EXACT SOLUTIONS FOR TWO-COMPONENT COSMOLOGICAL MODELS IN THE EINSTEIN – CARTAN THEORY

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In the framework of the Einstein–Cartan theory, closed homogeneous isotropic cosmological models with nonminimal coupled scalar field and ultrarelativistic gas are considered. General exact solutions are obtained for an arbitrary coupling constant. It is shown that singular models and a countable number of nonsingular ones are possible. For the obtained solutions, restrictions on the coupling constant are found. The role of a ultrarelativistic gas in the evolution of models is elucidated.

Introduction

In the framework of the problem of existence of exact cosmological solutions in the Einstein–Cartan theory with scalar field [1–6], closed homogeneous isotropic models with nonminimally coupled scalar field and ultrarelativistic gas are considered.

The interest to a nonminimally coupled scalar field in relativistic theories of gravity has been aroused by a number of circumstances: its role in scalar-tensor theories; its presence in GUT models and Kaluza–Klein theories; in connection with the Higgs mechanism and inflationary cosmology.

The account of the ultrarelativistic gas as an additional source of the gravitational field is caused by the fact that the Universe, generally speaking, is a multicomponent system, and the earlier results (see, e.g., [7–9]) obtained in the framework of General Relativity proved the importance of this component in the evolution of cosmological models.

Earlier in [4, 5], general exact cosmological solutions of the Einstein–Cartan equations for one- and two-component models with the ultrarelativistic gas, material scalar field ($\alpha_s = +1$), and "gravitational" one ($\alpha_s = -1$) [4–6, 10] have been obtained at a fixed value of the coupling constant: $\xi = 1/6$. In particular, it was shown that the system is not integrable for closed models containing only a scalar field for $\alpha_s = -1$, $\xi = 1/6$. In the present paper, we investigate the case of an arbitrary coupling ξ and the influence of this parameter on the character of the evolution of cosmological models.

1. Basic Equations

The Lagrangian Ł of the model is chosen as follows:

$$Ł = -\frac{R(\Gamma)}{2æ} + \frac{\alpha_s}{8\pi}\Big[\Phi_{,k}\Phi^{,k} + \xi R(\Gamma)\Phi^2\Big] + Ł_p\ , \qquad (1)$$

where $R(\Gamma)$ is the curvature scalar obtained from the full connection $\Gamma^k_{ij} = \{^{\,k}_{ij}\} + S_{ij\cdot}^{\ \ k} + S^k_{\cdot ij} + S^k_{\cdot ji}$; $\{^{\,k}_{ij}\}$ are Christoffel symbols of the second kind; $S_{ij\cdot}^{\ \ k} = \Gamma^k_{[ij]}$ is the torsion tensor; $æ = 8\pi G/c^4$ is Einstein's constant; $Ł_p$ is the Lagrangian of the ultrarelativistic gas.

The metric g_{ik} has signature $(-\ ,-\ ,-\ ,+\ )$, the Riemann and Ricci tensors are defined as $R_{ijk\cdot}^{\ \ \ m} = \Gamma^m_{jk,i} - \Gamma^m_{ik,j} + \Gamma^m_{ip}\Gamma^p_{jk} - \Gamma^m_{jp}\Gamma^p_{ik}$, $R_{jk} = R_{ijk\cdot}^{\ \ \ i}$. It follows from (1) that the torsion can interact with a scalar field only through its trace: $S_i = S_{ik\cdot}^{\ \ k}$ [6]. Hence, the curvature scalar can be presented as

$$R(\Gamma) = R(\{\}) + 4\nabla_k S^k - \frac{8}{3}S_k S^k\ , \qquad (2)$$

where $R(\{\})$ is the Riemannian part of the curvature built from Christoffel symbols; ∇_k is the covariant derivative in the Riemannian space.

