MAGNETORHEOLOGY OF ANISOTROPIC SUSPENSIONS OF NON-BROWNIAN AGGREGATES OF NANODISPERSED PARTICLES IN MAGNETIC FLUIDS

E.YU. TARAN, YU.V. PRIDATCHENKO, O.O. MELNYK

Taras Shevchenko Kyiv National University, Faculty of Mechanics and Mathematics
(64, Volodymyrs'ka Str., Kyiv 01033, Ukraine; e-mail: taran@univ.kiev.ua)

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A structure-phenomenological magnetorheological equation for the dilute suspensions of non-Brownian ellipsoidal microaggregates of nanoparticles of the dispersed phase of magnetic fluids is obtained. The magnetorheological characteristics of such suspensions in a simple shear flow in the presence of a transverse magnetic field are studied in the case of the formation of a structural anisotropy in suspensions that appears due to the stationary orientation of suspended microaggregates under the action of hydrodynamic forces and forces that act from the side of an external magnetic field.

Introduction

The aggregation of nanoparticles of the dispersed phase in magnetic fluids [1, 2] change their physical and rheological properties. The ability of anisometric microaggregates formed in this case to orient in an external magnetic field [3] was used in [4] for the development of the magnetooptical method of determination of their geometric and physical parameters. This method allows one to study only comparatively small Brownian microaggregates which loss the acquired orientation due to the action of the rotational Brownian motion after the termination of the orienting action of the external magnetic field.

For the study of non-Brownian microaggregates of nanoparticles of the dispersed phase in magnetic fluids insensitive to the disorienting action of the Brownian motion, when the magnetooptical method becomes useless, we propose the other method, the magnetorheological one, for the determination of the parameters of aggregates. As the theoretical basis of this new method, we take the structure-phenomenological theory of a stressed state constructed in this work. Such a theory describes arbitrary gradient flows of the dilute suspensions of non-Brownian anisometric microaggregates of nanoparticles of the dispersed phase in magnetic fluids with regard for the possible influence of an external magnetic field on their dynamics. The deduced constitutive equation for stresses in such suspensions is used to study the influence of a transverse magnetic field on the rheological behavior of suspensions in a simple shear flow in the case of the formation of a structural anisotropy in suspensions upon the stationary orientation of suspended anisometric non-Brownian microaggregates under the action of hydrodynamic forces and forces acting from the side of an external magnetic field. The use of the structure-phenomenological method [5, 6] upon the development of the theory of a stressed state in the dilute suspensions of non-Brownian aggregates allows us to get the explicit analytic dependence of the effictive shear viscosity of suspensions and the first and second differences of normal stresses on the physical and geometric parameters of suspended aggregates. This allows us to propose that the parameters of the non-Brownian microaggregates of nanoparticles of the dispersed phase in magnetic fluids can be found upon the comparison of theoretical and experimental values of the corresponding magnetorheological characteristics of suspensions.

1. The Structure-Continual Model of the Dilute Suspensions of non-Brownian Microaggregates of Nanoparticles of the Dispersed Phase in Magnetic Fluids

We assume that the suspensions of microaggregates formed in magnetic fluids have the following properties: 1) suspended microaggregates are nondeformable particles with the same form and sizes; 2) characteristic size d of suspended microaggregates is much less than the characteristic size $\bar{l}$ of the suspension macroflow region, but is much more than the characteristic size l of molecules of the Newtonian carrier fluid of a suspension:

$$l << d << \bar{l}; \qquad (1)$$

3) on the surface of suspended microaggregates, the no-slip condition is satisfied; 4) motion of the carrier

fluid relative to suspended aggregates is slow; 5) volume concentration of suspended microaggregates is small, the suspension is assumed to be diluted; 6) suspended microaggregates have zero buoyancy.

The left part, $l << d$, of the double inequality (1) and property 3) allow us to consider the interaction of the Newtonian carrier fluid of a suspension with suspended microaggregates as the hydrodynamic interaction. As a hydrodynamic model of nondeformable suspended microaggregates, we take an ellipsoid of revolution with the major axis $2a$ and the equatorial diameter $2b\,(a > b)$.

