ANNIHILATION MECHANISM OF DILEPTON EMISSION FROM FINITE FIREBALL

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Medium-induced modifications of the $\pi^+\pi^-$ and $q\bar{q}$ annihilation mechanisms of dilepton production during relativistic heavy ion collisions are considered. Due to the dense hadron environment, the pions produced during a collision are effectively confined in a finite volume, in which they live for a finite time which is scaled as the lifetime of a fireball. Keeping the vacuum mass and width of the ρ-meson formfactor, we compare two approaches to the description of the ρ-meson behavior. In the first approach, a ρ-meson has a zero mean free path due to the dense hadron environment. But, in the second approach, it propagates as in vacuum. Our results indicate that, due to the space-time finiteness of the pion system which generates the corresponding quantum randomization, the dilepton rates are finite in the invariant low-mass region $M \leqslant 2m_\pi$. It is found that the spatial finiteness of quark wave functions and the finiteness of the lifetime of excited states result in the same effect for the $q\bar{q}$ annihilation to dileptons. The breaking of the detailed energy-momentum conservation due to the broken translation invariance in a medium is discussed.

Introduction

Lepton pairs produced in high energy nucleus-nucleus collisions provide an information on the early high-temperature and high-density stage, where the quark-gluon plasma (QGP) formation is expected. The registration of dileptons gives important observables which probe the the pion dynamics in the dense nuclear matter that exists on the early stage of the collision. After the creation, e^+e^- and $\mu^+\mu^-$ pairs do not practically interact with the surrounding hadron matter, and the analysis of the final dilepton spectra experimentally obtained provides an excellent possibility to investigate the expansion dynamics of hadron fireballs. The enhancement in the production of dileptons with an invariant mass of 200—800 MeV observed by the CERES collaboration [1,2] has received a considerable attention recently and has been studied in the framework of various theoretical models. It was found that the most prominent channel of the dilepton production accounting a large part of the observed enhancement is the pion annihilation [3,4]. The sensitivity of the dilepton spectra to a variation in the initial conditions for a hadron fireball was investigated. Namely, an enhancement of the dilepton production was explained by a modification of pions in the medium, the pion-nuclear formfactors in the pion gas [3], or the pion dispersion relation modified in the medium [5,6]. The inclusion of the effects of baryon resonances [7,8] which couple directly to ρ-mesons seems to be able to substantially increase the yield, although the calculation in [9] found a much smaller effect due to baryons. In [10], the dilepton production was calculated as a result of the interaction between long-wavelength pion oscillations or disoriented chiral condensates and the thermal environment within the linear sigma model. It was shown that the dilepton yield with the invariant mass near and below $2m_\pi$ can be larger up to two orders of magnitude than the corresponding equilibrium yield due to soft pion modes. Finite pion width effects and their influence on ρ-meson properties and, subsequently,

on dilepton spectra were investigated in [11], and the influence of nonequilibrium processes and finite times were investigated in [12]. The conclusion of many works is that, in order to better fit the experimental data, an additional modifications, which are due to the medium and nonequilibrium processes, should be considered (for recent review, see [13]).

The purpose of the present paper is to revise the annihilation mechanism of dilepton production by taking into account that a multipion system or a multiquark system is confined to a finite space-time region. The standard consideration of the reaction $\pi^+ \pi^- \rightarrow \rho \rightarrow \gamma^* \rightarrow l\bar{l}$ or $q\bar{q} \rightarrow l\bar{l}$ [14–17] assumes an infinite space-time volume of the multiparticle system. This results immediately in the "sharp" energy-momentum conservation and presence of the δ-function $\delta^4(K-P)$ as a factor of the S-matrix element (here, $K = k_1 + k_2$ is the total momentum of the pion pair, and $P = p_+ + p_-$ is the total momentum of the lepton pair). As a consequence of the equality $K = P$, the invariant mass of the two-pion system is specified as $K^2 = M^2$, where M is the invariant mass of a registered lepton pair, i.e. $P^2 = M^2$. However, if the pion gas does not spread in all the space, but it occupies a finite space-time volume, and its lifetime is not long enough, the δ-function should be smeared to another distribution, e.g., $\rho(K-P)$ which represents the distribution of the two-pion total energy K^0 and total momentum $\mathbf{K}$ around the measurable quantities P^0 and $\mathbf{P}$, respectively. Effectively, this means the smearing of the pion pair invariant mass $M_\pi = \sqrt{K^2}$ around the mass M of a lepton pair. For instance, in the case of a finite time interval, one can use the Breit—Wigner energy distribution in place of $\delta(K^0 - P^0)$. In the case of a finite spatial volume, one can introduce a relevant formfactor $\rho(\mathbf{K} - \mathbf{P})$ instead of $\delta^3(\mathbf{K} - \mathbf{P})$. The consequence of smearing a distribution in the 4-momentum space is strong enough. The rate becomes measurable under the "standard" threshold, i.e. for the values $M < 2m_\pi$, and down to two lepton masses. For instance, for the electron-positron pair production, the rate becomes measurable even in the vicinity of $2m_e \approx 1$ MeV, where m_e is the electron mass. Obviously, this phenomenon is due to the uncertainty principle. In fact, if the life-time of a pion system is restricted, for instance, by $\tau = 2$ fm/c, then the energy (invariant mass) uncertainty is in the range up to $\Delta E = 100$ MeV. Actually, this means that lepton pairs with invariant masses down to $M \propto 2m_\pi - 100$ MeV can be registered. Indeed, in accordance with quantum mechanics, we can speak about this matter because the experimental data are taken from every particular event of the collision

of two relativistic nuclei. Reactions (quark-antiquark annihilation, pion-pion annihilation, etc.) attributed to a particular event take place during a finite time ($\tau = 4 \div 10$ fm/c) and in a bounded volume ($R = 4 \div 10$ fm) which are fixed by a number of other measurements characterizing the particular event.

Another important consequence of the finiteness of a reaction space-time volume is a cutting of the "exact connection" between the lepton-lepton c.m.s. and the pion-pion c.m.s. Indeed, the relation between the total momentum P of an outcoming lepton pair and the total momentum K of an incoming pion pair is weighted now by the distribution function $\rho(K - P)$ which can be regarded as a smeared pattern of $\delta^4(K - P)$. That is why, any quantity determined (evaluated) in the lepton-lepton c.m.s. which moves with the velocity $\mathbf{v}_P = \mathbf{P}/P_0$ in the lab system should be Lorentz-transformed to the pion-pion c.m.s. which, in turn, moves with the velocity $\mathbf{v}_K = \mathbf{K}/K_0$ in the lab system. As seen, such a transformation brings a sufficient increase of the electron-positron emission rate for small invariant masses M which are in the range $2m_e < M < 2m_\pi$.