We note that Lagrangian (1) in the torsionless case at $\xi = 1/6$ conforms to a conformally invariant scalar field. As shown in [6], when $\alpha_s = -1$, $\xi = -1/6$, the scalar field corresponding to Lagrangian (1) is the axion field in General Relativity. From the viewpoint of QCD, the interest to the axion field is based on the fact that it leads to a compensation of strong CP violation effects; from the viewpoint of cosmology, it is a cold dark matter candidate (see, e.g., [6, 11] and references therein).

Varying the action with Lagrangian (1) in g_{ij}, S_k, Φ, we obtain the following set of equations for the gravitational fields and matter:

$$G_{ij}(\{\}) = æ(T^p_{ij} + T^s_{ij}) + \Lambda_{ij} \ , \qquad (3)$$

$$S^k = (3/2)\xi\Psi\Phi\Phi^{,k} \ , \qquad (4)$$

$$\Box\Phi - \xi\Phi R(\Gamma) = 0 \ , \qquad (5)$$

where

$$T^p_{ij} = (\varepsilon_p + P_p)u_i u_j - P_p g_{ij} \ , \qquad (6)$$

$$T^s_{ij} = \frac{\alpha_s}{4\pi}\Big\{\Phi_{,i}\Phi_{,j} - \frac{1}{2}\Big[\Phi_{,m}\Phi^{,m} + \xi R(\{\})\Phi^2\Big]g_{ij} +$$
$$+\xi\Big[-2S_i\nabla_j - 2S_j\nabla_i + 2g_{ij}S^n\nabla_n -$$
$$-\nabla_i\nabla_j + g_{ij}\Box + R_{ij}(\{\}) - \Lambda_{ij}\Big]\Phi^2\Big\} \ , \qquad (7)$$

$$\Lambda_{ij} = \frac{8}{3}S_i S_j - \frac{4}{3}S_k S^k g_{ij} \ . \qquad (8)$$

Here, ε_p is the ultrarelativistic gas energy density; P_p is its pressure; u_i is the four-velocity; $\Box$ is the D'Alembertian operator in the Riemannian space, and $\Psi = æ(4\pi\alpha_s - \xiæ\Phi^2)^{-1}$.

For closed homogeneous isotropic models with the metric

$$ds^2 = a^2(\eta)[-dr^2 - \sin^2(r)(d\theta^2 + \sin^2\theta d\varphi^2) + d\eta^2], \qquad (9)$$

Eqs. (3) and (5) take the form:

$$2\frac{a''}{a} - \frac{a'^2}{a^2} + 1 = \Psi\Big[2\xi\Phi\Phi'' + 2\xi\frac{a'}{a}\Phi\Phi' +$$
$$+(-\frac{1}{2} + 2\xi + 3\xi^2\Phi^2\Psi)\Phi'^2\Big] - 4\pi\alpha_s\Psi P_p a^2 \ , \qquad (10)$$

$$\frac{a'^2}{a^2} + 1 = \Psi\Big[(\frac{1}{6} - \xi^2\Phi^2\Psi)\Phi'^2 + 2\xi\frac{a'}{a}\Phi\Phi'\Big] +$$
$$+\frac{4}{3}\pi\alpha_s\Psi a^2\varepsilon_p \ , \qquad (11)$$

$$(1 - 6\xi^2\Phi^2\Psi)(\Phi'' + 2\frac{a'}{a}\Phi') +$$
$$+6\xi(\frac{a''}{a} + 1)\Phi - 24\piæ^{-1}\alpha_s\xi^2\Psi^2\Phi\Phi'^2 = 0 \ , \qquad (12)$$

where the prime denotes differentiation with respect to η.