We assume that suspended microaggregates are insensitive to the influence of the Brownian motion on their rotational dynamics in the gradient flows of suspensions. For convenience, we will name such microaggregates as non-Brownian, in brief. According to [7], the effective radius $r = \sqrt[3]{ab^2}$ of ellipsoidal non-Brownian microaggregates (particles) satisfies the condition $r > 10^{-6}$ m and the condition $r << \bar{l}$ according to (1). The cited data [7] on the size of suspended ellipsoidal particles concern the suspensions, for which a carrier fluid is water.

We also assume that the suspended non-Brownian microaggregates of magnetic nanoparticles and their hydrodynamic model have constant magnetic moment $p_i = qn_i$, where q is the constant magnetic moment of microaggregates; n_i is the unit vector directed along the axis $2a$ of the ellipsoidal model of microaggregates characterizing their orientation. We suppose that the suspension of microaggregates is dilute so that the interaction of the magnetic fields of suspended microaggregates and the hydrodynamic interaction between the latter can be neglected.

It is known that a stressed state in the gradient flows of the dilute suspensions of axisymmetric particles depends on the rotational dynamics of suspended particles and their averaged orientation. According to the structure-phenomenological theory of a stressed state in the gradient flows of such suspensions [5, 6], the right part of the double inequality (1) allows one to simulate a dilute suspension of ellipsoidal non-Brownian aggregates by the structural continuum with two internal microparameters, namely the orientation vector n_i of suspended ellipsoidal microaggregates and the vector $N_i = \dot{n}_i - \omega_{ik} n_k$, which characterizes their angular velocity relative to the carrier fluid of the suspension. Here, the dot over n_i means the local differentiation with respect to time t, ω_{ik} is the velocity vortex tensor, and $\omega_{ik} = (1/2)(v_{i,k} - v_{k,i})$, where $v_{i,k}$ is the velocity gradient tensor. Here, we use the tensorial designations. In particular, a comma in indices means the differentiation along the direction of the axis distinguished by the index after the comma. As usual, we sum over the identical indices of a term.

2. The General Rheological Equation of a Dilute Suspension of Ellipsoidal Microaggregates

Within the framework of the structure-phenomenological method [5, 6], the rheological equation for stresses T_{ij} in gradient flows of a dilute suspension, which is modelled by a structural continuum, is phenomenologically postulated as the functional dependence of the tensor T_{ij} on the deformation rate tensor d_{ij}, $d_{ij} = (1/2)(v_{i,j} + v_{j,i})$, and on the internal microparameters n_i and N_i of the structural continuum. By [6], the most general phenomenological equation for the stress tensor T_{ij} is

$$T_{ij} = (a_0 + a_1 d_{km}\langle n_k n_m\rangle)\,\delta_{ij} + a_2\langle n_i n_j\rangle +$$

$$+a_3 d_{km}\langle n_k n_m n_i n_j\rangle + a_4 d_{ij} + +a_5 d_{ik}\langle n_k n_j\rangle +$$

$$+a_6 d_{jk}\langle n_k n_i\rangle + a_7\langle n_i N_j\rangle + a_8\langle n_j N_i\rangle, \tag{2}$$

where δ_{ij} is the Kronecker delta; a_i $(i = \overline{0,8})$ are phenomenological constants.

The transition from the microcharacteristics n_i and N_i for separate suspended particles to macrocharacteristics of suspensions occurs upon the execution of the averaging in relation (2) defining the stress tensor T_{ij} in the phase space of coordinates of the orientation vector n_i of suspended ellipsoidal microaggregates. Angular brackets $\langle\rangle$ in (2) mean such an averaging with the use of the distribution function F of angular positions of the vector n_i which satisfies the equation

$$\frac{\partial F}{\partial t} + \frac{\partial}{\partial n_i}(F\ \dot{n}_i) = 0. \tag{3}$$

