SIMULATION OF THREE-LAYER TRANSPARENT STRUCTURES BY THE METHOD OF ENVELOPING AMPLITUDE-PHASE FABRY—PEROT SPECTRA UPON THE NORMAL INCIDENCE OF A LIGHT BEAM ON THE INTERFACES

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A general approach to the description of amplitude-phase regularities of the spectra for the normal reflection from and the transmission of light by plane transparent three-layer structures with the use of the coefficients for the extrema of Fabry—Perot interference bands as enveloping contours is proposed. The correlations between the structure parameters and the reflectivities, transmittances, and phase relations for the corresponding waves are established.

Introduction

Despite the fact that the Fabry—Perot interferometry principle was discovered quite a long time ago [1], is considered to be well-investigated, and forms the basis of the well-known methods of non-destructive control over optical parameters of plane-parallel media [2—4], a number of recently carried out works [5—19] testifies to that this problem remains generally to be topical. First of all, it is conditioned by the 2π-uncertainty, which is related to the necessity to determine the exact value of the absolute order of an interference band. That is why, the method of envelopes seems to be of considerable interest [9,11,13], due to which not only a new approach to the description of the hardware-based properties of Fabry—Perot interferograms was suggested, but the method of reconstruction of the phase of a reflected wave and of the wave passed through the interferometer in terms of the reflectivity R and the transmittance T obtained experimentally was grounded in [18, 19]. Within this approach, it is suggested to determine the contour width not on the half of FWHM (the Full Width at a Half-Maximum), but at the level of the values $\frac{1}{2}(R_{\max}+R_{\min})$ and $\frac{1}{2}(T_{\max}+T_{\min})$.

In this work, the authors have generalized the application of the method of envelopes to the analysis of the data of Fabry–Perot amplitude-phase spectroscopy. This allows one not only to express the parameters of the media forming three-layer plane-parallel structure in terms of the values of R and T at the maxima and minima of interference bands, but also to ground the assumptions made earlier in works [18, 19]. Here, we do not go beyond the consideration of the geometry of a normal transmission of the interface of contacting transparent media by a ray. The influence of absorption and oblique reflection will be studied elsewhere.

Results and Their Discussion

As known [3], if a ray of light falls on a three-layer structure [the semibounded medium (index 1) with refractive index n_1 — interferometer Fabry—Perot (index 2) of d in thickness and with refractive index n — the semibounded medium (index 3) with refractive index n_2], then the amplitudes of the Fresnel reflectivity and transmittance are equal to, respectively,

$$\tilde{r} = \frac{r_{12} + r_{23}\exp(-i\,\delta)}{1 + r_{12}\,r_{23}\exp(-i\,\delta)}, \tag{1}$$

and

$$\tilde{t} = \frac{t_{12}t_{23}\exp(-i\,\delta/2)}{1 + r_{12}r_{23}\exp(-i\,\delta)} \tag{2}$$

due to the Fabry—Perot interference. Here, $\delta = \frac{2nd}{c_0}\omega$ is the phase thickness of a plane-parallel layer, and $\tilde{t}_{12,23} = 1 + \tilde{r}_{12,23}$ according to the boundary conditions of the amplitude. It is also known that the reflectivity $R = \tilde{r} * \tilde{r}^*$, the phase tangent of the reflected wave $\operatorname{tg}\phi = \operatorname{Im}\tilde{r}/\operatorname{Re}\tilde{r}$, transmittance $T = \frac{n_2}{n_1}\tilde{t} * \tilde{t}^*$, and the phase tangent of the wave passed through the structure $\operatorname{tg}\varphi = \operatorname{Im}\tilde{t}/\operatorname{Re}\tilde{t}$, are equal, respectively, to

$$R = \frac{\sigma_{12}^2 + \sigma_{23}^2 + 2\sigma_{12}\sigma_{23}\cos(\phi_{12} - \phi_{23} + \delta)}{1 + \sigma_{12}^2\sigma_{23}^2 + 2\sigma_{12}\sigma_{23}\cos(\phi_{12} + \phi_{23} - \delta)}, \tag{3}$$

