APPLICATION OF THE QUASICLASSICAL APPROXIMATION FOR THE ANALYSIS OF PROPERTIES OF LIGHT ATOMIC NUCLEI WITH HIGH EXCESS OF NEUTRONS

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The modified Thomas — Fermi method has been used for the computation of integral characteristics of the light nuclei with $Z = 4 \div 7$ placed near the neutron stability line. The basis of these calculations is that such nuclei are rather loose and nucleons move in smooth fields. For $N \approx Z$, the method leads to poor results, but it describes integral characteristics of the nuclei with high excess of neutrons with the same satisfactory precision as those of medium and heavy nuclei.

Nuclei with high excess of neutrons are β-nonstable and their life-time is too small, which hampers significantly their experimental study. However, the recent development of experimental methods allowed one to find a number of unusual properties of these nuclei which require a theoretical explanation.

For the description of lightest neutron-rich nuclei, the quantum microscopic and semimicroscopic approaches which are based on the idea of clusterization are usually used [1 — 11]. In their frame, the properties of nuclei ^{6}He, ^{8}He, and ^{11}Li were rather fully analyzed. In this case, especially in the use of semimicroscopic approaches where clusters are presented as structureless particles, one tries only to demonstrate the presence of a neutron halo in such nuclei and takes no care of the description of the integral characteristics of nuclei, including the binding energy. In addition, the calculations within these models are such complicated that their use is quite difficult for neutron-rich nuclei, beginning already from $Z = 4$.

In the region of medium and heavy nuclei, ordinary and exotic ones, most frequently are used the Hartree — Fock method, shell model, or Thomas — Fermi method. These methods are based on the idea of mean fields, in which the nucleons of a nucleus are moving, and are mainly realized in practice in the case of spherical symmetry. The first two approaches are employed sometimes for relatively light nuclei, whereas the latter is not practically used for nuclei with $A < 40$, though the number of nucleons does not enter the criterion of applicability of this quasiclassical method. In recent specific realizations of the Hartree — Fock and Thomas — Fermi methods, Skyrme forces are frequently used as the nucleon-nucleon interaction [12], which significantly simplifies calculations. Now a lot of parametrizations of Skyrme forces depending on the density is known [13 — 17]. Some of them were chosen to reproduce the integral characteristics of not only ordinary, but exotic heavy and medium nuclei.

In the present work, we use a nonlocal approximation of the modified Thomas — Fermi method (MTFM) with Skyrme forces for the description of the integral characteristics of light neutron-rich nuclei with $Z \geq 4$ which are in the vicinity of the neutron stability line. The comparison of the derived results with experimental data allows us to conclude on the applicability of such an approach in the above-indicated sense to the study of these nuclei and, in particular, the peculiarities of their structure.

Method

The equations of MTFM [15, 16]

$$\delta\varepsilon_n(\rho_n, \rho_p)/\delta\rho_p - \lambda_p = 0, \tag{1}$$

$$\delta\varepsilon(\rho_n, \rho_p)/\delta\rho_n - \lambda_n = 0 \tag{2}$$

can be deduced by using the variational principle, in which the total energy of a nucleus is considered as a functional $E(\rho_n, \rho_p)$ of the neutron ρ_n and proton ρ_p densities. The possibility of the construction of such a functional follows from the Hohenberg — Kohn theorem [18] valid for any many-component system. The functional of the total energy of a nucleus reads

$$E(\rho_n, \rho_p) = \int d\vec{r}(\tau + \varepsilon_{\rm pot} + \varepsilon_{\rm coul}), \tag{3}$$

