq 0$ of the system of linear equations (14) to exist leads to the equality

$$\det|T_{ab}| = 0 \tag{15}$$

which imposes certain restrictions on admissible values of the parameters related to the residual symmetry generator.

We define the order parameter operator for d-pairing, $\hat{\Delta}_{ik}(x)$, in terms of the operators of creation and annihilation of a Fermi-particle at a point x:

$$\begin{aligned}\hat{\Delta}_{ik}(x) &\equiv \nabla_i\hat{\psi}(x)\,\sigma_2\nabla_k\hat{\psi}(x) + \\ &+\nabla_k\hat{\psi}(x)\,\sigma_2\nabla_i\hat{\psi}(x) - \frac{2}{3}\delta_{ik}\nabla_j\hat{\psi}(x)\,\sigma_2\nabla_j\hat{\psi}(x),\end{aligned} \tag{16}$$

where σ_2 is the Pauli matrix. We introduce the operators of particle number $\hat{N}$, momentum $\hat{P}_k$, spin $\hat{S}_\alpha$, and orbital momentum $\hat{L}_k$ as

$$\hat{N} = \int d^3x\hat{n}(x),\quad \hat{P}_i = \int d^3x\hat{\pi}_i(x),$$

$$\hat{S}_i = \int d^3x\hat{s}_i(x),\quad \hat{L}_i = \int d^3x\hat{l}_i(x), \tag{17}$$

where the relevant densities of the integrals of motion $\hat{n}(x)$, $\hat{\pi}_i(x)$, $\hat{s}_i(x)$, and $\hat{l}_i(x)$ in terms of the operators of creation and annihilation $\hat{\psi}^+(x),\hat{\psi}(x)$ are as follows:

$$\hat{n}(x) = \hat{\psi}^+_\sigma(x)\,\hat{\psi}_\sigma(x),$$

$$\hat{s}_\alpha(x) = \hat{\psi}^+_\sigma(x)(s_\alpha)_{\sigma,\sigma'}\hat{\psi}_{\sigma'}(x),$$

$$\hat{\pi}_i(x) = -\frac{i}{2}\left\{\hat{\psi}^+_\sigma(x)\nabla_i\hat{\psi}_\sigma(x) - \nabla_i\hat{\psi}^+_\sigma(x)\,\hat{\psi}_\sigma(x)\right\},$$

$$\hat{l}_i(x) = \varepsilon_{ikl}x_k\hat{\pi}_l(x). \tag{18}$$

By using definitions (16)-(18), we arrive at the operator algebra:

$$\left[\hat{N},\hat{\Delta}_{ik}(x)\right] = -2\hat{\Delta}_{ik}(x),\quad \left[\hat{S}_\alpha,\hat{\Delta}_{ik}(x)\right]=0,$$

$$i\left[\hat{P}_l,\hat{\Delta}_{ik}(x)\right] = -\nabla_l\hat{\Delta}_{ik}(x),$$

$$i\left[\hat{L}_l,\hat{\Delta}_{ik}(x)\right] = -\varepsilon_{lij}\hat{\Delta}_{jk}(x) - \varepsilon_{lkj}\hat{\Delta}_{ji}(x) -$$

$$-\varepsilon_{lkj}x_k\nabla_j\hat{\Delta}_{ik}(x) \tag{19}$$

The mean value of the order parameter $\Delta_{ik}(x,\hat{\rho}) = \mathrm{Sp}\hat{\rho}\Delta_{ik}(x)$, where $\hat{\rho}$ is any statistical operator, has the properties $\Delta_{ik}(x,\hat{\rho}) = \Delta_{ki}(x,\hat{\rho})$ and $\Delta_{ii}(x,\hat{\rho}) = 0$ and contains 10 independent quantities. We choose the parametrization of Δ in the following form:

$$\Delta_{ik}(x,\hat{\rho}) \equiv Q_{ik}(x,\hat{\rho}) + i\underline{Q}_{ik}(x,\hat{\rho}),$$

$$Q_{ik} \equiv A\left[n_in_k - \frac{1}{3}\delta_{ik}\right] + B\left[m_im_k - \frac{1}{3}\delta_{ik}\right], \tag{20}$$

$$\underline{Q}_{ik} \equiv C\left(n_in_k - \frac{1}{3}\delta_{ik}\right) + D\left(m_im_k - \frac{1}{3}\delta_{ik}\right) +$$

