THERMODYNAMIC FUNCTIONS OF A RELATIVISTIC SYSTEM OF CHARGES IN THE RING-DIAGRAM APPROXIMATION

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A model of the gravitating electron gas with relativistic interaction in the first-order approximation in the coupling constant is considered. By means of the averaging of relativistic interaction over particle momenta, the effective potential is found. Using the standard diagram technique to the obtained potential, a partition function and thermodynamic functions are studied in the ring-diagram approximation.

Introduction

The traditional statistical mechanics that is based on the Gibbs distribution and the Liouville's equation needs the Hamiltonian formulation of the investigated system dynamics. Within the limits of such an approach to classical nonrelativistic systems of interacting particles, the powerful methods of diagrams [1–4] for the calculation of statistical sums, correlation functions, etc. were developed. The method of diagrams allows one to estimate the free energy and other thermodynamics functions of a system of charges with the long-range Coulomb interaction. At the same time, the first nonzero correction to the free energy describes a screening of charges and was found by summing ring diagrams as the contributions of pairwise interacting particles [2–4]. However, all fundamental interactions in physics are adequately described by field theory that has the relativistic and gauge nature. That is why, to apply the existing method of diagrams to a statistical description of systems with such interactions, it's needed to restate it in terms of canonical variables of particles. It turned out that, when you realize the transfer from field theory to the direct interaction theory, the potentials of interparticle relativistic interaction (unlike the nonrelativistic case) depends on canonical momenta of particles. However, we know nothing about the existence of any general regrouping scheme and the estimation of the diagrams for potentials depend on momenta.

In the pioneer work [5], the attempt to estimate the statistical sum of a system of charges with the Hamiltonian with a weakly relativistic Darwin interaction [6] by summing the series of ring-diagrams that are directly formed upon the potential dependent on momenta was made. Such a generalization of the nonrelativistic method of diagrams has faced with the problem of the free energy divergence. However, a number of free energy calculation methods [5,7,8] arose, that allowed one to avoid the mentioned divergence. Proposed calculation procedures cannot do without additional principles of the construction of relativistic statistical mechanics, which limits their applicability. On the other hand, the distinctions of underlying principles that have been proposed by the different authors came up for the discussion in the literature [9, 10].

Therefore, in the present work, we pick out the consequent approach that consists in applying the existing (nonrelativistic) method of diagrams to the effective potential obtained on the base of the relativistic potential by means of averaging over momenta, by concentrating at the calculation of the statistical sum and the thermodynamic functions of a relativistic system of charges with the Hamiltonian linear in the interaction.

1. Effective Potential

In the statistical investigation of a relativistic system of point charges, we make use of the results of works [11–13]. There, by means of the reduction of field degrees of freedom and taking relativistic kinematics into account accurately, Poincare-invariant Hamiltonians with a direct interparticle interaction in the first-order approximation in a coupling constant were found. In the general form, we write the Hamiltonian function of a system of N identical particles with a superposition of the electromagnetic interaction and the gravity interaction in such an approximation ($\sim e^2, G$) in terms of canonical particle variables (x_a^i, p_{ai}) as[1]

$$H = \sum_{a=1}^{N} p_a^0 + \frac{1}{2} {\sum_{a,b=1}^{N}}' U_{ab}, \quad p_a^0 = \sqrt{m^2 + \mathbf{p}_a^2}, \tag{1}$$

$$U_{ab} = \int \left[j_a^\mu a_\mu^b + \frac{1}{2} t_a^{\mu\nu} h_{\mu\nu}^b \right] d^3x =$$

$$= \frac{1}{V} \sum_{\mathbf{k}} \nu(\mathbf{k}) u_{ab} \mathrm{e}^{-i\mathbf{k}\mathbf{r}_{ab}}, \tag{2}$$

$$\mathbf{r}_{ab} = \mathbf{x}_a - \mathbf{x}_b, \quad \nu(\mathbf{k}) = \frac{4\pi}{\mathbf{k}^2}, \quad \mathbf{n} = \frac{\mathbf{k}}{|\mathbf{k}|}.$$