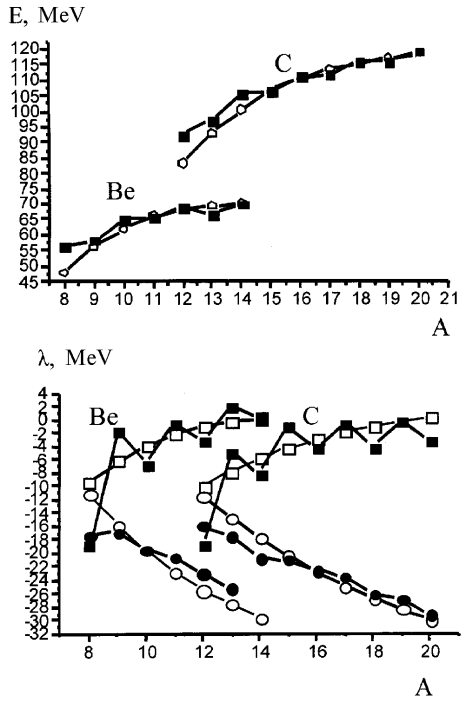

Fig. 1. Binding energies and neutron and proton chemical potentials for the isotopes of Be and C. Blank symbols correspond to the calculation with SLy4, and filled one are experimental data [19]

where τ, $\varepsilon_{\rm pot}$, and $\varepsilon_{\rm coul}$ are, respectively, the densities of kinetic, potential, and Coulomb energies. In Eqs. (1) and (2), λ_n and λ_p are Lagrange multipliers which are, in essence, the chemical potentials for neutrons and protons and are related to the conditions of preservation of the numbers of neutrons N and protons Z in the nucleus:

$$\int d\vec{r}\,\rho_{n(p)}(\vec{r}) = N(Z). \tag{4}$$

Given the expressions for the densities of kinetic, potential, and Coulomb energies in (3) which are presented, e.g., in works [19, 20], we can solve Eqs. (1) and (2) and find the chemical potentials and the distributions of the neutron and proton densities.

To within the terms of the second order in $\hbar$, the density of kinetic energy equals $\tau = \tau_{\rm TF} + \tau_2$, where $\tau = \tau_n + \tau_p$ is the sum of the densities of the kinetic energies of protons and neutrons. Here,

$$\tau_{{\rm TF},n(p)} = k\rho_{n(p)}^{5/3} \tag{5}$$

is the density of kinetic energy of neutrons (protons) in the Thomas — Fermi approximation, $k = \frac{5}{3}(3\pi^2)^{2/3}$, and $\tau_{2n(p)}$ is the gradient correction in $\hbar^2$ with regard for all possible gradient terms:

$$\tau_{2q} = b_1\frac{(\nabla\rho_q)^2}{\rho_q} + b_2\nabla^2\rho_q + b_3\frac{(\nabla f_q\rho_q)}{f_q} + b_4\rho_q\frac{\nabla^2 f_q}{f_q} +$$

$$+b_5\rho_q\left(\frac{\nabla f_q}{f_q}\right)^2 + b_6h_m^2\rho_q\left(\frac{W_0}{f_q}\right)^2, \tag{6}$$

where $b_1 = 1/36$, $b_2 = 1/3$, $b_3 = 1/6$, $b_4 = 1/6$, $b_5 = -1/12$, $b_6 = 1/2$, and W_0 are numerical coefficients, $h_m = \hbar^2/2m$, $f_{n(p)} = h_m(\beta+\gamma)\rho_{n(p)} + h_m\beta\rho_{p(n)}$, and the last term in (6) is connected with consideration of the spin-orbital interaction. Here, t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, α, and W_0 are the parameters of the Skyrme potential, β and γ are their combinations, and the variable q means n or p, respectively.