$$+E\left(m_in_k + m_kn_i\right) + F\left(n_il_k + n_kl_i\right) +$$

$$+G\left(l_im_k + l_km_i\right). \tag{21}$$

Here, $A, B, .., G$ are the order parameter moduli, $\vec{m},\vec{n},\vec{l}$ are the units, mutually orthogonal vectors oriented along the anisotropy axes: $\vec{n}^2 = \vec{m}^2 = \vec{l}^2 = 1$, $\vec{n}\vec{m} = 0$, $\vec{m}\vec{l} = 0$, $\vec{n}\vec{l} = 0$.

A condensed medium being in the normal equilibrium state is characterized by the following symmetry properties:

$$\left[\hat{w},\hat{P}_k\right]=0,\quad \left[\hat{w},\hat{L}_k\right]=0,\quad \left[\hat{w},\hat{S}_\alpha\right]=0. \tag{22}$$

These properties reflect the translational invariance, spatial and spin isotropy, and phase invariance of the normal phase of a condensed medium in the state of equilibrium. The condition of spatial isotropy in (22) and the algebra of quantum brackets (19) yield the equality $\mathrm{Sp}\hat{w}\hat{\Delta}_{ik}(x) = 0$ due to the absence of preferred directions in the state of equilibrium.

Below, we consider translation-invariant equilibrium superfluid states and determine possible equilibrium structures of the order parameter. The analysis of translation-invariant subgroups of the residual symmetry of equilibrium states will be realized on the basis of relations (10), (11). According (11), we have $\mathrm{Sp}\left[\hat{w},\hat{T}\right]\Delta_{ik}(x) = 0$. Therefore, with regard for algebra (19), we arrive at the system of linear homogeneous equations

$$F^{ik}_{jl}\Delta_{jl} = 0, \tag{23}$$

where

$$a_l\left(\varepsilon_{lij}\delta_{kl} + \varepsilon_{lkj}\delta_{il}\right) + 2ic\delta_{kl}\delta_{ji} \equiv F^{ik}_{jl}.$$

By passing in formula (23) from the double summation to the single one, where the summation indices α, β run the values $11 \equiv 1, 12 \equiv 2, ...33 \equiv 9$, we get the equation

$$F^{\beta}_{\alpha}\Delta_{\alpha} = 0, \quad \det\left|F^{\beta}_{\alpha}\right| = 0. \tag{24}$$

If the condition $\det\left|F^{\beta}_{\alpha}\right| = 0$ is valid, there exists a nontrivial solution of system (23). The inequality $\Delta_{ik}(x) \neq 0$ holds for $c = 0, \pm\frac{1}{2}a, \pm a$. Three solutions of the equation $\det\left|F^{\beta}_{\alpha}\right| = 0$ are

$$c = 0, \quad c = \pm\frac{1}{2}a, \quad c = \pm a. \tag{25}$$

Hence, the residual symmetry conditions for the equilibrium state of a superfluid with d-pairing can be written as

$$\left[\hat{w}, \frac{a_i}{a}\hat{L}_i - \frac{M}{2}\hat{N}\right] = 0. \tag{26}$$

Here, $M \equiv -c/2a$ is the quantum number taking the values $0, \pm1, \pm2$. In view of definition (21), relation (23) yields the equations defining the explicit form of the order parameter in the equilibrium state:

$$\frac{a_i}{a}\left(\varepsilon_{iuj}\underline{Q}_{jv} + \varepsilon_{ivj}\underline{Q}_{ju}\right) - MQ_{uv} = 0,$$

$$\frac{a_i}{a}\left(\varepsilon_{iuj}Q_{jv} + \varepsilon_{ivj}Q_{ju}\right) + M\underline{Q}_{uv} = 0. \tag{27}$$

While solving this system of equation, we expand the vector $\vec{a}$ in the above-mentioned reference frame $\vec{q} \equiv \vec{a}/a = \alpha\vec{n} + \beta\vec{m} + \gamma\vec{l}$. Here, the values α, β, γ are connected by the equality $\alpha^2 + \beta^2 + \gamma^2 = 1$. We set $M = 0$ in (27) and consider the following cases.