For the ultrarelativistic gas with metric (9), the following relations are true:

$$P_p = \varepsilon_p/3 \ , \quad \varepsilon_p = C_p a^{-4} \ , \quad C_p = \mathrm{const} \ . \qquad (13)$$

Note that, to solve the field equations, the requirement of positivity of the Einstein's effective constant was imposed:

$$æ_{\mathrm{eff}} = æ\{1 - (\alpha_s\xi/4\pi)æ\Phi^2\}^{-1} > 0 \ . \qquad (14)$$

2. Solutions with Material Scalar Field

2.1. Solution for $\xi > 0$

The exact general solution may be written as

$$a^2(\eta) = AF(\eta)\cos^{-2}u(\eta) \ , \quad \Phi(\eta) = B\sin u(\eta) \ , \qquad (15)$$

where $A = æ^{1/2}C_1/24\pi$, $F(\eta) = \sin 2(\eta + C_3) + \sigma\cos^2(\eta + C_3)$; $u(\eta) = (\lambda/2)\ln[C_2\lambda(2\,\mathrm{tg}(\eta + C_3) + \sigma)]$; $\sigma = 8\piæ^{1/2}C_pC_1^{-1}$; $\lambda = (6|\xi|)^{1/2}$; $B = (4\pi/æ|\xi|)^{1/2}$; C_1, C_2, C_3 are the integration constants ($C_1 > 0$), $2\,\mathrm{tg}(\eta + C_3) + \sigma > 0$.

A characteristic feature of solution (15) is that, for the scale factor $a(\eta)$, the limit for $2\,\mathrm{tg}(\eta + C_3) + \sigma \to 0$ does not exist. It is possible to exclude the constant C_3 by means of a displacement of coordinates. Note that, for $a(\eta)$, there are the points of discontinuities

$$\eta_m = \mathrm{arctg}\,\chi \ , \qquad (16)$$

where $\chi = [\exp((1 + 2m)\pi/\lambda)/\lambda C_2 - \sigma]/2$, $m = 0, \pm1, \pm2, \ldots$.

Therefore, under the transition to a synchronous own time t ($a(\eta)d\eta = cdt$), we can integrate over the intervals $\eta_m < \eta < \eta_{m+1}$. It is not difficult to show that the corresponding integrals diverge logarithmically. That is, for each interval $\eta_m < \eta < \eta_{m+1}$, we have a self-dependent cosmological model. Thus, solution (15) allows the countable number of cosmological models.

It is easy to verify that solution (15) describes nonsingular models in view of the fact that the strong energy condition is broken for minima of the scale factor:

$(\varepsilon_{\min} + 3P_{\min}) < 0$, where ε and P are, respectively, the energy density and pressure of a system.

Near the points of discontinuities η_m, the following asymptotics are true:

$$a|_{t\to\pm\infty} \sim \exp(\pm H_m t) \ , \quad \Phi|_{t\to\pm\infty} \simeq (-1)^m B \ , \qquad (17)$$

where $H_m = c\lambda^{5/4} C_2^2 A^{-1/2} e^{-3(1+2m)\pi/2\lambda}(1+\chi^2)^{3/2}$.

It is not difficult to show that, for minima of the scale factor and near the points of discontinuities, the contribution of the scalar-torsion field dominates. It is worth to note that the scalar-torsion field creates the effect similar to a curvature [12], since the evolution of models is characteristic of ones of the open types.

It is easy to see that, at fixed values of the parameter ξ, the Hubble‘s constant increases with m. As the models are considered in the framework of the classical theory of gravitation, they will be physically permissible provided that $\varepsilon < \varepsilon_{pl}$, where $\varepsilon_{pl} = c^7/\hbar G^2$. Consequently, the following restriction on m is valid at a fixed value of ξ: $m < m_{pl} = (\lambda/6\pi)\ln[(4C_2)^3 æ\lambda\varepsilon_{pl} A/3]$. The analysis showed that, at fixed values of m, the Hubble‘s constant can take great values at $\xi \ll 1$ and $\xi \gg 1$. As a result, we get the restrictions on ξ. For $\xi \ll 1$, we have $3\exp[3(1+2m)\pi(6\xi)^{-1/2}] < æA(4C_2)^3(6\xi)^{1/2}\varepsilon_{pl}$, and, for $\xi \gg 1$, we obtain $\xi < 6^{-1}(æA\varepsilon_{pl}/3C_2^3)^{2/5}(1 + 16æ\pi^2 C_p^2 C_1^{-2})^{-6/5}$.