Upon the structure-phenomenological study of the dilute suspensions of ellipsoidal microaggregates, the phenomenological constants a_i $\left(i = \overline{0,8}\right)$ in the rheological equation (2) can be theoretically derived, as in [5], with the use of the energetic method of Einstein [8]. But the results in [6] show that the application of the dynamical method of Landau [9] and the results of the Jeffery structural theory of the viscosity of the dilute suspensions of ellipsoidal particles [10] gives the advantages allowing the theoretical calculation of the phenomenological constants a_i $\left(i = \overline{0,8}\right)$ in the rheological equation (2) for stresses in a dilute suspension of ellipsoidal microaggregates prior to the

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study of their dynamics. The last varies depending on the size of microaggregates, i.e. on the type of aggregates which can be Brownian or non-Brownian, and on the system of forces defining the dynamics of such microaggregates in the carrier fluid of a suspension.

In order to determine the phenomenological constants a_i $\left(i=\overline{0,8}\right)$ in Eq. (2), we use the own results following from the structural part of the proposed structure-phenomenological theory. According to [6], we find firstly the stress tensor σ_{ij} in the carrier fluid of a suspension on the surface of a sphere S, whose center coincides with the center of an ellipsoidal microaggregate, and the radius R is much more than its size. In [10], Jeffery found the perturbation introduced in the flow of a Newtonian fluid by a nondeformable ellipsoid suspended in it. By using this result, we calculate the stress σ_{ij} on the surface of a sphere S in the moving coordinate system Ox_i with the axes x_1, x_2, x_3, coinciding with the major axes of the ellipsoidal microaggregate, as

$$\sigma_{ij}=-p\delta_{ij}+2\mu\, d_{ij}+10\mu\times$$

$$\times\left(\frac{5}{R^2}\Phi\delta_{ij}+\frac{4x_ix_j}{R^7}-\frac{x_i}{R^5}\frac{\partial\Phi}{\partial x_j}-\frac{x_j}{R^5}\frac{\partial\Phi}{\partial x_i}\right),$$

$$\Phi=A_{km}x_kx_m,$$

$$A_{11}=\frac{d_{11}}{6\beta_0''},\quad A_{12}=\frac{\alpha_0d_{12}+b^2\beta_0'\left(\omega_{12}+\omega_3\right)}{2\beta_0'B},$$

$$A_{13}=\frac{\alpha_0d_{13}+b^2\beta_0'\left(\omega_{13}-\omega_2\right)}{2\beta_0'B},$$

$$A_{21}=\frac{\beta_0d_{21}+a^2\beta_0'\left(\omega_{21}-\omega_3\right)}{2\beta_0'B},$$

$$A_{22}=\frac{d_{22}}{4b^2\alpha_0'}+\frac{d_{11}\left(\beta_0''-\alpha_0''\right)}{12b^2\beta_0''\alpha_0'},\quad A_{23}=\frac{d_{23}}{4b^2\alpha_0'},$$

$$A_{31}=\frac{\beta_0d_{31}+a^2\beta_0'\left(\omega_{31}+\omega_2\right)}{2\beta_0'B},$$

$$A_{32}=\frac{d_{32}}{4b^2\alpha_0'},\quad A_{33}=\frac{d_{33}}{4b^2\alpha_0'}+\frac{d_{11}\left(\beta_0''-\alpha_0''\right)}{12b^2\beta_0''\alpha_0'},$$

$$B=a^2\alpha_0+b^2\beta_0,$$

where p is the pressure; μ is the dynamical coefficient of viscosity of the carrier fluid; α_0, β_0, α_0', β_0', α_0'', and β_0'' are functions defined in [10]; ω_2 and ω_3 are the components of the angular velocity of ellipsoidal microaggregates.