Here, j_a^μ is the partial free-particle 4-current of the a-th particle; $t_a^{\mu\nu}$ is the energy-momentum tensor of the free a-th particle; V is the system volume; the prime on the first sum indicates that $a \neq b$. The 4-potential of the electromagnetic field, a_μ^b, and one of the gravity field $h_{\mu\nu}^b$ that are produced by the particle b can be found from the corresponding field equations with a point source (see [11–13]). However, a specific form of the momentum part u_{ab} of the interparticle interaction potential (2) depends on a reduction method of the gauge freedom of electromagnetic field equations and gravity field equations. So, by applying the Lorentz gauge condition, we get

$$u_{ab} = \frac{p_b^0/p_a^0}{(p_b^0)^2 - (\mathbf{n}\mathbf{p}_b)^2} \left\{ e^2 p_a^\mu p_{b\mu} - G \left[2(p_a^\mu p_{b\mu})^2 - m^4 \right] \right\}. \tag{3}$$

By applying the Coulomb gauge, we get for the resultant momentum part of a potential:

$$u_{ab} = e^2 \left[1 - \frac{\mathbf{p}_a\mathbf{p}_b - (\mathbf{n}\mathbf{p}_a)(\mathbf{n}\mathbf{p}_b)}{(p_b^0)^2 - (\mathbf{n}\mathbf{p}_b)^2} \frac{p_b^0}{p_a^0} \right] -$$
$$-G \left\{ p_a^0 p_b^0 - (\mathbf{n}\mathbf{p}_a)(\mathbf{n}\mathbf{p}_b) + \right.$$
$$+ \left(2[\mathbf{p}_a^2 - (\mathbf{n}\mathbf{p}_a)^2] + \frac{[\mathbf{p}_a\mathbf{p}_b - (\mathbf{n}\mathbf{p}_a)(\mathbf{n}\mathbf{p}_b)]^2}{(p_b^0)^2 - (\mathbf{n}\mathbf{p}_b)^2} \right) \frac{p_b^0}{p_a^0} -$$
$$\left. -4[\mathbf{p}_a\mathbf{p}_b - (\mathbf{n}\mathbf{p}_a)(\mathbf{n}\mathbf{p}_b)] \right\}. \tag{4}$$

Regardless of external differences of the obtained potentials, the obtained Hamiltonians are physically equivalent, which follows from the independence of values of the observable quantities on the choice of a gauge, on a gauge freedom reduction method [14]. Within the limits of the direct interaction theory, a physical equivalence means that the Hamiltonians from (3) and (4) are connected by a canonical transformation (see. [12, 13]). For example, a statistical sum (a free energy) is a gauge-invariant quantity [15] therefore, the Hamiltonians with momentum parts like (3) and (4) must result in equal values of statistical and thermodynamic quantities.We write the statistical sum of a weakly non-ideal system of N identical particles as

$$Z_N = \frac{1}{N!} \int \mathrm{e}^{-\beta H} \prod_{a=1}^{N} \frac{d^3x_a d^3p_a}{(2\pi)^3}, \tag{5}$$

where $\beta^{-1} = T$ is the system temperature.

Now, to find the effective potential, we integrate over canonical momenta of particles. We represent expression (5) as

$$Z_N = \frac{Z_N^{\mathrm{id}}}{V^N} \int \langle \mathrm{e}^{-\beta U} \rangle \prod_{a=1}^{N} d^3x_a, \tag{6}$$

where the statistical sum Z_N^{id} of free relativistic particles looks like [16]

$$Z_N^{\mathrm{id}} = \frac{1}{N!} (\xi V)^N, \quad \xi = \int \mathrm{e}^{-\beta p_a^0} \frac{d^3p_a}{(2\pi)^3}, \tag{7}$$

and the average over momenta (with the distribution function of free particles) is given by the formula

$$\langle \mathrm{e}^{-\beta U} \rangle = \frac{1}{\xi^N} \int \mathrm{e}^{-\beta(H_0 + U)} \prod_{a=1}^{N} \frac{d^3p_a}{(2\pi)^3}; \tag{8}$$

$$H_0 = \sum_{a=1}^{N} p_a^0, \quad U = \frac{1}{2} {\sum_{a,b=1}^{N}}' U_{ab}.$$

For the analysis of properties of the average and its calculation, it is convenient to introduce a symmetrized pairwise interaction potential $V_{ab} (= V_{ba})$:

$$V_{ab} = \frac{1}{2}(U_{ab} + U_{ba}) = \frac{1}{V} \sum_{\mathbf{k}} \nu(\mathbf{k}) v_{ab} \mathrm{e}^{-i\mathbf{k}\mathbf{r}_{ab}}. \tag{9}$$

Moreover, it is obvious that

$$U = \frac{1}{2} {\sum_{a,b=1}^{N}}' U_{ab} = \frac{1}{2} {\sum_{a,b=1}^{N}}' V_{ab}.$$

[1] The Greek indices $\mu, \nu, \ldots$ run from 0 to 3; the Latin indices i, j, k run from 1 to 3. The summation over such a type of indices is performed according to conventions with the metric $\|\eta_{\mu\nu}\| = \mathrm{diag}(1, -1, -1, -1)$. The indices a, b denote particles and run from 1 to N. Light speed is $c = 1$.