2.2. Solution for $\xi < 0$

The exact general solution differs from solution (15) by the replacement of trigonometric functions of u by hyperbolic ones:

$$a^2(\eta) = AF(\eta)\cosh^{-2} u(\eta) \ , \quad \Phi(\eta) = B\sinh u(\eta) \ . \quad (18)$$

Solution (18) describes singular models with asymptotics

$$a|_{t\to 0} \sim t^{\gamma(1+\lambda)} \ , \quad \Phi|_{t\to 0} \sim t^{-\lambda\gamma} \ ,$$

$$a|_{t\to t_0} \sim (t_0 - t)^{\gamma(1+\lambda)} \ , \quad \Phi|_{t\to t_0} \sim -(t_0 - t)^{-\lambda\gamma} \ , \quad (19)$$

where $\gamma = (3+\lambda)^{-1}$.

It follows from (19) that the parameter ξ determines the asymptotics of the scale factor for $t \to 0 : a|_{|\xi|\ll 1} \sim t^{1/3}$, $a|_{|\xi|\gg 1} \sim t$ (precisely the same asymptotics are valid for $t \to t_0$).

The analysis showed that the scale factor has a sole maximum. It is interesting to observe that, under the condition that $(4+\sigma^2)\lambda^2 C_2^2 = 1$, $a_{\max}$ does not depend on ξ: $a_{\max} = \{2A((4+\sigma^2)^{1/2} - \sigma)^{-1}\}^{1/2}$. In this case, the account of the ultrarelativistic gas results in the increase in $a_{\max}$: $a_{\max}(C_p = 0) < a_{\max}(C_p \neq 0)$.

It follows from (15) and (18) that, for $\alpha_s = +1$, the presence of the ultrarelativistic gas does not tell decisively about the qualitative character of the evolution of models.

3. Solutions with “Gravitational” Scalar Field

3.1. Solution for $\xi > 0$

The exact general solution may be written as follows:

$$a^2(\eta) = A_1(1 + V^2(\eta))\cos^2(n\eta + C_3\lambda^{-1})\cosh^{-2} u(\eta) \ ,$$

$$\Phi(\eta) = B\sinh u(\eta) \ , \qquad (20)$$

where $A_1 = æC_p/6$, $V(\eta) = (2/\sigma)\,\mathrm{tg}(n\eta + C_3\lambda^{-1}) + (1 - 4\sigma^{-2})^{1/2}$; $u(\eta) = n\lambda(C_2 + \mathrm{arctg}\, V(\eta))$; $n = \mp 1$; $\sigma \geq 2$.

In order that the scale factor have no discontinuities at the points $\eta_m = \pi/2 + m\pi$, it is necessary to put $C_2 = 0$. It is possible to exclude the constant C_3 by means of a displacement of coordinates.

The analysis showed that solution (20) describes the singular models at $\sigma = 2$ and $\sigma \to \infty$, and the nonsingular ones at $2 < \sigma < \infty$ ($\xi < \infty$).

Note that, for $\sigma = 2$, solution (20) can be expressed in elementary functions of t:

$$a^2(t) = A_1 \sin^2(\beta\xi^{1/2} t) \ , \ \Phi(t) = -B\mathrm{ctg}(\beta\xi^{1/2} t) \ , \quad (21)$$

where $\beta = 3C_1 c/2æ\pi C_p^{3/2}$.

It is easy to see from (21) that $a_{\max} = A_1^{1/2}$, $\Phi_{\max} = \Phi(a_{\max}) = 0$, and the model is singular at $t = 0$ ($a = 0$, $\Phi = -\infty$) and $t = \pi/\beta\xi^{1/2}$ ($a = 0$, $\Phi = \infty$). We note that, as $t \to 0$, the expansion rate of the model $\dot{a} = da/dt \sim \xi^{1/2}$ increases with ξ. In this model, it is important that the evolution time $T = 2\pi^2 æC_p^{3/2}/3C_1 c\xi^{1/2}$ increases with decrease in the parameter ξ.