$$\operatorname{tg}\phi = \frac{\sigma_{23}\left(1 - \sigma_{12}^2\right)\sin(\phi_{23} - \delta)}{\sigma_{12}\left(1 + \sigma_{23}^2\right)\cos\phi_{12} + \sigma_{23}\left(1 + \sigma_{12}^2\right)\cos(\phi_{23} - \delta)}, \tag{4}$$

$$T = \frac{n_2}{n_1}\frac{\left(1 + \sigma_{12}^2 + 2\sigma_{12}^2\cos\phi_{12}\right)\left(1 + \sigma_{23}^2 + 2\sigma_{23}^2\cos\phi_{23}\right)}{1 + \sigma_{12}^2\sigma_{23}^2 + 2\sigma_{12}\sigma_{23}\cos(\phi_{12} + \phi_{23} - \delta)}, \tag{5}$$

$$\operatorname{tg}\varphi = -\frac{\sin(\delta/2) + \sigma_{12}\sigma_{23}\sin(\phi_{12} + \phi_{23} - \delta/2)}{\cos(\delta/2) + \sigma_{12}\sigma_{23}\cos(\phi_{12} + \phi_{23} - \delta/2)}, \tag{6}$$

where we take the complex amplitudes in the form $\tilde{r} = \sigma\exp(\,i\,\phi)$ and $\tilde{t} = \rho\exp(\,i\,\varphi)$.

Our approach is based on that the right parts of relations (3)–(6) should be expressed in terms of the reflectivity and transmittance at the extrema of interference bands. To this end, we use the transformation well-known in mathematics, namely

$$\cos\beta = \begin{cases} 2\cos^2\frac{\beta}{2} - 1, \\ 1 - 2\sin^2\frac{\beta}{2}. \end{cases} \tag{7}$$

In this work, we analyze only structure-forming transparent media. Therefore, the imaginary parts of the amplitudes of $r_{12,23}$ are $\operatorname{Im}\tilde{r}_{12} = \operatorname{Im}\tilde{r}_{23} = 0$, and their real parts $\operatorname{Re} r_{12,23} = \sigma_{12,23}\cos\,\phi_{12,23} \neq 0$, where the values $\phi_{12,23}$ are equal to π or 2π depending on the relations between n_1, n, and n_2. It is very convenient to join the possible types of structures with arbitrary relations between of n_1, n, and n_2 into three main groups which have common regularities of the amplitude-phase spectroscopy of reflection and transmission of light, namely: asymmetric ($n_1 \neq n_2$) structures such as group **1** with ($n_1 \lessgtr n \lessgtr n_2$) and group **2** with ($n_1 \lessgtr n \gtrless n_2$) and symmetric ones, group **3** with ($n_1 = n_2 \lessgtr n$). For the first group, $\sigma_{12}\cos\phi_{12} \lessgtr 0$ and $\sigma_{23}\cos\phi_{23} \lessgtr 0$. Therefore, according to (3) and (7), their reflectivity is described by the expression

$$R_I = 1 - \frac{1 - R_{\min}}{1 + b^2\cos^2\frac{\delta_I}{2}} = 1 - \frac{1 - R_{\max}}{1 - a^2\sin^2\frac{\delta_I}{2}}. \tag{8}$$

Thus, under the above-mentioned conditions, the reflection spectra of three-layer structures of this group can be described by the relations, in which the functions $R_{\max} = \left(\frac{\sigma_{12} + \sigma_{23}}{1 + \sigma_{12}\sigma_{23}}\right)^2$, $R_{\min} = \left(\frac{\sigma_{12} - \sigma_{23}}{1 - \sigma_{12}\sigma_{23}}\right)^2$ are the enveloping ones, $a^2 = \frac{4\sigma_{12}\sigma_{23}}{(1 + \sigma_{12}\sigma_{23})^2}$, and $b^2 = \frac{4\sigma_{12}\sigma_{23}}{(1 - \sigma_{12}\sigma_{23})^2}$.