By the Landau structural theory [9] of the dilute suspensions of nondeformable particles, the stress tensor of a suspension of ellipsoidal microaggregates is the tensor σ_{ij} averaged over the volume of the above-mentioned sphere S surrounding a suspended microaggregate. By passing from the integration over the sphere volume to that over its surface, we found the components of the stress tensor in a dilute suspension of ellipsoidal microaggregates as

$$\langle\sigma_{11}\rangle_{\rm vol}=-p+\left(2\mu+\frac{4\mu V}{3ab^2\beta_0''}\right)d_{11},$$

$$\langle\sigma_{22}\rangle_{\rm vol}=-p+\left(2\mu+\frac{2\mu V}{ab^4\alpha_0'}\right)d_{22}+$$

$$+\frac{2\mu V\left(\beta_0''-\alpha_0''\right)}{3ab^4\beta_0''\alpha_0'}d_{11},$$

$$\langle\sigma_{33}\rangle_{\rm vol}=-p+\left(2\mu+\frac{2\mu V}{ab^4\alpha_0'}\right)d_{33}+$$

$$+\frac{2\mu V\left(\beta_0''-\alpha_0''\right)}{3ab^4\beta_0''\alpha_0'}d_{11},$$

$$\langle\sigma_{12}\rangle_{\rm vol}=\left(2\mu+\frac{4\mu\alpha_0V}{ab^2\beta_0'B}\right)d_{12}+\frac{4\mu Vb^2\left(\omega_{12}+\omega_3\right)}{ab^2B},$$

$$\langle\sigma_{21}\rangle_{\rm vol}=\left(2\mu+\frac{4\mu\beta_0V}{ab^2\beta_0'B}\right)d_{21}+\frac{4\mu Va^2\left(\omega_{21}-\omega_3\right)}{ab^2B},$$

$$\langle\sigma_{13}\rangle_{\rm vol}=\left(2\mu+\frac{4\mu\alpha_0V}{ab^2\beta_0'B}\right)d_{13}+\frac{4\mu Vb^2\left(\omega_{13}-\omega_2\right)}{ab^2B},$$

$$\langle\sigma_{31}\rangle_{\rm vol}=\left(2\mu+\frac{4\mu\beta_0V}{ab^2\beta_0'B}\right)d_{31}+\frac{4\mu Va^2\left(\omega_{31}+\omega_2\right)}{ab^2B},$$

$$\langle\sigma_{23}\rangle_{\rm vol}=\left(2\mu+\frac{2\mu V}{ab^4\alpha_0'}\right)d_{23},$$

$$\langle\sigma_{32}\rangle_{\rm vol}=\left(2\mu+\frac{2\mu V}{ab^4\alpha_0'}\right)d_{32},\qquad(4)$$

where V is the volume concentration of suspended microaggregates; and angular brackets $\langle\rangle_{vol}$ mean the above-indicated averaging over the volume of the sphere S.

The coefficients a_i $\left(i=\overline{0,8}\right)$ in the phenomenological rheological equation (2) are determined upon the comparison of elements of the stress tensor T_{ij} in the suspension with the corresponding elements $\langle\sigma_{ij}\rangle_{\rm vol}$ calculated according to the structural part of the theory. To this end, we firstly pass to the moving coordinate system $Ox_1x_2x_3$, connected with the suspended ellipsoidal aggregate, in Eq. (2). For such a transition, $n_1 = 1$, $n_2 = 0$, $n_3 = 0$, $\dot{n}_1 = 0$, $\dot{n}_2 = \omega_3$, $\dot{n}_3 = -\omega_2$. Therefore, the elements of the stress tensor T_{ij}, which is defined by relation (2), take the form

$$T_{11} = a_0 + a_1 d_{11} + a_2 + (a_3 + a_4 + a_5 + a_6)\, d_{11},$$

$$T_{22} = a_0 + a_1 d_{11} + a_4 d_{22}, \quad T_{33} = a_0 + a_1 d_{11} + a_4 d_{33},$$

$$T_{12} = (a_4 + a_6)\, d_{12} + a_7 (\omega_3 + \omega_{12}),$$

$$T_{21} = (a_4 + a_5)\, d_{21} + a_8 (\omega_3 - \omega_{21}),$$

$$T_{13} = (a_4 + a_6)\, d_{13} + a_7 (-\omega_2 + \omega_{13}),$$

$$T_{31} = (a_4 + a_5)\, d_{31} + a_8 (-\omega_2 - \omega_{31}),$$

$$T_{23} = a_4 d_{23}, \quad T_{32} = a_4 d_{32}. \tag{5}$$

The comparison of (4) and (5) allows us to determine the phenomenological rheological constants a_i $(i = \overline{0,8})$ in the rheological equation (2) for the stress in a dilute suspension of ellipsoidal aggregates as