For $2 < \sigma < \infty$, solution (20) describes the oscillating models. For the maxima of the scale factor, the following is true:

$$a^2_{\max} = 2A_1 \frac{1 - 2\sigma^{-2}}{\cosh^2(\tau\lambda)} \ , \quad \Phi_{\max} = nB\sinh(\tau\lambda) \ , \qquad (22)$$

where $\tau = \mathrm{arctg}(1 - 4\sigma^{-2})^{1/2}$.

It follows from (22) that the parameter ξ essentially influences $a_{\max}$ and $\Phi_{\max}$:

$$a^2_{\max}|_{\xi\ll 1} \simeq A_1(2 - \frac{4}{\sigma^2}) \ , \ \Phi_{\max}|_{\xi\ll 1} \simeq n\tau\sqrt{\frac{24\pi}{æ}} \ . \quad (23)$$

$$a^2_{\max}|_{\xi\gg 1} \sim e^{-2\tau(6\xi)^{1/2}} \ , \quad \Phi_{\max}|_{\xi\gg 1} \sim e^{\tau(6\xi)^{1/2}} \ . \qquad (24)$$

For the minima of the scale factor, the following is true:

$$a^2_{\min} = \frac{A_1\sigma^2}{4\cosh^2(\pi\lambda/2)} \ , \quad \Phi_{\min} = nB\sinh(\pi\lambda/2) \ . \qquad (25)$$

From (25), we have the following asymptotics:

$$a^2_{\min}|_{\xi\ll 1} \simeq \frac{A_1\sigma^2}{4} \ , \quad \Phi_{\min}|_{\xi\ll 1} \simeq \frac{n\pi}{2}\sqrt{\frac{24\pi}{æ}} \ . \qquad (26)$$

$$a^2_{\min}|_{\xi\gg 1} \sim e^{-\pi\tau(6\xi)^{1/2}} \ , \quad \Phi_{\min}|_{\xi\gg 1} \sim e^{(\pi/2)(6\xi)^{1/2}} . \ (27)$$

It follows from (23)–(27) that $\Phi_{\min} > \Phi_{\max}$, and values of $a_{\max}$ and $a_{\min}$ decrease with increase in the parameter ξ. It is not difficult to show that, for $a_{\min}$, the contribution of the scalar-torsion field dominates. Whereas the contribution of the ultrarelativistic gas dominates for $a_{\max}$.

We note that, in this case, nothing can be said about the dependence of the evolution time on the parameter ξ.

3.2. Solution for $\xi < 0$

The exact general solution differs from solution (20) by the replacement of hyperbolic functions of u by trigonometric ones:

$$a^2(\eta) = A_1(1 + V^2(\eta))\cos^2(n\eta + C_3\lambda^{-1})\cos^{-2}u(\eta) \ ,$$

$$\Phi(\eta) = B\sin u(\eta) \ . \qquad (28)$$

Solution (28) belongs to the type studied above. Therefore, we present only the ascertaining part. The points of discontinuity of the scale factor are defined by the expression (we remove the constant C_3 by a displacement of coordinates):

$$\eta_m = n\,\mathrm{arctg}[(\sigma/2)\,\mathrm{tg}\,\varphi - (1 - 4\sigma^{-2})^{1/2}] \ , \qquad (29)$$

where $\varphi = (1 + 2m)n\pi/2\lambda - C_2$. Hence, solution (28) describes the countable number of regular models.

For $\sigma = 2$, this solution can be expressed in elementary functions of t:

$$a(t) = A_1^{1/2}\cosh(\beta_1 t) \ , \quad \Phi(t) = B\tanh(\beta_1 t) \ , \qquad (30)$$

where $\beta_1 = 6c(|\xi|/æC_p)^{1/2}$.