For the second group of structures, $\sigma_{12}\cos\phi_{12} \lessgtr 0$ and $\sigma_{23}\cos\phi_{23} \lessgtr 0$. For them, we get the similar relation

$$R_{II} = 1 - \frac{1 - R_{\max}}{1 - a^2\cos^2\frac{\delta_{II}}{2}} = 1 - \frac{1 - R_{\min}}{1 + b^2\sin^2\frac{\delta_{II}}{2}}. \tag{9}$$

For the third group of symmetric structures, we have $\sigma_{12}\cos\phi_{12} \lessgtr 0$ and $\sigma_{23}\cos\phi_{23} \lessgtr 0$ and, because $R_{\min} = 0$ for them,

$$R_{III} = 1 - \frac{1 - R_{\max}}{1 - a^2\cos^2\frac{\delta_{III}}{2}} = 1 - \frac{1}{1 + b^2\sin^2\frac{\delta_{III}}{2}}. \tag{10}$$

Thus, relations (8)—(10) describe the corresponding spectra with the use of the enveloping values of the factors in the extrema of interference bands. We note that their lower indices I, II, III are of no physical sense and show only the belonging of a structure to some group.

Relations (8) — (10) yield

$$\frac{a^2}{b^2}\operatorname{tg}^2\frac{\delta_I}{2} = \frac{R_{\max} - R_I}{R_I - R_{\min}}, \quad \frac{b^2}{a^2}\operatorname{tg}^2\frac{\delta_{II}}{2} = \frac{R_{II} - R_{\min}}{R_{\max} - R_{II}}, \quad \frac{b^2}{a^2}\operatorname{tg}^2\frac{\delta_{III}}{2} = \frac{R_{III}}{R_{\max} - R_{III}}. \tag{11}$$

Their right parts are varied within the limits $[0, +\infty]$ going on through 1 at a certain frequency ω_Σ

$$\frac{R_{\max} - R_I}{R_I - R_{\min}} = \frac{R_{II} - R_{\min}}{R_{\max} - R_{II}} = \frac{R_{III}}{R_{\max} - R_{III}} = 1. \tag{12}$$

So, we have substantiated that the reflectivity on both sides of a band maximum at this frequency ω_Σ is equal to

$$R = \frac{1}{2}(R_{\max} + R_{\min}) = \Sigma_R. \tag{13}$$

Exactly at this level, it was suggested to determine the parameters of the contours for reflection and transmission bands with the purpose to get a more correct description of the hardware-based properties of Fabry—Perot interferometers [18].

At the frequency ω_Σ for the first group of structures, $\mathrm{tg}\frac{\delta_{I\,\Sigma}}{2} = \frac{1+\sigma_{12}\sigma_{23}}{1-\sigma_{12}\sigma_{23}}$, whereas $\mathrm{tg}\frac{\delta_{II,III\,\Sigma}}{2} = \frac{1-\sigma_{12}\sigma_{23}}{1+\sigma_{12}\sigma_{23}}$ for the second and third groups, which allows us to determine the phase thickness of interferometers at an arbitrary frequency as

$$\delta = \delta_\Sigma \frac{\omega}{\omega_\Sigma}, \tag{14}$$

where $\delta_{I\,\Sigma} = 2\,\mathrm{arctg}\frac{b}{a}$, $\quad \delta_{II,III\,\Sigma} = 2\,\mathrm{arctg}\frac{a}{b}$ for the all groups of structures.

Now we pass to the analysis of the phase spectra of a reflected wave. According to (4) and (7), we get

$$\mathrm{tg}\phi_{III} = \mathrm{tg}\frac{\delta_{III}}{2}\frac{-\frac{1-\sigma^2}{2\sigma}a^2\sin^2\frac{\delta_{III}}{2}}{\sqrt{R_{\max}} - \frac{1+\sigma^2}{2\sigma}a^2\cos^2\frac{\delta_{III}}{2}} =$$

$$= \mathrm{ctg}\frac{\delta_{III}}{2} = \frac{1-\sigma^2}{1+\sigma^2}\mathrm{ctg}\frac{\delta_{III}}{2} = \sqrt{\frac{R_{\max}}{R_{III}} - 1}, \tag{15}$$

for symmetric structures, whereas

$$\mathrm{tg}\phi_I = \mathrm{tg}\frac{\delta_1}{2}\frac{-\frac{1-\sigma_{12}^2}{2\sigma_{12}}b^2\cos^2\frac{\delta_I}{2}}{\sqrt{R_{\min}} + \frac{1+\sigma_{12}^2}{2\sigma_{12}}b^2\cos^2\frac{\delta_I}{2}} =$$