$$a_0 = -p, \quad a_1 = \frac{2\mu V (\beta_0'' - \alpha_0'')}{3ab^4 \beta_0'' \alpha_0'}, \quad a_2 = 0,$$

$$a_3 = \frac{2\mu V}{ab^2}\left[\frac{\alpha_0'' + \beta_0''}{b^2 \alpha_0' \beta_0''} - \frac{2\,(\alpha_0 + \beta_0)}{\beta_0' (a^2\alpha_0 + b^2\beta_0)}\right],$$

$$a_4 = 2\mu\left(1 + \frac{V}{ab^4\alpha_0'}\right), \quad a_5 = \frac{4\mu V}{ab^2}\left(\frac{\beta_0}{\beta_0' B} - \frac{1}{2b^2\alpha_0'}\right),$$

$$a_6 = \frac{4\mu V}{ab^2}\left(\frac{\alpha_0}{\beta_0'(a^2\alpha_0 + b^2\beta_0)} - \frac{1}{2b^2\alpha_0'}\right),$$

$$a_7 = \frac{4b^2\mu V}{ab^2(a^2\alpha_0 + b^2\beta_0)}, \quad a_8 = -\frac{4a^2\mu V}{ab^2(a^2\alpha_0 + b^2\beta_0)}. \tag{6}$$

The use of the results of the Jeffery structural theory [10] allows us to find the functions α_0, β_0, α_0', β_0', α_0'', and β_0'' and to get

$$ab^2\alpha_0 = 2 - 2A, \quad ab^2\beta_0 = A, \quad ab^4\alpha_0' = \frac{2p_0 - 3A}{4\,(p_0^2 - 1)},$$

$$ab^4\beta_0' = \frac{3A - 2}{p_0^2 - 1}, \quad ab^2\alpha_0'' = \frac{(4p_0^2 - 1)\, A - 2p_0^2}{4\,(p_0^2 - 1)},$$

$$ab^2\beta_0'' = \frac{2p_0^2 - (2p_0^2 + 1)\, A}{p_0^2 - 1}, \tag{7}$$

where $p_0 = a/b$ and

$$A = \frac{p_0^2}{p_0^2 - 1} - \frac{p_0 \ln\left(p_0 + \sqrt{p_0^2 - 1}\right)}{(p_0^2 - 1)^{3/2}}$$

for $p_0 > 1$.

Formulas (6) and (7) show that the rheological constants a_1, a_3, $\ldots a_8$ in Eq. (2) depend only on the dynamical coefficient of viscosity μ of the carrier fluid of a suspension, volume concentration V of suspended microaggregates, and the ratio p_0 of the axes of the ellipsoid of revolution simulating the microaggregates suspended in a suspension.

Within the framework of the structure-phenomenological method employed in the present work, the constitutive equation for the internal microparameters n_i and N_i of the structural continuum simulating a real dilute suspension follows from the equation of the rotational dynamics of suspended ellipsoidal microaggregates in gradient flows of the suspension.

3. A Rheological Equation for the Dilute Suspensions of non-Brownian Ellipsoidal Microaggregates of Nanoparticles of the Dispersed Phase in Magnetic Fluids

The constitutive equation for the internal parameters n_i and N_i of the structural continuum simulating a dilute suspension of non-Brownian ellipsoidal microaggregates can be derived upon the vector multiplication of the equation of rotational motion

$$\frac{dL_i}{dt} = M_i^{(hf)} + M_i^{(mf)}, \tag{8}$$

of suspended non-Brownian ellipsoidal microaggregates in the gradient flows of a suspension in the presence of an external magnetic field H_i by the vector n_i. In (8), L_i is the moment of momentum of suspended microaggregates, $L_i = I\varepsilon_{ikm} n_k \dot{n}_m$; I is the inertia moment of an ellipsoidal microaggregate relative to the axis passing through the microaggregate center normally to its symmetry axis, $I = (m/5)\left(a^2 + b^2\right)$, m is the microaggregate mass; ε_{ikm} is the Levi-Civita tensor; $M_i^{(hf)}$ and $M_i^{(mf)}$ are the angular moments of the hydrodynamic forces and forces acting from the side of the external magnetic field on suspended microaggregates.