1. $\alpha \neq 0, \beta \neq 0, \gamma \neq 0$. In this case, the order parameter amplitudes $A = 0, B = 0$ and, hence, $\Delta_{uv} = i\underline{Q}_{uv}$:

$$\Delta_{uv} = \frac{iE}{\alpha\beta}\left(q_u q_v - \frac{1}{3}\delta_{uv}\right). \tag{28}$$

2. $\gamma = 0, \beta \neq 0, \alpha \neq 0$ (the cases $\beta = 0, \alpha \neq 0, \gamma \neq 0$ and $\alpha = 0, \beta \neq 0, \alpha \neq 0$ are equivalent to the case under study). Here, we also have $A = 0, B = 0$, and $\Delta_{uv} = i\underline{Q}_{uv}$:

$$\Delta_{uv} = \frac{i\widetilde{C}}{\alpha^2}\left(q_u q_v - \frac{1}{3}\delta_{uv}\right). \tag{29}$$

3. $\alpha \neq 0, \beta = \gamma = 0$ (the cases $\beta \neq 0, \alpha = \gamma = 0$ и $\gamma \neq 0, \beta = \alpha = 0$ are equivalent to the case under study). Here, we have $A \neq 0, C \neq 0$, i.e., the real and imaginary parts of the order parameter are not equal to zero: $\Delta_{uv} = Q_{uv} + i\underline{Q}_{uv}$:

$$\Delta_{uv} = (A + iC)\left\{q_u q_v - \frac{1}{3}\delta_{uv}\right\}. \tag{30}$$

In all three cases, we got the order parameter structure for a uniaxial liquid crystal with complex amplitude.

If the quantum number $M \neq 0$, a solution of Eq. (27) has the form

$$\Delta_{uv} = A(n_u n_v - \frac{1}{3}\delta_{uv}) + B(m_u m_v - \frac{1}{3}\delta_{uv}) +$$

$$+ i\{E(m_v n_u + m_u n_v) + F(l_v n_u + l_u n_v) +$$

$$G(l_v m_u + l_u m_v)\}, \tag{31}$$

where $E = (\gamma/M)(B - A)$, $G = -(\alpha/M)B$, $F = (\beta/M)A$.

Like to the case where $M = 0$, we consider all possible values of the coefficients α, β, γ.

1. $\alpha \neq 0, \beta \neq 0, \gamma \neq 0$. In this case, the order parameter amplitudes $A = 0, B = 0$ and, hence, $\Delta_{uv} = 0$.

2. $\gamma = 0, \beta \neq 0, \alpha \neq 0$ (the cases $\beta = 0, \alpha \neq 0, \gamma \neq 0$ and $\alpha = 0, \beta \neq 0, \alpha \neq 0$ are equivalent to the case under study). Here, we have $A \neq 0, B \neq 0$. For those values of the parameters, there exists only the solution for $M = \pm1$:

$$\Delta_{uv} = A(n_u n_v - m_u m_v) \pm$$

$$\pm\frac{iA}{\sqrt{2}}\left(l_v(n_u+m_u)+l_u(n_v+m_v)\right). \quad (32)$$

In this case, $\beta^2=\alpha^2=1/2$.

3. $\gamma=0$, $\beta=0$, $\alpha=1$ (the cases $\beta=0$, $\alpha=0$, $\gamma=1$ и $\alpha=0$, $\gamma=0$, $\beta=1$ are equivalent to the case under study). There exists a solution only for $M=\pm2$:

$$\Delta_{uv}=A(m_um_v-l_ul_v)\mp iA\left(l_vm_u+l_um_v\right). \quad (33)$$

To compare the results obtained with those in [5], [14], we normalize the order parameter by the relation

$$\Delta_{ik}\Delta^*_{ik}=1. \quad (34)$$

Then, following [5], we introduce the average

$$\Delta\equiv k_i\Delta_{ij}k_j, \quad (35)$$

where the unit vector is defined by the equality

$$\vec{k}\equiv\sin\theta\sin\varphi\vec{m}+\cos\theta\vec{n}+\sin\theta\cos\varphi\vec{l}. \quad (36)$$

For solutions(28), (29), and (30) with $M=0$, we found

$$\Delta_1^{(0)}=\frac{iE}{\alpha\beta}\left\{(\beta k_y+\alpha k_z+\gamma k_x)^2-\frac{1}{3}\right\},$$

$$\Delta_2^{(0)}=\frac{i\widetilde{C}}{\alpha^2}\left\{(\beta k_y+\alpha k_z)^2-\frac{1}{3}\right\},$$

$$\Delta_3^{(0)}=(A+iC)\left(k_z^2-1/3\right), \quad (37)$$

in correspondence with (35), (36). Here, $E^2=(3/2)\alpha^2\beta^2$, $\widetilde{C}^2=(3/2)\alpha^4$, $A^2+C^2=3/2$.