Actually, a restriction of the space-time volume in the process $a\bar{a} \rightarrow b\bar{b}$ induces several effects which should be taken into account. Basically, the nonequilibrium behavior results in a correction of the standard spectrum [17] obtained in the infinite space-time. On the other hand, this correction can be conventionally separated into three different contributions: a) First, it is a nonstationary behavior of a multiparticle system. If, for instance, the multipion system which provides pions for the process $\pi^+ \pi^- \rightarrow l\bar{l}$ has a finite lifetime, then individual pion states are also nonstationary, and a decay of the states should be taken into account. b) The second contribution comes from the direct presence of the formfactor of the multipion system which practically determines the finiteness of a space-time volume occupied by the system. c) At least, as we discussed above, the third contribution comes from smearing of the δ-function connection between the total 4-momentum of incoming particles (pions) and the total 4-momentum of outgoing particles (leptons).

The goal of the present paper is to estimate the influence of the finite space-time volume of a hadron reaction zone on the dilepton emission rate.

1. Two-pion Annihilation in a Dense Hadron Medium

We assume that the pion plasma which has been formed after the equilibration process exists in a finite volume,

and the confinement of pions to this volume is a direct consequence of the dense hadron environment which prevents the escape of pions during some mean lifetime τ. We sketch a possible geometry in Fig. 1. A small circle of radius R represents the subsystem of pions which is in the local equilibrium and moves with collective velocity $\mathbf{v}$. If the fireball size is of the order of $R_0 \propto 4 \div 10$ fm, then the mean size of a small subsystem is of the order of $R \propto 1 \div 5$ fm. So, we assume that pion states $\phi_\lambda(x)$ (λ accumulates relevant quantum numbers) which can take part in the annihilation reaction are effectively bounded in a finite volume marked by radius R in Fig. 1.

The pion-pion annihilation reaction in accordance with vector meson dominance runs in the following way (see Figs. 2): pions annihilate through a ρ-meson which, in turn, transforms to a virtual photon creating a lepton-antilepton pair, i.e. $\pi^+ \pi^- \to \rho \to \gamma^* \to l\bar{l}$. Let us point out that the system of pions under discussion has been formed in ultrarelativistic nucleus-nucleus collisions and, together with other species, represents a superhigh dense state of matter, where the created pions are bounded during the lifetime of the fireball by the hadron environment. To represent the finite lifetime and spatial squeezing of pion states, which can take part in the annihilation reaction, we adopt the ansatz for pion wave functions

$$\phi_\lambda(\mathbf{x}) = \sqrt{\rho(\mathbf{x})}e^{i\alpha_\lambda(\mathbf{x})}, \tag{1}$$

where $\rho(\mathbf{x})$ is the density of pions in the volume V_π, in which pions being in a local thermodynamic equilibrium (marked by radius R in Fig. 1). Assuming a single-particle wave function in the form (1), we should like to note that particles which are squeezed approximately to a dense packing create a strongly correlated quantum system like that observed in quantum liquids. One of the recent results of HBT interferometry (the investigation of two-particle spectra) indicates that the pions registered as secondaries in heavy ion collisions are emitted from the region of homogeneity of a fireball. We take this as a basis for the assumption that the volume V_π during the evolution inside a fireball is approximately equal to that of the homogeneity region. Hence, one can assume a smooth behavior of the phase of the wave function (1) inside this volume and expand the phase functions $\alpha_\lambda(\mathbf{x})$ in the coordinate space at the center of the region

$$\alpha_\lambda(\mathbf{x}) = \alpha_k + \mathbf{k} \cdot \mathbf{x} + \ldots, \tag{2}$$

where $\alpha_k \equiv \alpha_\lambda(\mathbf{x} = 0)$, $\mathbf{k} = \partial\alpha_\lambda(\mathbf{x} = 0)/\partial\mathbf{x}$, and we start to label states by the index k in place of λ. For
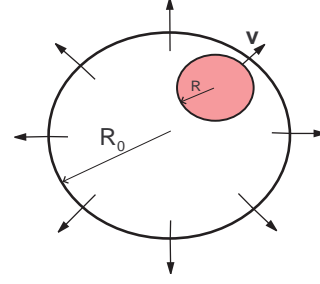

Fig. 1. Sketch of an expanding fireball. The small circle of radius R represents the subsystem of pions which is in a local equilibrium state and moves with the collective velocity $\mathbf{v}$. If the fireball size is of the order of $R_0 \propto 4 \div 10$ fm, then the size of the small subsystem is $R \propto 1 \div 5$ fm

the sake of clearness of the effects under investigation, we assume the free dispersion law $\omega_k = \sqrt{m^2 + \mathbf{k}^2}$. The quantities $\mathbf{k}$ and ω_k can be interpreted as quasi-momentum and quasi-energy. The amplitude $A_{\mathrm{fi}}(k_1, k_2; p_+, p_-)$ of the annihilation process depicted in Fig. 2 reads

$$A_{\mathrm{fi}}(k_1, k_2; p_+, p_-) = \langle \mathbf{p}_+, \mathbf{p}_- | S^{(2)} | \mathbf{k}_1, \mathbf{k}_2 \rangle =$$

$$= -\int d^4x_1\, d^4x_2 \langle \mathbf{p}_+, \mathbf{p}_- | T\left[H_I^\pi(x_1) H_I^l(x_2)\right] | \mathbf{k}_1, \mathbf{k}_2 \rangle =$$

$$= ie^2 \int d^4x_1 \langle 0,\ 0 | j_\nu^\pi(x_1) | \mathbf{k}_1, \mathbf{k}_2 \rangle \times$$

$$\times \int d^4x_2 \langle \mathbf{p}_+, \mathbf{p}_- | j_\mu^l(x_2) | 0,\ 0 \rangle D_F^{\mu\nu}(x_1 - x_2), \tag{3}$$

where

$$j_\mu^\pi(x) = -i\phi(x) \overset{\leftrightarrow}{\partial_\mu} \phi^+(x), \quad j_\mu^l(x) = \overline{\psi}(x)\gamma_\mu\psi(x),$$

and the photon propagator is

$$D_F^{\mu\nu}(x_1 - x_2) = \int \frac{d^4P}{(2\pi)^4} \frac{g^{\mu\nu}}{P^2 + i\epsilon} e^{-iP\cdot(x_1-x_2)}.$$