Taking into account that

$$M_i^{(mf)} = q\varepsilon_{ikm} n_k H_m, \tag{9}$$

$$\left[\frac{dL_i}{dt}\times n_i\right] = I\ (\ddot{n}_i + \dot{n}_k\dot{n}_k n_i)\,, \tag{10}$$

and, according to [6],

$$\left[M_i^{(hf)}\times n_i\right] = W\ (\lambda\ (d_{ik}n_k - d_{km}n_k n_m n_i) - N_i)\,, \tag{11}$$

we get the required constitutive equation for n_i and N_i as

$$I\ (\ddot{n}_i + \dot{n}_k\dot{n}_k n_i) =$$

$$= W\ (\lambda\ (d_{ik}n_k - d_{km}n_k n_m n_i) - N_i) + q\,(H_i - n_k H_k n_i)\,. \tag{12}$$

In (11) and (12), $\lambda = \left(p_0^2-1\right)/\left(p_0^2+1\right)$; W is the coefficient of rotational friction of an ellipsoidal microaggregate in the Newtonian carrier fluid which is defined by the relation [11]

$$W = 4\upsilon\,\mu\frac{p_0^4-1}{p_0^2\left(\frac{2p_0^2-1}{2p_0\sqrt{p_0^2-1}}\ln\frac{p_0+\sqrt{p_0^2-1}}{p_0-\sqrt{p_0^2-1}}-1\right)},$$

if $p_0 > 1$. Here, υ is the volume of an ellipsoidal microaggregate and $\upsilon = 4\pi\,ab^2/3$.

Without regard for the inertia moment of suspended microaggregates, as it usually is in the rheology of suspensions, the constitutive equation (12) becomes

$$N_i = \lambda\ (d_{ik}n_k - d_{km}n_k n_m n_i) + \frac{q}{W}\,(H_i - n_k H_k n_i)\,. \tag{13}$$

The substitution of N_i defined by Eq. (13) in Eq. (2) allows us to get, in view of (6), the rheological equation for the dilute suspensions of non-Brownian ellipsoidal microaggregates of nanoparticles of the dispersed phase in a magnetic fluid flowing in the presence of a magnetic field H_i as

$$T_{ij} = -p\delta_{ij} + 2\mu\left(1+\frac{V}{ab^4\alpha_0'}\right)d_{ij} +$$

$$+2\mu\frac{V}{ab^2}\left(\frac{\alpha_0''}{b^2\alpha_0'\beta_0''}+\frac{1}{b^2\alpha_0'}-\frac{4}{\beta_0'\,(a^2+b^2)}\right)\times$$

$$\times d_{km}\langle n_k n_m n_i n_j\rangle + 2\mu\frac{V}{ab^2}\left(\frac{2}{\beta_0'\,(a^2+b^2)}-\frac{1}{b^2\alpha_0'}\right)\times$$

$$\times\left(d_{jk}\langle n_k n_i\rangle + d_{ik}\langle n_k n_j\rangle\right) + \frac{V}{\upsilon}\frac{q}{p_0^2+1}\times$$

$$\times\left(H_j\langle n_i\rangle - p_0^2 H_i\langle n_j\rangle\right) + \frac{V}{\upsilon}\lambda q H_m\langle n_m n_i n_j\rangle. \tag{14}$$

In (14), the averaging is realized with the use of the distribution function F which is defined, according to (3) and (13), by the equation

$$\frac{\partial F}{\partial t} + \frac{\partial}{\partial n_i}(F(\omega_{ik}n_k + \lambda\,(d_{ik}n_k - d_{km}n_k n_m n_i) +$$

$$+\frac{q}{W}\,(H_i - n_k H_k n_i))) = 0. \tag{15}$$

4. Magnetorheological Behavior of the Dilute Suspensions of non-Brownian Ellipsoidal Microaggregates

Consider a simple shear flow

$$v_x = 0,\quad v_y = Kx,\quad v_z = 0\qquad (K - \text{const}) \tag{16}$$

of a dilute suspension of non-Brownian ellipsoidal microaggregates of nanoparticles of the dispersed phase in a magnetic fluid in the presence of an external magnetic field

$$H_x = H,\quad H_y = H_z = 0\quad (H - \text{const}) \tag{17}$$

with the use of the constitutive equations (13) and (14).