In terms of symmetrized potential V_{ab}, the interaction between two particles a and b can be interpreted as an interaction by means of a virtual photon loop and a graviton loop. A similar one-loop interaction appears in quantum field theory in a first-order approximation in the coupling constant, which provides a general nature of the fundamental interactions under study. We represent the symmetrized momentum part v_{ab} of the potential in the quadratic form:

$$v_{ab} = f_a^{\bar{\alpha}} A_{\bar{\alpha}\bar{\beta}} f_b^{\bar{\beta}}. \qquad (10)$$

Here, $f_a^{\bar{\alpha}}$ is "a vector" dependent on the canonical impulse $\mathbf{p}_a$ only; $A_{\bar{\alpha}\bar{\beta}}$ is a constant symmetric matrix; $\bar{\alpha}$, $\bar{\beta}$ are multiindices. A presentation v_{ab} by expression (10) is not unambiguous. One of the possible definitions of $f_a^{\bar{\alpha}}$ and $A_{\bar{\alpha}\bar{\beta}}$ for the impulse part (3) is shown in Appendix.

Expression (10) can be rewritten as

$$v_{ab} = \frac{1}{2}\Big[f_a^{\bar{\alpha}} A_{\bar{\alpha}\bar{\beta}} f_a^{\bar{\beta}} + f_b^{\bar{\alpha}} A_{\bar{\alpha}\bar{\beta}} f_b^{\bar{\beta}} -$$
$$-(f_a^{\bar{\alpha}} - f_b^{\bar{\alpha}}) A_{\bar{\alpha}\bar{\beta}} (f_a^{\bar{\beta}} - f_b^{\bar{\beta}})\Big] =$$
$$= \frac{1}{2}\Big[u_{aa} + u_{bb} - (f_a^{\bar{\alpha}} - f_b^{\bar{\alpha}}) A_{\bar{\alpha}\bar{\beta}} (f_a^{\bar{\beta}} - f_b^{\bar{\beta}})\Big]. \qquad (11)$$

After the direct substitution (3) and (4), it turned out that the quantity u_{aa} is the same for initial Hamiltonians:

$$u_{aa} = (e^2 - Gm^2)\frac{m^2}{(p_a^0)^2 - (\mathbf{n}\mathbf{p}_a)^2}. \qquad (12)$$

Therefore, we present the momentum part v_{ab} as

$$v_{ab} = v_{ab}^{\rm inv} + v_{ab}^{\rm gauge}, \qquad (13)$$

where

$$v_{ab}^{\rm inv} = \frac{1}{2}(u_{aa} + u_{bb}),$$

$$v_{ab}^{\rm gauge} = -\frac{1}{2}(f_a^{\bar{\alpha}} - f_b^{\bar{\alpha}}) A_{\bar{\alpha}\bar{\beta}} (f_a^{\bar{\beta}} - f_b^{\bar{\beta}}). \qquad (14)$$

The component $v_{ab}^{\rm inv}$ is an invariant with respect to the group of a canonical transformations, which defines the class of equivalent Hamiltonians (1). At the same time, the component $v_{ab}^{\rm gauge}$ of the momentum part of the potential depends upon a fixation technique of a gauge freedom of field equations and changes relatively to the indicated group of canonical transformations. It's easy to see that $v_{ab}^{\rm gauge} < 0$ and $|v_{ab}^{\rm gauge}| < |v_{ab}^{\rm inv}|$. Moreover, the quantity $v_{ab}^{\rm gauge}$ disappears in the nonrelativistic limit (where the dependence of the interaction on momenta vanishes). In view of the fact that statistical sum shouldn't depend on the choice of gauge conditions, we must eliminate the contribution to the interaction that is determined by $v_{ab}^{\rm gauge}$, using a canonical transformation or on the ground of a smallness of $v_{ab}^{\rm gauge}$ as compared to $v_{ab}^{\rm inv}$.