To reflect explicitly the finiteness of the space-time volume of the pion system, one can extract the pion 4-density $\rho(x_1)$ and write it separately, so that the vertex x_1 in Fig. 2 is weighted by this density. Then, the amplitude can be rewritten in the form

$$A_{\mathrm{fi}}(k_1, k_2; p_+, p_-) = 4\pi\alpha \int \rho(x_1)d^4x_1 \int d^4x_2\, \phi_{k_1}^*(x_1) \times$$

$$\times \phi_{k_2}^*(x_1) D_F(x_1 - x_2) F_\pi\left((k_1 + k_2)^2\right) \phi_{p_+}(x_2) \phi_{p_-}(x_2) \times$$

$$\times m_{\mathrm{fi}}(k_1, k_2; p_+, p_-), \tag{4}$$

where $\alpha = e^2/4\pi$ and $\phi_k(x) = [(2\pi)^3 2E_k]^{-1/2} \times \exp(-ik \cdot x)$ with $k = k_1$, $k = k_2$ for pion states and

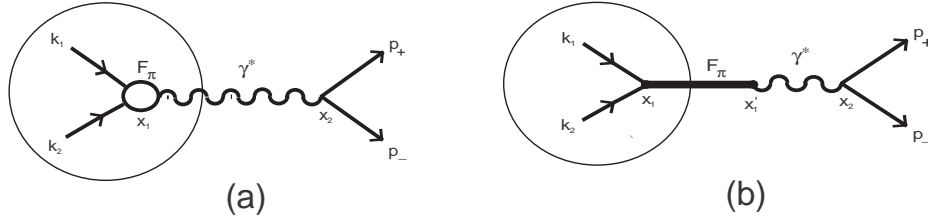Fig. 2. The first-order nonvanishing Feynman graph of the lepton pair creation in the process $\pi^+ \pi^- \to \rho \to \gamma^* \to l\bar{l}$. The circle which surrounds the vertex x_1 sketches the finite space-time region of the pion-pion interactions

with $k = p_+$, $k = p_-$ for final lepton states (the lepton spinor part is hidden in the matrix element m_{fi}), where $p_\pm^0 = E_\pm = \sqrt{m_e^2 + \mathbf{p}_\pm^2}$. The quasi-energy of incoming pions is defined as $k_i^0 = E_i = \sqrt{m_\pi^2 + \mathbf{k}_i^2}$ with $i = 1, 2$. A ρ-meson formfactor $F_\pi(M)$ is taken in the form [16, 18]

$$\left| F_\pi(M^2) \right|^2 = \frac{m_\rho^4}{(M^2 - m_\rho'^2)^2 + m_\rho^2 \, \Gamma_\rho^2} \tag{5}$$

with $m_\rho = 775\,\text{MeV}$, $m'_\rho = 761\,\text{MeV}$, $\Gamma_\rho = 118\,\text{MeV}$. The standard calculation of the matrix element m_{fi} gives

$$|m_{\text{fi}}|^2 = \sum_{\text{spins}} t_\nu^+ t_\mu (k_1 - k_2)^\nu (k_1 - k_2)^\mu =$$
$$= 8(k_1 - k_2) \cdot p_+ \ (k_1 - k_2) \cdot p_- -$$
$$-4(k_1 - k_2)^2(m_e^2 + p_+ \cdot p_-) \ , \tag{6}$$

where $t_\nu = \bar{u}(p_-)\gamma_\nu v(p_+)$ with $u(p)$ and $v(p)$ being the electron and positron spinors, respectively. In the momentum space representation, amplitude (4) looks like

$$A_{\text{fi}}(k_1, k_2; p_+, p_-) = \frac{\alpha F_\pi\left((k_1 + k_2)^2\right)}{2\pi^2 \sqrt{2E_1 2E_2}} \times$$
$$\times \rho(k_1 + k_2 - p_+ - p_-) \, D_F(p_+ + p_-) \frac{m_{\text{fi}}(k_1, k_2; p_+, p_-)}{(2\pi)^3 \sqrt{2E_+ 2E_-}}, \tag{7}$$

where $\rho(k) = \int d^4x \rho(x) e^{ik \cdot x}$ is the Fourier transform of the pion density. The quantity $\rho(k_1 + k_2 - p_+ - p_-)$ in (7) stands obviously in place of the delta function $(2\pi)^4 \delta^4(k_1 + k_2 - p_+ - p_-)$ as a reflection of the finiteness of the space-time volume occupied by the pion system.

We consider two models (a) and (b) of the pion-pion annihilation, and each of them can be put in correspondence to Fig. 2,a or Fig. 2,b, respectively. Because we concentrate on the investigation of the influence of the features of annihilating pion states on the dilepton emission, we take the vacuum width and mass of the ρ-meson formfactor in (5) for both models. At the same time, to reflect in some way the influence

of the dense hadron environment on the propagation of a ρ-meson, we adopt in model (a) a zero mean free path of ρ-mesons what is reflected in the diagram, see Fig. 2,a. Amplitude (4) was written in accordance with model (a). On the other hand, in the standard vector meson dominance treatment reflected by Fig. 2,b, the explicit propagation of a ρ-meson is taken into account. The formal difference of these two approaches reveals itself just as the difference of the arguments of the ρ-meson propagator $F_\pi(k^2)$ which is $F_\pi\left((k_1 + k_2)^2\right)$ and $F_\pi\left((p_+ + p_-)^2\right)$ for models (a) and (b), respectively. It is worth to note that models (a) and (b) coincide when there is no restriction on the 4-volume of the reaction zone. But it is not the case when the reaction takes place in a finite space-time volume. Below, we analyze the consequences of the difference of these two approaches for the rate of dilepton emission.