The study of the rotational motion of non-Brownian ellipsoidal microaggregates in the simple shear flow (16) of a suspension in the presence of the external magnetic field (17) with the use of Eq. (13) shows that such microaggregates are rotating under the action of the hydrodynamic forces and forces acting from the side of the magnetic field with the angular velocity $\omega_i = \{\dot\varphi,\ \dot\theta\}$ so that

$$\dot\varphi = \frac{K}{2}\,(1+\lambda\cos 2\varphi) - \frac{qH}{W}\frac{\sin\varphi}{\sin\theta}, \tag{18}$$

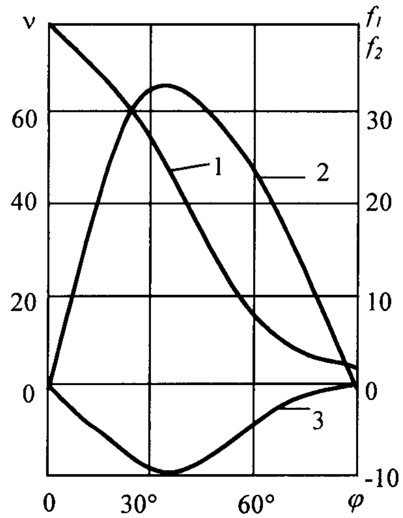
$$\dot\theta = \frac{K}{4}\lambda\sin 2\varphi\,\sin 2\theta + \frac{qW}{H}\cos\varphi\cos\theta, \tag{19}$$

where φ and θ are angles of the spherical coordinate system, in which

$$n_x = \cos\varphi\sin\theta,\quad n_y = \sin\varphi\sin\theta,\quad n_z = \cos\theta. \tag{20}$$

Here, φ is the angle between the xOz plane and the plane passing through the coordinate axis Oz and the unit vector n_i; θ is the angle between the Oz axis and the unit vector n_i.

Solving Eqs. (18), (19) and analyzing their solutions show that suspended non-Brownian ellipsoidal microaggregates can stationarily hover under the action of the hydrodynamic forces and forces acting from the side of the external magnetic field in the shear plane Oxy



Dependences of ν, f_1, and f_2 on φ in the case where non-Brownian ellipsoidal aggregates hover; $p_0=10$; curves *1, 2, 3* correspond to ν, f_1, and f_2

at an angle φ with respect to the Ox axis which is defined by the equation

$$\alpha\,(\lambda\cos 2\varphi+1)-2\sin\varphi=0, \tag{21}$$

where

$$\alpha=\frac{KW}{qH}. \tag{22}$$

The calculation of the elements T_{xy}, T_{yx}, T_{xx}, T_{yy}, and T_{zz} of the tensor T_{ij} with the use of (14) and (20) leads to the following formulas for the characteristic viscosity of the suspension,

$$\nu\equiv\frac{\mu_a-\mu}{\mu V}=\frac{1}{ab^2}\frac{1}{\beta_0'(a^2+b^2)}+ +\frac{2}{ab^2}\left(\frac{\alpha_0''}{b^2\alpha_0'\beta_0''}+\frac{1}{b^2\alpha_0'}-\frac{4}{\beta_0'\,(a^2+b^2)}\right)\times \times\sin^2\varphi\,\cos^2\varphi+\frac{2}{\alpha}\frac{a^2-b^2}{ab^2\,(a^2\alpha_0+b^2\beta_0)}\cos 2\varphi\sin\varphi \tag{23}$$