Thus, we find the effective potential of an interparticle interaction from average (8) by keeping only $v_{ab}^{\rm inv}$ in v_{ab}:

$$W = -\frac{1}{\beta}\ln\langle {\rm e}^{-\beta U}\rangle = -\frac{1}{\beta}\ln\left(1 - \beta\langle U\rangle + ...\right). \qquad (15)$$

In the first-order approximation in an interaction constant by which we are limited, W coincides with $\langle U\rangle$. Therefore,

$$W = \frac{1}{2}\sum_{a,b=1}^{N}{}' W_{ab}, \quad W_{ab} = \frac{1}{V}\sum_{\mathbf{k}} \nu(\mathbf{k})\langle u_{aa}\rangle {\rm e}^{-i\mathbf{k}\mathbf{r}_{ab}}. \qquad (16)$$

We introduce an effective coupling constant $Q^2 = \langle u_{aa}\rangle$. Direct calculations give

$$Q^2(m\beta) = (e^2 - Gm^2)\frac{K_0(m\beta)}{K_2(m\beta)}, \qquad (17)$$

where $K_n(z)$ is the McDonald's function. Here, the electromagnetic effective interaction and the gravity effective interaction are characterized by the common function of $m\beta$. This fact can be explained by a similarity of the corresponding field equations in the first-order approximation in a coupling constant [12, 13].

Let us analyze the limits of a range of $Q^2(m\beta)$:

$$\lim_{z\to 0} Q^2(z) = 0, \quad \lim_{z\to\infty} Q^2(z) = e^2 - Gm^2.$$

The first case corresponds to the limit $T \to \infty$. That is, the interaction between particles goes down at high temperatures, and a difference between the electromagnetic and gravity interactions disappears at the same time. However, in our approach the temperature values permitted must be bounded above by the condition $T < m$. The second case is the nonrelativistic limit ($c \to \infty$).

When examining the model where the kinematic part of a Hamiltonian is relativistic, but an interaction is nonrelativistic, one needs to set $Q^2 = e^2 - Gm^2$ with regard to the previous formula.

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2. Thermodynamic Functions

After integrating over canonical momenta in the expression for the statistical sum (5) of a weakly nonideal single-sorted system of particles (an electron gas) with the electromagnetic and gravity interactions, we represent Z_N as

$$Z_N = Z_N^{\rm id} Q_N, \tag{18}$$

where

$$Q_N = \frac{1}{V^N} \int {\rm e}^{-\beta W} \prod_{a=1}^{N} d^3 x_a \tag{19}$$

is a configuration integral obtained from the expression for the effective potential (16).

To find the Helmholtz free energy $F = -T \ln Z_N$, we use its representation as a series taken from [4]

$$F = F^{\rm id} + F' + F'' - \frac{1}{\beta}(B_2 + B_3 + \ldots). \tag{20}$$

Here,

$$F^{\rm id} = -T \ln Z_N^{\rm id} \tag{21}$$

is the Helmholtz free energy of non-interacting relativistic particles; the correction F' corresponds to the high-temperature approximation for an interaction (under an uniform distribution of particles); F'' characterizes the work on a charge screening and consists of a ring diagram series; B_n are virial coefficients.

In the present work, we will focus on the determination of the corrections F', F'' only, neglecting a contribution of virial coefficients. Since the charge system's statistical equilibrium description is available because of the screening effect [3], first of all, let us investigate the correction F''. For a effective pairwise interaction potential which in the coordinate representation and the Fourier representation looks like

$$W(r) = \frac{1}{V} \sum_{\mathbf{k}} \nu(\mathbf{k}) Q^2(m\beta) {\rm e}^{-i\mathbf{kr}} = \frac{Q^2(m\beta)}{r}, \tag{22}$$

a value of the quantity F'' can be obtained by replacing the coupling constant e^2 with $Q^2(m\beta)$ at the corresponding known contribution for the Coulomb plasma [2–4]. We get

$$F'' = -\frac{æ^3 V}{12\pi\beta} \left[\left(1 - \frac{r_g}{r_0}\right) \frac{K_0(m\beta)}{K_2(m\beta)} \right]^{3/2}, \tag{23}$$

where $æ^2 = 4\pi e^2 \beta N/V$ is the Debye parameter; $r_0 = e^2/m$ is the classical radius of an electron, and $r_g = Gm$ is its gravitational radius.