The number of lepton pairs produced per event in the element of the dilepton 4-momentum space d^4P from the pion momentum space elements d^3k_1 and d^3k_2 reads

$$\frac{dN^{(\rho)}}{d^3k_1 \, d^3k_2 \, d^4P} = \int d^3p_+ \, d^3p_- \, \delta^4(p_+ + p_- - P) \times$$
$$\times \left| A_{\text{fi}}(k_1, k_2; p_+, p_-) \right|^2 , \tag{8}$$

where the super index (ρ) denotes the presence of the pion source formfactor $\rho(K - P)$. Taking an explicit form of amplitude (7), the expression on the r.h.s. of (8) can be represented as

$$\frac{dN^{(\rho)}}{d^3k_1 d^3k_2 d^4P} = |\rho(k_1 + k_2 - P)|^2 \, \frac{dN}{d^3k_1 d^3k_2 d^4P} , \tag{9}$$

where $dN/d^3k_1 d^3k_2 d^4P$ is the "standard" rate of particle pair production with the total momentum P,

$$\frac{dN}{d^3k_1 \, d^3k_2 \, d^4P} = \frac{\alpha^2 \, |F_\pi\left((k_1 + k_2)^2\right)|^2}{4\pi^4 \, 2E_1 \, 2E_2 \, P^4} \times$$
$$\times \int \frac{d^3p_+}{(2\pi)^3 \, 2E_+} \frac{d^3p_-}{(2\pi)^3 \, 2E_-} \, |m_{\text{fi}}|^2 \, \delta^4(p_+ + p_- - P). \tag{10}$$

Meanwhile, there are new important features relevant to (10) which are the consequence of the space-time restrictions on the vertex x_1 in the diagrams depicted in Fig. 2. Indeed, in the evaluation of the expression on the r.h.s. of (10), the relative movement of the pion-pion c.m.s. and the lepton pair c.m.s. should be taken into account. For instance, the velocity of the pion-pion c.m.s. is $\mathbf{v}_K = \mathbf{K}/K_0$, where $K = k_1 + k_2$, whereas the dilepton c.m.s. moves with velocity $\mathbf{v}_P = \mathbf{P}/P_0$. That is why, the velocities $\mathbf{v}_K$ and $\mathbf{v}_P$ are not identical now, as it would have been in the presence of the delta-function $\delta^4(K - P)$ when the annihilation reaction takes place in the infinite space-time. These two center-off-mass systems are "disconnected" now, and the relation between the total momenta K and P is weighted by the formfactor $|\rho(K - P)|^2$ which stands as a prefactor on the r.h.s. of (9).

Actually, the violation of the energy-momentum conservation is typical of quantum physics, when the process or reaction under consideration takes place during a finite time interval (energy uncertainty) or in a finite coordinate space region (momentum uncertainty). In essence, any system with finite lifetime reveals a resonance-like behavior, and the energy-smearing function $\rho(K^0 - P^0)$ can look, hence, like the Breit—Wigner function (Lorentz shape) $\rho(K^0 - P^0) = \frac{1}{\pi} \frac{\Gamma}{(K^0 - P^0)^2 + \Gamma^2}$ with $\Gamma = 1/\tau$ as the width of the energy probability distribution (τ is the lifetime) or like the Gaussian function $\rho(K^0 - P^0) = (\tau/\sqrt{\pi}) \exp[-(K^0 - P^0)^2 \tau^2]$. In these probability distributions, P^0 represents the mean value as an external quantity, and K^0 is a running value of energy which is distributed around P^0. In our case, the total energy of the pion pair K^0 is distributed around a fixed value P^0 (experimentally measured dilepton total energy), and the smearing is a consequence of the finiteness of the pion system lifetime.

Substituting the result of the evaluation of the integral on the r.h.s. of (10) in (9), we obtain

$$\frac{dN^{(\rho)}}{d^3k_1\, d^3k_2\, d^4P} = |\rho(k_1 + k_2 - P)|^2 \frac{(\mathbf{k}_1 - \mathbf{k}_2)^2_P}{3(2\pi)^5} \times$$

$$\times \frac{\alpha^2 \left|F_\pi\left((k_1 + k_2)^2\right)\right|^2}{4\pi^4\, 2E_1\, 2E_2\, P^2} \left(1 + \frac{2m_e^2}{P^2}\right)\left(1 - \frac{4m_e^2}{P^2}\right)^{1/2}, \quad (11)$$

where integration was done in the lepton pair c.m.s. Subindex P denotes that the quantity $(\mathbf{k}_1 - \mathbf{k}_2)^2$ is taken in the P-system which is, by definition, the lepton pair c.m.s., whereas, subindex K means that the quantity is taken in the K-system which is the pion pair c.m.s. As we have discussed above, such a separation is due

to the fact that the velocity $\mathbf{v}_K$ of the K-system and that $\mathbf{v}_P$ of the P-system do not coincide. For the further calculation, the quantity $(\mathbf{k}_1 - \mathbf{k}_2)^2_P$ will be Lorentz-transformed to $(\mathbf{k}_1 - \mathbf{k}_2)^2_K$. For this purpose, we start from the covariant relation $q_P^2 = q_K^2$, where the shorthand notation $q = k_1 - k_2$ is adopted. Since $\left(q_P^0\right)^2 = (q \cdot P)^2 / P^2$, one can write

$$\mathbf{q}_P^2 = \mathbf{q}_K^2 \left(1 + \frac{\mathbf{P}_K^2}{P^2} \cos^2\theta\right), \quad (12)$$

where θ is the angle between vectors $\mathbf{q}$ and $\mathbf{P}$ when they are considered in the pion pair c.m.s. Taking into account that $\mathbf{P}_K^2 = \left(P_K^0\right)^2 - P^2 = \frac{(P \cdot K)^2}{K^2} - P^2$, and substituting this relation in (12), we get the relation

$$(\mathbf{k}_1 - \mathbf{k}_2)^2_P = (\mathbf{k}_1 - \mathbf{k}_2)^2_K \left[1 + \cos^2\theta \left(\frac{(P \cdot K)^2}{P^2\, K^2} - 1\right)\right]. \quad (13)$$

So, the expression on the r.h.s. of (13) is related to the pion pair c.m.s.