We do not give the full formula for the potential energy density because of its awkwardness and present only the Coulomb energy density with regard for the exchange term:

$$\varepsilon_{\rm Coul} = \frac{1}{2}e^2\rho_p(r)\int d\vec{r}\prime\,\frac{\rho_p(r\prime)}{|\vec{r}-\vec{r}\prime|} - \frac{3}{4}e^2\left(\frac{3}{\pi}\right)^{1/3}\rho_p^{4/3}(r).$$

As a result, Eqs. (1) and (2) transform into a system of nonlinear integro-differential equations in the case of spherical symmetry. The system is solved numerically by the method of successive approximations with the boundary conditions

$$\rho_q(r)\big|_{\to\infty} = 1/r^2\exp(-\sqrt{|\lambda_q|/(h_m b_1)}\,r),$$

and ρ_q should be limited at $r \to 0$.

Results

The integral characteristics of the light atomic nuclei with $Z \geq 4$ were calculated with the use of Skyrme forces depending on the density. We use the parametrizations Sly4 [17] and SkP [16] which are successfully used for the analysis of the properties of ordinary and exotic medium and heavy nuclei.

Fig. 1 presents the results of calculations of the binding energies and chemical potentials of the isotopes of Be and C with potential Sly4 (the results derived with the use of potential SkP are practically identical) as functions of the mass number. For the dependence of the binding energies on the mass number, the differences between the theoretical and experimental values for nuclei with the approximately identical numbers of neutrons and protons are very large. But, with increase in the mass number of isotopes, the theoretical curves become to reproduce the experimental data rather well. In this case, it is worth noting that nuclei ^{8}Be and

^{12}C in the ground state are composed, respectively, from two α-clusters which do not overlap practically and three well-separated α-clusters. The differences in the binding energies which are observed for nuclei with a great excess of neutrons can be related to not only the underestimation of shell effects, but also with the specificity of the determination of parameters of the used nucleon-nucleon potentials. The latter and the constant of spin-orbital interaction, in particular, were fitted by the binding energies of heavier nuclei.

As for chemical potentials, we note that they are described on the average. Respectively, it is impossible to exactly determine a position of the neutron stability line (if this term would be literally applied to light nuclei). Though the differences are not too high, but we can say nothing about the position of a specific isotope relative to the stability line, because this is defined, as usual, by quantum effects which are not considered to a full extent in the used approach.

Table 1 presents the results of calculations of the binding energies, neutron chemical potentials, proton chemical potentials, and neutron, proton, and mass mean square radii for the isotopes of nuclei of Be, B, C, and N which lie in the close vicinity of the neutron stability line. For the isotopes of the first two nuclei, the difference of the calculated and experimental energies can be several percents. But, for the heavier isotopes of C and N, this value is at most 1%, i.e., such as MTFM gives usually for medium and heavy nuclei. We note that the full account of gradient corrections for the kinetic energy of the nuclei under study plays a more significant role as compared to that for medium and heavy nuclei

T a b l e 1. Binding energies, neutron chemical potentials, and neutron, proton, and mass mean square radii

Nucleus	E, MeV	$E_{\exp}$, MeV	λ_n, MeV	$\lambda_{n,\exp}$, MeV	RMSN, fm	RMSP, fm	RMS, fm
^{12}Be	67.89	68.70	–0.95	–3.17	2.58	2.15	2.45
^{13}Be	69.14	66.64	–0.03	2.01	2.72	2.17	2.57
^{15}B	91.19	88.19	–0.96	–2.77	2.72	2.29	2.58
^{16}B	93.13	(88.10)	–0.12	(0.10)	2.81	2.31	2.67
^{18}C	115.40	115.67	–0.95	-4.18	2.83	2.41	2.70
^{19}C	116.99	115.83	–0.21	–0.16	2.92	2.44	2.78
^{21}N	139.66	138.79	–0.94	–4.61	2.94	2.53	2.81
^{22}N	141.45	140.01	–0.27	–1.22	3.02	2.54	2.88

N o t e: the values in parentheses are the results of nuclear systematics rather that experimental data [19].