$$= \mathrm{ctg}\frac{\delta_I}{2}\frac{-\frac{1-\sigma_{12}^2}{2\sigma_{12}}a^2\sin^2\frac{\delta_I}{2}}{\sqrt{R_{\max}} - \frac{1+\sigma_{12}^2}{2\sigma_{12}}a^2\sin^2\frac{\delta_I}{2}}$$

and

$$\mathrm{tg}\phi_{II} = \mathrm{tg}\frac{\delta_{II}}{2}\frac{\frac{1-\sigma_{12}^2}{2\sigma_{12}}a^2\cos^2\frac{\delta_{II}}{2}}{\sqrt{R_{\max}} - \frac{1+\sigma_{12}^2}{2\sigma_{12}}b^2\cos^2\frac{\delta_{II}}{2}} =$$

$$= \mathrm{ctg}\frac{\delta_{II}}{2}\frac{\frac{1-\sigma_{12}^2}{2\sigma_{12}}b^2\sin^2\frac{\delta_{II}}{2}}{\sqrt{R_{\min}} + \frac{1-\sigma_{12}^2}{2\sigma_{12}}b^2\sin^2\frac{\delta_{II}}{2}},$$

for asymmetric structures. Whence, by excluding δ, we obtain

$$\mathrm{tg}\phi_I = \frac{1-\sigma_{12}^2}{1+\sigma_{23}^2}\frac{b}{a}\sqrt{\frac{R_I - R_{\min}}{R_{\max} - R_I}}\frac{-1+\left(\frac{a}{b}\right)^2\sqrt{\frac{R_{\min}}{R_{\max}}}}{1+\left(\frac{R_I - R_{\min}}{R_{\max}-R_I}\right)\sqrt{\frac{R_{\min}}{R_{\max}}}}, \tag{16}$$

$$\mathrm{tg}\phi_{II} = \frac{1-\sigma_{12}^2}{1+\sigma_{23}^2}\frac{b}{a}\sqrt{\frac{R_{II} - R_{\min}}{R_{\max} - R_{II}}}\frac{-1+\left(\frac{b}{a}\right)^2\sqrt{\frac{R_{\max}}{R_{\min}}}}{1+\left(\frac{R_{II} - R_{\min}}{R_{\max}-R_{II}}\right)\sqrt{\frac{R_{\max}}{R_{\min}}}}. \tag{17}$$

Thus, for symmetric structures at the frequency ω_Σ, the phase tangent of a reflected wave $\mathrm{tg}\phi_{III\,\Sigma} = 1$, and, for asymmetric structures, $\mathrm{tg}\phi_{II,III\,\Sigma} \neq 1$.

Now we consider the spectra of transmission. According to (5), the energy factor of light transmission by a three-layer structure is

$$T = \frac{n_2}{n_1}\frac{T_{12}T_{23}}{1+\sigma_{12}^2\sigma_{23}^2 + 2\sigma_{12}\sigma_{23}\cos\left(\phi_{12}+\phi_{23}-\delta\right)}, \tag{18}$$

where $T_{12,23} = \tilde{t}_{12,23}\cdot\tilde{t}^*_{12,23}$. We see that the argument $(\phi_{12}+\phi_{23}-\delta)$ is lacking in the numerator, and, taking into account that $\phi_{12,23} = \pi$ or 2π and using transformation (7), we get

$$T_I = \frac{T_{\min}}{1 - a^2\sin^2\frac{\delta_I}{2}} = \frac{T_{\max}}{1+b^2\cos^2\frac{\delta_I}{2}} \tag{19}$$

for the first group of structures,

$$T_{II} = \frac{T_{\min}}{1 - a^2\cos^2\frac{\delta_{II}}{2}} = \frac{T_{\max}}{1+b^2\sin^2\frac{\delta_{II}}{2}} \tag{20}$$

for the second group, and

$$T_{III} = \frac{T_{\min}}{1 - a^2\sin^2\frac{\delta_{III}}{2}} = \frac{1}{1+b^2\cos^2\frac{\delta_{III}}{2}} \tag{21}$$

for the third one. Here, the parameters $T_{\max}$ and $T_{\min}$ are defined with regard for the phase shifts $\phi_{12,23}$ as $T_{\max} = \frac{n_2}{n_1}\frac{T_{12}}{(1-\sigma_{12}\sigma_{23})^2}$, $\quad T_{\min} = \frac{n_2}{n_1}\frac{T_{23}}{(1+\sigma_{12}\sigma_{23})^2}$.