The next step of evaluations needs the averaging of the quantity $dN^{(\rho)}/d^3k_1\, d^3k_2\, d^4P$ (11) over the pion momentum space with the distribution function which reflects the model of a pion system created in a relativistic heavy ion collision. For our estimation, we take the widely used thermodynamic model: the subsystem of pions which is under consideration is in the thermal equilibrium characterized by the temperature T. Then, one has to weight the incoming pion momenta by a thermal distribution function $f_{\text{th}}(E)$ and make integration with respect to these momenta:

$$\left\langle \frac{dN^{(\rho)}}{d^4P} \right\rangle = \int d^3k_1 f_{\text{th}}(E_1) \int d^3k_2\, f_{\text{th}}(E_2) \times$$

$$\times \frac{dN^{(\rho)}}{d^3k_1 d^3k_2 d^4P}. \quad (14)$$

It is pedagogical to represent this equation in the form where the explicit factorization of the pion source formfactor $|\rho(K - P)|^2$ is made:

$$\left\langle \frac{dN^{(\rho)}}{d^4P} \right\rangle = \int d^4K\ |\rho(K - P)|^2 \left\langle \frac{dN}{d^4K\ d^4P} \right\rangle. \quad (15)$$

Here, we introduce the auxiliary quantity

$$\left\langle \frac{dN}{d^4K\ d^4P} \right\rangle \equiv \int d^3k_1\ f_{\text{th}}(E_1) \int d^3k_2\ f_{\text{th}}(E_2) \times$$

$$\times \frac{dN}{d^3k_1 d^3k_2 d^4P} \delta^4(k_1 + k_2 - K), \quad (16)$$

which is the averaged dilepton number density with respect to the total 4-momenta K and P. In this formula, the spectrum $dN/d^3k_1 d^3k_2 d^4P$ equals the expression on the r.h.s. of (11) without the first factor $|\rho(k_1 + k_2 - P)|^2$. The meaning of the averaging on the r.h.s. of (16) is quite transparent: we integrate over pion momenta with weights which are the thermal distribution functions of pions in the momentum space, keeping meanwhile the total energy-momentum of the pion pair as a constant equal to K. The resulting quantity on the l.h.s. of (16) looks like we cut the s-channel of the reaction $\pi^+ \pi^- \to l \bar{l}$ into two independent blocks (vertices) (see the diagrams depicted in Fig. 2) and calculate the distribution $\langle dN/d^4K d^4P \rangle$ with respect to the incoming total momentum K and outgoing total momentum P independently. Then, Eq. (15) gives the connection of these vertices with a weight function, the formfactor $|\rho(K - P)|^2$, by reflecting the fact that the pion system lives in a finite space-time volume. On the other hand, one can regard Eq. (15) as an averaging of the *random* quantity $\langle \frac{dN}{d^4K d^4P} \rangle$ with the help of the distribution function $|\rho(K - P)|^2$, which is centered around the mean value P. In this sense, the hadron medium temporary holding pions in a local spatial region (this generates the local pion distribution $\rho(x)$) randomizes the system of pions on the quantum level. This randomization of the pion pair total momentum K is really a quantum one as a particular manifestation of the uncertainty principle. Note that the thermal randomization of the multipion system was included already to the quantity $\langle \frac{dN}{d^4K d^4P} \rangle$ (see (16)) through the thermal distribution functions $f_{\text{th}}(E)$.

Taking into account a result of integration of the expression on the r.h.s. of (16), we can rewrite (15) as

$$\left\langle \frac{dN^{(\rho)}}{d^4P} \right\rangle = \frac{\alpha^2 C^2}{3(2\pi)^8} \left(1 - \frac{4m_e^2}{P^2}\right)^{1/2} \left(1 + \frac{2m_e^2}{P^2}\right) \times$$

$$\times \int d^4K \, |\rho(K - P)|^2 \, e^{-\beta u \cdot K} \frac{K^2}{P^2} |F_\pi(K^2)|^2 \times$$

$$\times \left(1 - \frac{4m_\pi^2}{K^2}\right)^{3/2} \left[1 + \frac{1}{3}\left(\frac{(P \cdot K)^2}{P^2 K^2} - 1\right)\right], \quad (17)$$

where u is the collective 4-velocity of the pion system, which is an element of the fireball. We take the Boltzmann distribution $f_{\text{th}}(E) = C \exp(-\beta E)$ as a pion thermal distribution function with inverse temperature $\beta = 1/T$, where C is a normalization constant. The factor $1/3$ in the square brackets on the r.h.s. of (17) appears after the integration over angles. Expression (17) is a final result for the distribution of the number of

particles as a function of dilepton 4-momentum P. The further evaluation can be made for a particular form of the pion system formfactor $|\rho(K - P)|^2$.

We add for completeness that, for the process which corresponds to the diagram depicted in Fig. 2,*b* [model (b)], one should correct the last expression (17) by shifting the pion formfactor which is now taken in the form $|F_\pi(P^2)|^2$, in front of the integral.

The convergence of (17) to the standard result [17] can be recognized, when one changes the pion system formfactor to the δ-function, namely $\rho(K - P) \to (2\pi)^4 \delta(K - P)$. It is easy to see that the expression in square brackets can be immediately transformed to unity when $K = P$, because there is no difference between the P- and K-systems in this case. They both move now with the velocity $\mathbf{v}_K = \mathbf{v}_P = \mathbf{P}/P_0$. Then, dividing by the 4-volume $\tilde{T}V$ ($\tilde{T}$ is the time interval and V is the volume), we obtain the rate ($R \equiv N/\tilde{T}V$)

$$\left\langle \frac{dR}{d^4P} \right\rangle_{\substack{V \to \infty \\ \tilde{T} \to \infty}} = \frac{\alpha^2 C^2}{3(2\pi)^8} e^{-\beta P_0} |F_\pi(P^2)|^2 \times$$

$$\times \left(1 - \frac{4m_\pi^2}{P^2}\right)^{3/2} \left(1 - \frac{4m_e^2}{P^2}\right)^{1/2} \left(1 + \frac{2m_e^2}{P^2}\right). \quad (18)$$

To obtain the distribution of the number of lepton pairs with respect to the invariant mass M of a pair, it is necessary to integrate distribution (17), $\langle dN^{(\rho)}/d^4P \rangle$, over the 4-momentum space P setting the value P^2 to M^2, namely

$$\left\langle \frac{dN^{(\rho)}}{dM^2} \right\rangle = \int d^4P \left\langle \frac{dN^{(\rho)}}{d^4P} \right\rangle \delta(P^2 - M^2) \, \theta(P_0). \quad (19)$$