T a b l e 2. Proton experimental mean square radii of the isotopes of Be [5]

Nucleus	^{7}Be	^{8}Be	^{9}Be	^{10}Be
RMSP, fm	2.36	—	2.34	2.24

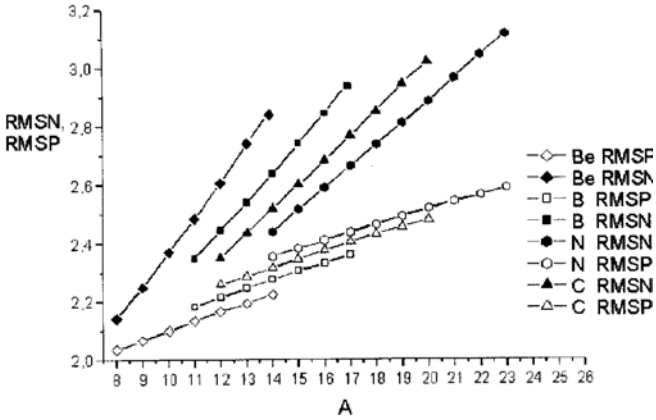

Fig. 2. Neutron, proton, and mass mean square radii vs the mass number for the isotopes of Be, B, C, and N

and can contribute up to several percents to the binding energy. The experience of calculations in the frame of MTFM in the region of medium and heavy nuclei shows that the values of mean square radii can be somewhat underestimated. This is related to the fact that the nucleon density distributions obtained in this approach decrease somewhat more rapidly in the diffusion region than it occurs in reality. That is, the derived values of mean square radii should be considered as some lower bounds of these quantities. Moreover, the large differences of neutron and proton radii can indicate the presence of a "neutron coat".

Fig. 2 displays the mean square radii versus mass number. The neutron radii increase rather rapidly by almost a linear law. But the proton radii vary slowly. Such a behavior of these quantities is characteristic of the isotopes of most nuclei. Though, for light nuclei near $N = Z$, the clearly pronounced clusterization inherent in them can give contribution. This is demonstrated by Table 2 containing the available experimental values of proton mean square radii for the isotopes of ^{8}Be. In addition, these data can indicate a certain compression of the substructure of ^{8}Be by excessive neutrons in the sufficiently heavy isotopes.

As the examples of the distribution of neutron and proton densities, Fig. 3 shows the plots for nuclei ^{16}B and ^{22}N. For these nuclei, the large excess of neutrons yields the longer neutron distributions as compared to the proton ones, which allows one to say about the presence of a "neutron coat".

Conclusions

A certain success of the spherical-symmetry-based quasiclassical calculations of the integral characteristics

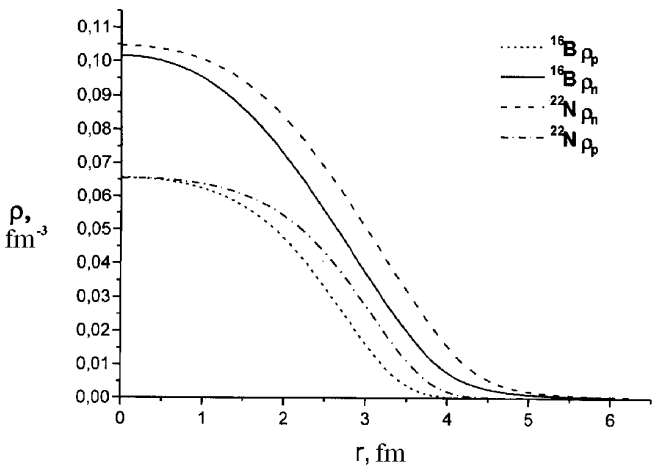

Fig. 3. Distributions of the densities of neutrons and protons in the nuclei of ^{16}B and ^{22}N

of light atomic nuclei lying in the close vicinity of the neutron stability line confirms that the nucleons in these nuclei move, indeed, in relatively smooth fields. That is, the clearly pronounced clusterization inherent in their isotopes with the approximately identical numbers of protons and neutrons is significantly leveled in the case where the number of neutrons strongly prevails. In this case, the nuclei whose basic isotopes are strongly deformed due to clusterization can take a form close to the spherical one. As for quantum effects which were practically neglected in the quasiclassical approximation used by us, they can be taken into account in the frame of the Hartree — Fock method whose quasiclassical limit is MTFM.