and for the differences of normal stresses σ_1 and σ_2,

$$\sigma_1\equiv T_{yy}-T_{zz}=K\mu V f_1, \tag{24}$$

$$\sigma_2\equiv T_{xx}-T_{zz}=K\mu V f_2 \tag{25}$$

for the dilute suspensions of non-Brownian ellipsoidal microaggregates which hover under the action of the hydrodynamic forces in the simple shear flow (16) of a suspension and forces acting from the side of the external magnetic field (17) on microaggregates. In (23)–(25), μ_a is the effective viscosity of the suspension which is defined as

$$\mu_a=\frac{T_{xy}+T_{yx}}{2K},$$

and f_1 and f_2 have the form

$$f_1=\frac{1}{ab^2}\left(\frac{\alpha_0''}{b^2\alpha_0'\beta_0''}+\frac{1}{b^2\alpha_0'}-\frac{4}{\beta_0'\,(a^2+b^2)}\right)\times \times\sin 2\varphi\,\sin^2\varphi+\frac{1}{ab^2}\left(\frac{2}{\beta_0'\,(a^2+b^2)}-\frac{1}{b^2\alpha_0'}\right)\times \times\sin 2\varphi\,+\frac{2}{\alpha}\frac{a^2-b^2}{ab^2\,(a^2\alpha_0+b^2\beta_0)}\sin 2\varphi\sin\varphi. \tag{26}$$

$$f_2=\frac{1}{ab^2}\left(\frac{\alpha_0''}{b^2\alpha_0'\beta_0''}+\frac{1}{b^2\alpha_0'}-\frac{4}{\beta_0'\,(a^2+b^2)}\right)\times \times\sin 2\varphi\cos^2\varphi+\frac{1}{ab^2}\left(\frac{2}{\beta_0'\,(a^2+b^2)}-\frac{1}{b^2\alpha_0'}\right)\times \times\sin 2\varphi-\frac{2}{\alpha}\frac{a^2-b^2}{ab^2\,(a^2\alpha_0+b^2\beta_0)}\sin 2\varphi\sin\varphi. \tag{27}$$

According to (23), (26), and (27), ν, f_1, and f_2 are explicit functions of p_0, φ, and α. It follows from (21), (23), (26), and (27) that for every value of α defined by relation (22) corresponds to a certain hovering angle φ of microaggregates and, as a consequence, to certain values of the functions ν, f_1, and f_2 depending only on p_0. The dependence of ν, f_1, and f_2 on the hovering angle φ at $p_0=10$ is shown in the figure.

Conclusions

The magnetorheological equation of the dilute suspensions of non-Brownian ellipsoidal microaggregates of nanoparticles of the dispersed phase in magnetic fluids derived with the use of the structure-phenomenological method [5,6] ensures the possibility to theoretically study the behavior of such suspensions in arbitrary gradient flows in the presence of an external magnetic field.

The explicit dependence of the effective viscosity μ_a of these suspensions and the first and second differences of the normal stresses σ_1 and σ_2 in the simple shear flow (16) in the presence of a transverse magnetic field (17) on the geometric characteristics of suspended non-Brownian ellipsoidal microaggregates and their magnetic moment allows us to consider the derived results as a theoretical model for the magnetorheological experimental method of determination of the indicated parameters of suspended non-Brownian microaggregates upon the comparison of the calculated and experimental values of the magnetorheological characteristics of suspensions.

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МАГНІТОРЕОЛОГІЯ АНІЗОТРОПНИХ СУСПЕНЗІЙ НЕБРОУНІВСЬКИХ АГРЕГАТІВ НАНОДИСПЕРСНИХ ЧАСТИНОК МАГНІТНИХ РІДИН

Є.Ю. Таран, Ю.В. Придатченко, О.О. Мельник

Р е з ю м е

Одержано структурно-феноменологічне магнітореологічне рівняння розведених суспензій неброунівських еліпсоїдальних мікроагрегатів наночастинок дисперсної фази магнітних рідин. Досліджуються магнітореологічні характеристики таких суспензій у простій зсувній течії за наявності поперечного магнітного поля у випадку формування у суспензіях структурної анізотропії, що виникає завдяки стаціонарній орієнтації зважених мікроагрегатів під дією гідродинамічних сил і сил, які діють з боку зовнішнього магнітного поля.