Hence, using our result (17), we can write the distribution of the number of created lepton pairs $N^{(\rho)}$ with respect to the invariant mass M of a lepton pair as

$$\left\langle \frac{dN^{(\rho)}}{dM^2} \right\rangle = \frac{\alpha^2 C^2}{3(2\pi)^8} \left(1 - \frac{4m_e^2}{M^2}\right)^{1/2} \left(1 + \frac{2m_e^2}{M^2}\right) \times$$

$$\times \int \frac{d^3P}{2\,\Omega(\mathbf{P})} \int d^4K \, \theta(K_0) \, \theta\left(K^2 - 4m_\pi^2\right) |\rho(K - P)|^2 \times$$

$$\times e^{-\beta K_0} \frac{K^2}{M^2} |F_\pi(K^2)|^2 \left(1 - \frac{4m_\pi^2}{K^2}\right)^{3/2} \times$$

$$\times \left[1 + \frac{1}{3}\left(\frac{(P \cdot K)^2}{M^2 K^2} - 1\right)\right] \quad (20)$$

with $P_0 = \Omega(\mathbf{P}) \equiv \sqrt{M^2 + \mathbf{P}^2}$. As for the presence of the θ-function $\theta\left(K^2 - 4m_\pi^2\right)$ on the r.h.s. of (20), we would like to stress that the integration with respect

to K is going on over the domain where the invariant mass of a pion pair $M_\pi \equiv \sqrt{K^2}$ is not lesser than two pion masses. On the other hand, possible finite values of the distribution $\langle dN^{(\rho)}/dM^2 \rangle$ below the two pion mass threshold can occur due to the smearing of the standard rate which is generated by the quantum fluctuations of the pion pair total momentum K as a realization of the uncertainty principle. These quantum fluctuations are visualized in (20) by the presence of the pion system formfactor $|\rho(K-P)|^2$. Note, if one follows model (b) which corresponds to the process depicted in Fig. 2b, then the ρ-meson formfactor on the r.h.s. of (20) should be taken in the form $|F_\pi(M^2)|^2$ and shifted to the front of the integral. Equation (20) is the main theoretical result of the present paper. The next steps in the investigations of the rate (20) can be made by taking a particular source formfactor $|\rho(K-P)|^2$ of the many-particle (multipion) system.

1.1. Gaussian Pion System Formfactor

To make an access to the effects under investigation transparent as much as possible, we take, as a model of the pion source, the Gaussian distribution in space and the Gaussian decay of the multipion system

$$\rho(x) = \exp\left(-\frac{t^2}{2\tau^2} - \frac{\mathbf{r}^2}{2R^2}\right), \qquad (21)$$

which transforms to 1 for the large enough mean lifetime τ and mean radius R of the system, i.e., when $\lim_{\substack{\tau \to \infty \\ R \to \infty}} \rho(x) = 1$, amplitude (4) comes to the standard form. The Fourier transformation of $\rho(x)$ reads

$$\rho(Q) = (2\pi)^2 \tau R^3 \exp\left[-\frac{1}{2}\left(Q_0^2 \tau^2 + \mathbf{Q}^2 R^2\right)\right]. \qquad (22)$$

The correspondence with the formfactor in case of the infinite space-time volume should be installed as

$$\lim_{\tau, R \to \infty} \rho^2(Q) = \tilde{T}V\,(2\pi)^4 \delta^4(Q), \qquad (23)$$

where, on the r.h.s., one δ-function is transformed to the 4-volume, $\tilde{T}$ is a time interval, and V is a spatial volume. On the other hand, for the l.h.s. of (23), we have explicitly

$$\lim_{\tau, R \to \infty} \rho^2(Q) = (\pi^2 \tau R^3)(2\pi)^4\,\delta^4(Q). \qquad (24)$$

Hence, by comparing the right-hand sides of (23) (24), we can conclude that the quantity $\pi^2 \tau R^3$ should represent the 4-volume $\tilde{T}V$ for the source parametrization (21).

With the use of the above source distribution function, Eq. (20) gives the spectrum with respect to the lepton pair invariant mass during a unit time interval and from the unit volume of the multipion system

$$\left\langle \frac{dN^{(\rho)}}{\tilde{T}V\,dM^2} \right\rangle = \frac{\alpha^2 C^2}{3(2\pi)^4}\left(1 - \frac{4m_e^2}{M^2}\right)^{1/2}\left(1 + \frac{2m_e^2}{M^2}\right) \times$$

$$\times \int d^4K\,\theta(K_0)\,\theta\left(K^2 - 4m_\pi^2\right)\,e^{-\beta K_0}\,\frac{K^2}{M^2}\,\left|F_\pi(K^2)\right|^2 \times$$

$$\times \left(1 - \frac{4m_\pi^2}{K^2}\right)^{3/2} \int \frac{d^3P}{2\sqrt{M^2 + \mathbf{P}^2}}\,\frac{\tau}{\pi^{1/2}}e^{-(K_0 - P_0)^2\tau^2} \times$$

$$\times \frac{R^3}{\pi^{3/2}}\,e^{-(\mathbf{K} - \mathbf{P})^2 R^2}\left[1 + \frac{1}{3}\left(\frac{(P \cdot K)^2}{M^2 K^2} - 1\right)\right], \qquad (25)$$

where $P_0 = \Omega(\mathbf{P}) \equiv \sqrt{M^2 + \mathbf{P}^2}$ and $\tilde{T}V = \pi^2 \tau R^3$. We write the source formfactor exponents together with relevant coefficients in the form which shows the evident transformation to the δ-functions $\delta(K_0 - P_0)$ and $\delta^3(\mathbf{K} - \mathbf{P})$ when $\tau \to \infty$ and $R \to \infty$, respectively.

After the integration over angle variables in both integrals on the r.h.s. of Eq. (25), we obtain the rate $(R^{(\rho)} \equiv N^{(\rho)}/\tilde{T}V)$ of dilepton production in the reaction $\pi^+ \pi^- \to l\bar{l}$

$$\left\langle \frac{dR^{(\rho)}}{dM^2} \right\rangle = \frac{\alpha^2 C^2}{3(2\pi)^3}\left(1 - \frac{4m_e^2}{M^2}\right)^{\frac{1}{2}}\left(1 + \frac{2m_e^2}{M^2}\right)\int_{-\infty}^{\infty} \frac{\bar{P}d\bar{P}}{P_0} \times$$

$$\times \int_0^{\infty} \bar{K}d\bar{K}\,\frac{R}{\pi^{1/2}}e^{-(\bar{K} - \bar{P})^2 R^2}\int_{\sqrt{4m_\pi^2 + \bar{K}^2}}^{\infty} dK_0\,e^{-\beta K_0} \times$$

$$\times \frac{\tau}{\pi^{1/2}}e^{-(K_0 - P_0)^2\tau^2}\,\frac{K^2}{M^2}\,\left|F_\pi(K^2)\right|^2\left(1 - \frac{4m_\pi^2}{K^2}\right)^{\frac{3}{2}} \times$$

$$\times \left[1 + \frac{1}{3M^2 K^2}\left((P \cdot K)^2 - M^2 K^2 + \frac{P \cdot K}{R^2} + \frac{1}{2R^4}\right)\right], \quad (26)$$

where we use the notations

$$\bar{K} \equiv |\mathbf{K}|,\quad \bar{P} \equiv |\mathbf{P}|,\quad P_0 = \Omega(\bar{P}) = \sqrt{M^2 + \bar{P}^2},$$

$$K^2 = K_0^2 - \bar{K}^2,\quad P \cdot K = \Omega(\bar{P})K_0 - \bar{P}\bar{K}.$$