Equation (23) yields the condition for a statistical equilibrium description applicability to the systems with the gravity interaction: $r_0 > r_g$, i.e. gravitationally collapsing objects can't be described by means of equilibrium statistics. The given condition is met automatically in the case of an electron gas ($r_g/r_0 \sim 10^{-6}$).

In addition, we note that, even in the nonrelativistic case, a statistical sum computation for a system of gravitational objects (in the absence of a repulsion that has an electromagnetic field nature) loses a physical sense, as far as such a system can't be in equilibrium because of a collapse possibility. On a calculative level, this appears in a divergence of integrals, as shown in [17]. That is why for systems, where the gravity interaction dominates, it is necessary to use the formalism of non-equilibrium statistical mechanics [18]. It is possible to describe the processes, which are the subjects of astrophysical investigations [19] with its help.

Another expression for F'' is based on using a screened potential [4]:

$$G(r) = \frac{1}{V} \sum_{\mathbf{k}} G(\mathbf{k}) {\rm e}^{-i\mathbf{kr}},$$

$$G(\mathbf{k}) = \frac{4\pi Q^2(m\beta)}{\mathbf{k}^2 + 4\pi Q^2(m\beta)\beta N/V}, \tag{24}$$

which is in accord with the Ornstein-Zernike equation

$$-\beta G(r_{12}) = -\beta W(r_{12}) + \beta^2 \frac{N}{V} \int W(r_{13}) G(r_{32}) d^3 r_3. \tag{25}$$

Then

$$F'' = \frac{1}{2} \frac{N^2}{V} \int d^3 r \int_0^1 d\lambda W(r) G(r, \lambda), \tag{26}$$

where $G(r,\lambda)$ is obtained from $G(r)$ by replacing $Q^2(m\beta)$ with $\lambda Q^2(m\beta)$; λ is the interaction switching parameter.

If we neglect the gravity interaction ($G = 0$), we will obtain an equation for the Helmholtz free energy in the ring-diagram approximation

$$F = F^{\rm id} - \frac{æ^3 V}{12\pi\beta} \left(\frac{K_0(m\beta)}{K_2(m\beta)} \right)^{3/2}, \tag{27}$$

because $F' = 0$ for electromagnetic interaction. A lack of F' comes from the electrical neutrality condition, which requires the introduction of an effective background with an opposite sign while considering a single-sorted charge system [3]. Note that the zero value of the correction F' follows rigorously when considering a two-component system of opposite charges [4].

One should take into account that, in our approach which is based on the application of the standard method of diagrams to an effective potential, a finit value of the Helmholtz free energy for a system of electromagnetically interacting particles is obtained without additional construction principles of relativistic statistical mechanics, in contrast to approaches in [7,8].

Returning to the consideration of an electron gas with the gravity interaction, according to the definition of F' (see [4]), we must write

$$F' = \frac{1}{2}\frac{N^2}{V}\int\left(-\frac{Gm^2}{r}\frac{K_0(m\beta)}{K_2(m\beta)}\right)d^3r. \qquad (28)$$

If we integrate over all the space, a nonzero value and, generally speaking, an infinite value of the correction F' is caused by the absence of repulsion between masses in nature. However, in a weakly nonideal electron gas, when the average free particle energy is higher than the interaction energy (where electrostatic repulsion forces give the main contribution), values of F' must be limited by a proper (physical) choice of the limits of integration.

Assuming that gravitation forces take part only in the formations of a charge screening, we rewrite the last as formula

$$F' = -\frac{1}{2}\frac{N^2}{V}\frac{K_0(m\beta)}{K_2(m\beta)}\int\limits_0^d \frac{4\pi Gm^2}{r}r^2dr = -\frac{1}{4}NT\frac{r_g}{r_0}, \qquad (29)$$

where $d^{-2} = æ^2K_0(m\beta)/K_2(m\beta)$. One can easily see that, in the absence of the electromagnetic interaction when $r_0 \to 0$, $F' \to \infty$ due to Eq. (28) and conclusions from [17].

After substitution (29), (23) into (20), we finally obtain the Helmholtz free energy of a gravitating electron gas:

$$F = F^{\rm id} - \frac{1}{4}NT\frac{r_g}{r_0} - \frac{æ^3V}{12\pi\beta}\left[\left(1-\frac{r_g}{r_0}\right)\frac{K_0(m\beta)}{K_2(m\beta)}\right]^{3/2}. \qquad (30)$$

It is clear that this expression loses meaning in the absence of the electromagnetic interaction.