For the spectra of reflection and transmission, we get

$$\frac{a^2}{b^2}\mathrm{tg}^2\frac{\delta_I}{2} = \frac{T_I - T_{\min}}{T_{\max} - T_I}, \quad \frac{b^2}{a^2}\mathrm{tg}^2\frac{\delta_{II}}{2} = \frac{T_{II} - T_{\min}}{T_{\max} - T_{II}},$$

$$\frac{a^2}{b^2}\mathrm{tg}^2\frac{\delta_{III}}{2} = T_{III} - T_{\min}, \tag{22}$$

analogously to relations (11). Based on the same substantiation, we also obtain that, at the same frequency ω_Σ, a condition similar to (13) is fulfilled [18,19]:

$$T = \frac{1}{2}\left(T_{\max} + T_{\min}\right) = \Sigma_T. \tag{23}$$

The phase tangent of a wave which passed through an interferometer is equal to $\mathrm{tg}\varphi_I = -\frac{1-\sigma_{12}\sigma_{23}}{1+\sigma_{12}\sigma_{23}}\mathrm{tg}\frac{\delta_I}{2}$ for the first group of structures, and $\mathrm{tg}\varphi_{II} =$

$-\frac{1+\sigma_{12}\sigma_{23}}{1-\sigma_{12}\sigma_{23}}\text{tg}\frac{\delta_{II}}{2}$ for the second one. Therefore, the condition $\text{tg}\varphi_{I\,\Sigma} = -1$ holds true for them at the frequency ω_Σ. For symmetric structures, $\sigma_{12} = \sigma_{23} = \sigma$ and $\text{tg}\varphi_{III} = -\frac{1+\sigma^2}{1-\sigma^2}\text{tg}\frac{\delta_{III}}{2}$. In the general case for symmetric structures, $\text{tg}\,\phi \cdot \text{tg}\,\varphi = -1$, since the ratio $\frac{\tilde{r}}{\tilde{t}}$ is an imaginary number [20].

The cases of complex dispersion and oblique incidence of a light ray on a surface will be studied in a separate work.

Conclusion

1. We have suggested an analytical algorithm of the description of amplitude-phase spectra with the use of envelopes in terms of the reflectivity $R_{\max,\min}$ and the transmittance $T_{\max,\min}$ of light in three-layer transparent structures at the maxima of Fabry—Perot interference bands.

2. We have grounded the expediency to describe the hardware-based properties of Fabry—Perot interferometers at the level of reflection $\Sigma_R = \frac{1}{2}(R_{\max} + R_{\min})$ and transmission $\Sigma_T = \frac{1}{2}(T_{\max} + T_{\min})$, at which the factors do not depend on the phase thickness of an interferometer.

3. We have analyzed the correlation of phases of the reflected light and that passed through an interferometer with values of the reflectivity and transmittance at the extrema of Fabry—Perot interference bands.

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МОДЕЛЮВАННЯ МЕТОДОМ ОБВІДНИХ АМПЛІТУДНО-ФАЗОВИХ СПЕКТРІВ ФАБРІ—ПЕРО ТРИШАРОВИХ ПРОЗОРИХ СТРУКТУР ПРИ НОРМАЛЬНОМУ ПАДІННІ ПРОМЕНЯ

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Р е з ю м е

Запропоновано загальний підхід до опису амплітудно-фазових закономірностей спектрів відбиття і пропускання світла плоскими прозорими тришаровими структурами через значення коефіцієнтів в екстремумах смуг інтерференції Фабрі—Перо як обвідних контурів. Встановлено кореляції між значеннями коефіцієнтів відбиття, пропускання і фази з параметрами середовищ, що утворюють структуру.