These solutions correspond to a “real” state derived in [5].

By virtue of (35), (36), solution (32) leads to the equality

$$\Delta^{(1)}=A\left(k_x+k_y\right)\left(\pm i\sqrt{2}k_z+k_y-k_x\right), \quad (38)$$

for $M=\pm1$. Here, $A^2=1/4$.

In the last case for solution (33) ($M=\pm2$), we get

$$\Delta^{(2)}=A\left(k_x\pm ik_y\right)^2,$$

where $A^2=1/4$. This solution exactly corresponds to an “axial” state in [5].

Comparing the results obtained for the order parameter with those in [5], [14], we see that there is no full equivalence of solutions for the “cyclic” state (38).

1. *Barton G., Moore M.*// J. Phys. C. — 1974. — **7**. — P. 2989—3000.
2. *Capel H.W.* // Proc. 5th Intern. Symp. on Selected Topics in Statistical Mechanics. — Singapore: World Scientific, 1990. — P. 73—83.
3. *Vollhardt D., Wolfle P.* The Superfluid Phases of Helium 3.— Taylor Francis, 1990.
4. *Kovalevsky M.Y., Peletminsky S.V., Chekanova N.N.* // Low Temper. Phys. — 2002. — **28**, N 4. — P. 327—339.
5. *Mermin N.D.*// Superfluidity of ^{3}He. — Moscow: Mir, 1977 (in Russian). — P. 110—120.
6. *Pines D., Alpar A.* //Nature. — 1985. — **316**. — P. 27.
7. *Baldo M., Cugnon J., Lejeune A., Lombardo U.* // Nucl. Phys. A. — 1992. — **536**. — P. 349—365.
8. *Schaab V.C., Weber F., Weigel M., Glendenning N.* // Ibid. — 1996. — **605**. — P. 531—565.
9. *Elgaroy O., Engvik L., Hjorth-Jensen M., Osnes E.* // Ibid. — 1996. — **607**. — P. 425—441.
10. *Mermin N.D., Stare C.C.* // Phys.Rev.Lett. — 1973. — **30**. — P. 1135.
11. *Van Harlingen D.J.* //Rev. Mod. Phys. — 1995. — **67**. — P. 515.
12. *Tsuei C.C., Kirtley J.R.*// Physica C. — 1997. — **67**. — P. 515.
13. *Won H., Maki K.*// Phys. Rev. B. — 1994. — **49**. — P. 1397.
14. *Ho T.L., Yip S.* // Phys. Rev. Lett. — 1999. — **82**, N 2. — P. 247—250.
15. *Bogolyubov N.* // Proc. Steklov Inst. of Math. — 1988. — Iss. 2. — P. 3—45.
16. *Bogolyubov N.N. (jr.), Kovalevsky M.Yu., Kurbatov A.M. et al.* // Uspekhi Fiz. Nauk. — 1989. — **159**. — P. 585—620.
17. *Mineev V.P.* // Soviet Sci. Rev., Sect. A. — 1980. — **2**. — P. 173.
18. *Kovalevsky M.Yu., Rozhkov A.A.* // Teor. Mat. Fiz. — 1997. — **113**, N 2. — P. 313.

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КЛАСИФІКАЦІЯ СТАНІВ РІВНОВАГИ КВАНТОВОЇ РІДИНИ З ТЕНЗОРНИМ ПАРАМЕТРОМ ПОРЯДКУ

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Р е з ю м е

Проведено класифікацію рівноважних станів квантової рідини з d-спарюванням на основі концепції квазісередніх. Оператор параметра порядку подано у термінах фермі-операторів. Показано, що множина таких станів може бути кваліфікована у термінах квантового числа, що відповідає проекції орбітального момента куперівської пари на напрямок анізотропії. Знайдено явний вигляд трьох допустимих генераторів залишкової симетрії та відповідні рівноважні значення параметра порядку.