On base of Eq. (30) for the Helmholtz free energy, we obtain the average value of the energy and pressure for an electron gas:

$$E = E^{\rm id} - \frac{æ^3V}{8\pi\beta}\left[\left(1-\frac{r_g}{r_0}\right)\frac{K_0(m\beta)}{K_2(m\beta)}\right]^{3/2} \times$$

$$\times\left\{3 - \frac{2K_1^2(m\beta)}{K_2(m\beta)K_0(m\beta)}\right\}, \qquad (31)$$

$$P = \frac{TN}{V}\left\{1 - \frac{e^2\beta}{6}æ\left[\left(1-\frac{r_g}{r_0}\right)\frac{K_0(m\beta)}{K_2(m\beta)}\right]^{3/2}\right\}. \qquad (32)$$

If $r_g = 0$, the formulae obtained result in the non-relativistic limit in the well-known values of thermodynamic functions for the Coulomb plasma [3].

APPENDIX

The momentum part v_{ab} of a symmetrized relativistic potential can be represented as a quadratic form

$$v_{ab} = f_{a(\xi)(\alpha)}^{[\mu][\nu]} A_{[\mu\lambda][\nu\sigma]}^{(\xi\zeta)(\alpha\beta)} f_{b(\zeta)(\beta)}^{[\lambda][\sigma]}, \qquad (A.1)$$

where the indices $\xi,\zeta = \overline{1,2}$ number interactions — the electromagnetic and gravity ones; the indices $\alpha,\beta = \overline{1,2}$ correspond to the subjects of interactions — "current"and "field".

For an interaction written on the base of Eq. (3), the vector components f_a and the matrix A are defined as

$$f_{a(1)(1)}^{[\mu][\nu]} = \frac{p_a^\mu}{p_a^0}\ell^\nu, \quad f_{a(1)(2)}^{[\mu][\nu]} = \frac{p_a^\mu p_a^0}{(p_a^0)^2 - (\mathbf{n}\mathbf{p}_a)^2}\ell^\nu, \qquad (A.2)$$

$$f_{a(2)(1)}^{[\mu][\nu]} = \frac{p_a^\mu p_a^\nu}{p_a^0}, \quad f_{a(2)(2)}^{[\mu][\nu]} = \frac{2p_a^\mu p_a^\nu - \eta^{\mu\nu}m^2}{(p_a^0)^2 - (\mathbf{n}\mathbf{p}_a)^2}p_a^0, \qquad (A.3)$$

$$A_{[\mu\lambda][\nu\sigma]}^{(\alpha\beta)(\xi\zeta)} = \frac{1}{2}g_\xi\delta_{\xi\zeta}\eta_{\mu\lambda}\eta_{\nu\sigma}\delta(3-\alpha-\beta), \qquad (A.4)$$

where ℓ^μ is an arbitrary constant 4-vector for which $\ell^\mu\ell_\mu = 1$; $g_1 = e^2$, $g_2 = -G$.

For short, the notations of indices' assembly are denoted as

$$\bar\alpha = \{(\xi)(\alpha)[\mu][\nu]\}, \quad \bar\beta = \{(\zeta)(\beta)[\lambda][\sigma]\}. \qquad (A.5)$$

Then we come to expression (10), that is,

$$v_{ab} = f_a^{\bar\alpha}A_{\bar\alpha\bar\beta}f_b^{\bar\beta}, \quad A_{\bar\alpha\bar\beta} = A_{\bar\beta\bar\alpha}. \qquad (A.6)$$

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ТЕРМОДИНАМІЧНІ ФУНКЦІЇ РЕЛЯТИВІСТСЬКОЇ СИСТЕМИ ЗАРЯДІВ У НАБЛИЖЕННІ КІЛЬЦЕВИХ ДІАГРАМ

А.Назаренко

Резюме

Розглянуто модель гравітуючого електронного газу з релятивістською взаємодією в лінійному наближенні за константою взаємодії. За допомогою усереднення релятивістської взаємодії за імпульсами частинок знайдено ефективний потенціал. Застосовуючи стандартну діаграмну техніку до одержаного потенціалу, досліджено статистичну суму та термодинамічні функції у наближенні кільцевих діаграм.