Let us recall that the expression in square brackets on the r.h.s. of (26) is a relativistic correction which arises when we perform the Lorentz transformation of the quantity $(\mathbf{k}_1 - \mathbf{k}_2)^2$ from the lepton pair c.m.s. to the pion pair c.m.s. The influence of the finite spatial size of the system on the correction discussed is explicitly represented by two terms $\propto R^{-2}$ and $\propto R^{-4}$ which reflect the fact that the relativistic correction is larger for a smaller pion system. We note again that, within model

(b) corresponding to the process depicted in Fig. 2,*b*, just one correction should be done in expressions (25) and (26). Formally, it is necessary to change the pion formfactor argument from K^2 to M^2, and then it can be written in front of the integral in the form $\left|F_\pi(M^2)\right|^2$.

1.2. Passage to the Limits $R \to \infty$ and $\tau \to \infty$

First, we consider the limit $R \to \infty$, which means that the multipion system occupies the infinite volume in the coordinate space, but lives for a finite lifetime in this state. By performing the integration in Eq. (26) with the use of the δ-function $\delta(\bar{K} - \bar{P})$, we obtain

$$\left\langle \frac{dR^{(\rho)}}{dM^2} \right\rangle_{R \to \infty} = \frac{\alpha^2 C^2 \tau}{24 \pi^{7/2}} \left(1 - \frac{4m_e^2}{M^2}\right)^{1/2} \left(1 + \frac{2m_e^2}{M^2}\right) \times$$
$$\times \int_0^\infty \frac{\bar{P}^2 d\bar{P}}{\Omega(\bar{P})} \int_{\sqrt{4m_\pi^2 + \bar{P}^2}}^\infty dK_0 e^{-[K_0 - \Omega(\bar{P})]^2 \tau^2} e^{-\beta K_0} \frac{K^2}{M^2} \times$$
$$\times \left|F_\pi(K^2)\right|^2 \left(1 - \frac{4m_\pi^2}{K^2}\right)^{3/2} \left[1 + \frac{\bar{P}^2 \left(K_0 - \Omega(\bar{P})\right)^2}{3M^2 K^2}\right], \quad (27)$$

where $\Omega(\bar{P}) = \sqrt{M^2 + \bar{P}^2}$, $K^2 = K_0^2 - \bar{P}^2$. It is reasonable to note that, due to a finite lifetime τ of the multipion system and, thus, a finite lifetime of annihilating pion states, the off-shell behavior of the two-pion system is seen pretty well. Indeed, it is reflected by the presence of the distribution $(\tau/\sqrt{\pi}) e^{-[K_0 - \Omega(\bar{P})]^2 \tau^2}$ of the fluctuating total energy K_0 of the pion pair around the nominal quantity $\Omega(\bar{P}) = \sqrt{M^2 + \mathbf{P}^2}$ in the integrand on the r.h.s. of (27). At the same time, the discrepancy of K_0 and P_0 causes the difference of velocity $\mathbf{P}/K_0$ of the K-system and velocity $\mathbf{P}/P_0$ of the P-system. Hence, it determines the relativistic correction which is manifested by the presence of square brackets on the r.h.s. of (27). Let us consider now another limit. We take the infinite lifetime of the pion system ($\tau \to \infty$), but a finite volume of the reaction region. This leads to the equality of the total energies of lepton and pion pairs. The result of integration over K_0 on the r.h.s. of (26) with the help of the δ-function $\delta\left(K_0 - \Omega(\bar{P})\right)$ is not zero when the following inequality holds:

$$\sqrt{4m_\pi^2 + \bar{K}^2} \leqslant \Omega\left(\bar{P}\right) . \quad (28)$$

If the measured dilepton invariant mass M is larger than $2m_\pi$, inequality (28) results in a restriction from above of the integration over $\bar{K}$. Then the integration is carried out in the limits $[0, \bar{K}_{max}]$, where $\bar{K}_{max} = \sqrt{\bar{P}^2 + M^2 - 4m_\pi^2}$. On the other hand, if the measured dilepton invariant mass $M \leqslant 2m_\pi$, it is necessary to

introduce a restriction on the limits of integration over the total momentum of the lepton pair $\bar{P}$. By keeping all this together, we write the rate of the reaction $\pi^+ \pi^- \to l\bar{l}$ in the case of a finite spatial volume of the reaction region as

$$\left\langle \frac{dR^{(\rho)}}{dM^2} \right\rangle_{\tau \to \infty} = \int_{-\infty}^\infty d\bar{P} \, J\left(M, R, \bar{P}\right) -$$
$$- \theta\left(2m_\pi - M\right) \int_{-\bar{P}_{min}}^{\bar{P}_{min}} d\bar{P} J\left(M, R, \bar{P}\right) , \quad (29)$$

where $\bar{P}_{min} = \sqrt{4m_\pi^2 - M^2}$ and

$$J\left(M, R, \bar{P}\right) = \frac{\alpha^2 C^2 R}{24(\pi)^{7/2}} \frac{\bar{P} \, e^{-\beta \Omega(\bar{P})}}{\Omega\left(\bar{P}\right)} \int_0^{\bar{K}_{max}} \bar{K} d\bar{K} \times$$
$$\times e^{-(\bar{K} - \bar{P})^2 R^2} \frac{K^2}{M^2} \left|F_\pi(K^2)\right|^2 \left(1 - \frac{4m_\pi^2}{K^2}\right)^{3/2} \times$$
$$\times \left[1 + \frac{1}{3M^2 K^2} \left(\Omega^2\left(\bar{P}\right) \left(\bar{P} - \bar{K}\right)^2 + \frac{P \cdot K}{R^2} + \frac{1}{2R^4}\right)\right] \quad (30)$$

with $\Omega(\bar{P}) = \sqrt{M^2 + \bar{P}^2}$, $K^2 = \Omega^2\left(\bar{P}\right) - \bar{K}^2$, $P \cdot K = \Omega^2(\bar{P}) - \bar{P}\bar{K}$ and $\bar{K}_{max} = \sqrt{\bar{P}^2 + M^2 - 4m_\pi^2}$.