1. *Filippov G.F.* //Riv. Nuovo cim.— 1989. — **9**. — P.1.
2. *Wurzer J., Hofman H.M.* // Phys. Rev. C. — 1997. — **55**.— P.688.
3. *Suzuki Y., Varga K., Lovas R.G.*// Nucl. Phys. A.— 1994.— **571**.—P.447.
4. *Karataglidis S., Dortmans P.J., Amos K., Bennhold C.* Alternative evaluations of halos in nuclei (Nucl-th/9811045).
5. *Navratil P., Barrett B.R.*//Phys. Rev. C.—1998.— **57**.— P.3119.
6. *Zhukov M.V., Danilin B.V., Fedorov D.V. et al.*// Phys. Repts.— 1993.— **231**.— P.151
7. *Zhukov M.V., Danilin B.V., Ershov S.N. et al.*// Phys. Rev. C. — 1991.— **43**.— P.2835.
8. *Csoto A.* //Ibid.— 1993.— **48**.— P.165.
9. *Obayashy Y., Varga K., Suzuki Y.*//Ibid.—1994.— **50**.— P.189.
10. *Danilin B.V., Tompson I.J., Zhukov M.V. et al.*//ЯФ.— 2001.— **64**.— С.1290.
11. *Filippov G.F.*//Там же.— С.1311.
12. *Skyrme T.H.R.*//Nucl. Phys.— 1959.— **9**.— P.635.
13. *Nguyen Van Giai, Sagawa N.*// Nucl. Phys. A.— 1980.— **371**.— P.1
14. *Bartel J., Quentin P., Brack M. et al.*// Ibid.— 1982.— **386**.— P.79.
15. *Dobaczewski J., Flocard H., Treiner J.*//Ibid.— 1984.— **422**.— P.103.
16. *Tondeur F., Brack M., Farine M., Pearson J.M.*//Ibid.— **420**.— P.297.
17. *Chabanat E.* Interactions effectives pour des conditions extremes d'isospin: Thesisi 1995, Univ. Claude Bernard Lyon-1, LYCEN T 9501 (unpublished).
18. *Hohenberg P., Kohn W.*//Phys. Rev. B.— 1964.— **136**, N2.— P.864; *Brack M., Guet C., Hakanson H.-B.*//Phys. Repts.— 1985.— **123**.— P.275.
19. *Brack M., Bhaduri R.K.* Semiclassical Physics.— Addison-Wesley, 1997.
20. *Audi G., Wapstra A.H.*//Nucl. Phys. A.— 1993.— **565**.— P.1

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ЗАСТОСУВАННЯ КВАЗІКЛАСИЧНОГО НАБЛИЖЕННЯ ДЛЯ РОЗГЛЯДУ ВЛАСТИВОСТЕЙ ЛЕГКИХ АТОМНИХ ЯДЕР З ВЕЛИКИМ НАДЛИШКОМ НЕЙТРОНІВ

В. О. Нестеров

Р е з ю м е

Модифікований метод Томаса — Фермі використано для розрахунків інтегральних характеристик легких атомних ядер $Z = 4 \div 7$, що лежать у безпосередній близькості від межі нейтральної стабільності. Обгрунтуванням для проведення таких розрахунків може служити те, що розглядувані ядра є пухкими, і відповідно нуклони в них рухаються у досить гладеньких полях. Якщо у вказаній області заряду ядер при $N \approx Z$ даний метод є поганим наближенням, то при великому надлишку нейтронів він відтворює інтегральні характеристики ядер практично так само добре, як і при застосуванні його до дослідження середніх та важких ядер.