It follows from (30) that $a_{\min} = A_1^{1/2}$, $\Phi_{\min} = 0$, and the expansion rate increases with the parameter ξ. This results in a restriction on ξ: $|\xi| < æ^2 C_p\varepsilon_{pl}/108$.

For $\sigma > 2$, near the points of discontinuity, solution (28) has asymptotics of the type (17), where $H_m = (2c/\sigma)A_1^{-1/2}(6|\xi|)^{1/2}W^{3/2}$, $W = \cos^2\varphi + (\sigma^2/4)[\sin\varphi - (1 - 4\sigma^{-2})^{1/2}\cos\varphi]^2$.

It is easy to show that, for $|\xi| \gg 1$, the Hubble‘s constant may attain Planck‘s values. So, the restriction on ξ is: $|\xi| < æ^2 C_p\varepsilon_{pl}\sigma^2/432W_1^3$, where $W_1 = \cos^2 C_2 + (\sigma^2/4)[\sin C_2 + (1 - 4\sigma^{-2})^{1/2}\cos C_2]^2$.

For the minima of the scale factor, we have ($C_2 = 0$): $a_{\min} = 2A_1^{1/2}\sigma^{-1}$, $\Phi_{\min} = 0$, i.e. $a_{\min}$ decreases with increase in σ.

Finally, we note that all solutions in the case $\alpha_s = -1$ have been obtained only when the ultrarelativistic gas is taken into account.

Conclusion

In this article, exact general solutions in the Einstein – Cartan cosmology with scalar field and ultrarelativistic gas have been obtained. These solutions generalize the results which have been got in [4, 5] in the case of an arbitrary coupling constant ξ. It is worth to note that all solutions have been obtained analytically.

It is shown that, for $\alpha_s = +1, \varepsilon < 0$, $\xi > 0$ and $\alpha_s = -1$, $\xi < 0$, solutions admit the countable number of cosmological models. When the Hubble‘s constant may grow down to Planck‘s values, the restrictions on ξ have been found.

It has been demonstrated that, for $\alpha_s = +1$, $\xi < 0$, and $\alpha_s = -1$, $\xi > 0$ ($\sigma = 2$), solutions describe singular models, and their evolution is characteristic of closed models. It is shown that, for $\alpha_s = +1$, $\xi > 0$ and $\alpha_s = -1$, $\xi < 0$, the scalar-torsion field creates the effect similar to a curvature.

It is shown that, for $\alpha_s = -1$, $\xi < 0$, $\xi > 0$ ($\sigma > 2$), the oscillating models are possible.

It is found that, for $\alpha_s = +1$, $\xi > 0$ and $\alpha_s = -1$, $\xi < 0$ ($\sigma > 2$), a scalar-torsion field averts singularities.

The analysis showed that, for all the solutions obtained above, there are no specific values of the parameter ξ such that, for a fixed sign of the parameter α_s, the character of the evolution of cosmological models would be qualitatively changed.

It is indicated that the ultrarelativistic gas does not play the determining role in the cosmological evolution for the models with material scalar field, but it plays a decisive role in two-component models with “gravitational” scalar field.

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ТОЧНІ РОЗВ'ЯЗКИ ДЛЯ ДВОКОМПОНЕНТНИХ КОСМОЛОГІЧНИХ МОДЕЛЕЙ У ТЕОРІЇ ЕЙНШТЕЙНА–КАРТАНА

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Р е з ю м е

Досліджуються закриті однорідні ізотропні космологічні моделі з немінімально зв'язаним скалярним полем та ультрарелятивістським газом у теорії Ейнштейна–Картана. Одержано точні загальні розв'язки для довільних значень сталої зв'язку. Показано, що можливі сингулярні та скінченна кількість несингулярних моделей. Для отриманих розв'язків знайдено обмеження на сталу зв'язку. Визначено роль ультрарелятивістського газу в еволюції моделей.