For completeness, let us consider the rate when we take both limits, $R \to \infty$ and $\tau \to \infty$. Equation (27) represents the rate in the case of infinite spatial volume. If, in addition, we consider a big enough lifetime of the multipion system, i.e. if we take the limit $\tau \to \infty$, this results in the appearance of the δ-function $\delta\left(K_0 - \Omega(\bar{K})\right)$ on the r.h.s. of (27), and the integration gives

$$\left\langle \frac{dR^{(\rho)}}{dM^2} \right\rangle_{\substack{R \to \infty \\ \tau \to \infty}} = \frac{\alpha^2 C^2}{3(2\pi)^3} \left(1 - \frac{4m_e^2}{M^2}\right)^{1/2} \left(1 + \frac{2m_e^2}{M^2}\right) \times$$
$$\times \left|F_\pi(M^2)\right|^2 \left(1 - \frac{4m_\pi^2}{M^2}\right)^{3/2} \frac{M}{\beta} K_1(\beta M) , \quad (31)$$

where $K_1(z)$ is a McDonald function of the first order. So, we come to the standard result [17] for the rate of dilepton emission in $\pi^+ \pi^-$ annihilation in the case of the Boltzmann distribution of pions in the stationary infinite multipion system.

1.3. Evaluations of the Dilepton Emission Rate

Before going to numerically evaluate the rate of dilepton emission, let us briefly consider the geometry of a multipion system.

After the nuclei collision, the highly excited nuclear matter goes through several stages. At the same time, all steps of the particle-particle transformations take place against the background of the spatial dynamics of the
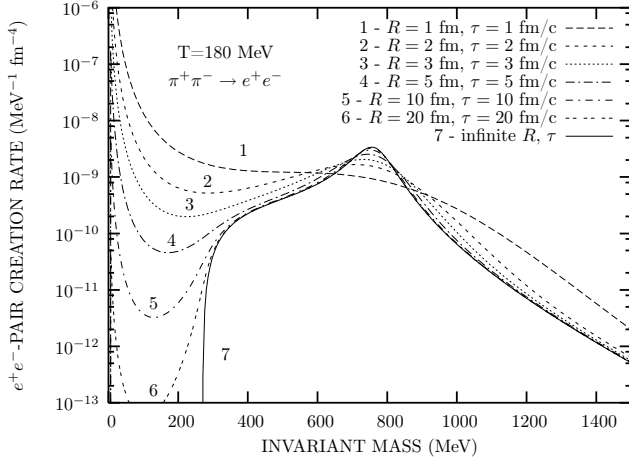
Fig. 3. The rate $dR^{(\rho)}_{e^+e^-}/dM$ of electron-positron pair creation in pion-pion annihilation. Different curves correspond to the different spatial sizes R and different lifetimes τ of a hot pion system, $T = 180$ MeV

Fig. 4. The rate $dR^{(\rho)}_{\mu^+\mu^-}/dM$ of $\mu^+\mu^-$ pair creation in pion-pion annihilation. Different curves correspond to the different spatial sizes R and different lifetimes τ of a hot pion system, $T = 180$ MeV

system. It is believed that the total system of pions which were created after hadronization expands in all directions, and it is a hard task to find even quasi-equilibrated pions in the fireball taken as a whole.

Let us, however, consider a locally equilibrium system of pions which is a small part of the total system (fireball) and moves with a collective velocity $\mathbf{v}$. We draw it schematically in Fig. 1, where the arrows mean the radially directed velocities of the expanding hadron matter. Just to be transparent in the consideration of finite space-time effects, we adopt a quasi-static picture. If, for heavy colliding nuclei, the fireball is of the mean radius $R_0 \propto 4 \div 10$ fm, what is known from interferometry, then the spatial region occupied by pions which are in local equilibrium is of the size $R \propto 1 \div 5$ fm. This small region is depicted in Fig. 1 as a circle of radius R on the body of the fireball. Hence, the consideration of the previous paragraphs can be attributed to the multipion subsystem which is in the local equilibrium in a small spatial region (on the scale of the fireball size) characterized by radius R. The evolution of this multipion subsystem in time is restricted by the mean lifetime τ which is not bigger than the total lifetime of the fireball. We evaluate numerically integrals (26) for different sets of parameters R and τ. Fig. 3 shows the creation rate $dR^{(\rho)}_{e^+e^-}/dM$ for electron-positron pairs appearing from a hot multipion subsystem characterized by the temperature $T = 180$ MeV. There are seven curves in this figure which are labelled by numbers $1, \ldots, 7$ in the following correspondence to the values of the mean size of the pion subsystem and its lifetime: 1) $R = 1$ fm, $\tau = 1$ fm/c; 2) $R = 2$ fm, $\tau = 2$ fm/c;
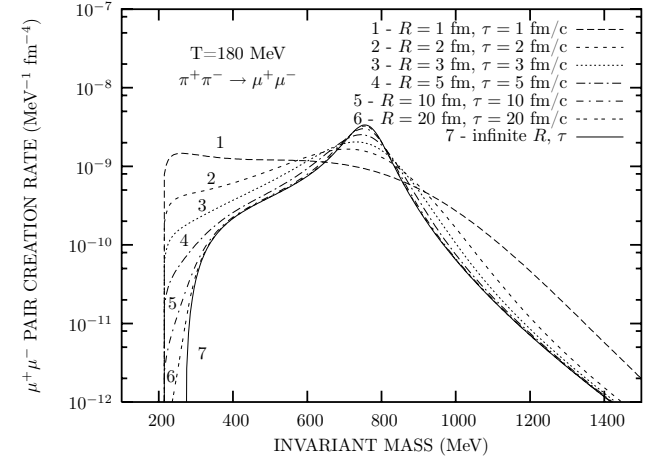
3) $R = 3$ fm, $\tau = 3$ fm/c; 4) $R = 5$ fm, $\tau = 5$ fm/c; 5) $R = 10$ fm, $\tau = 10$ fm/c; 6) $R = 20$ fm, $\tau = 20$ fm/c; 7) $R = \infty$, $\tau = \infty$. The evaluation for the infinite space-time volume (see (31)), corresponds to curve 7. Notice how the e^+e^- emission rate deviates from the standard one (curve 7): it is finite in the region of the lepton invariant mass $M \leqslant 2m_\pi$. As we discussed above, it is a consequence of the quantum fluctuations of the total pion pair momentum K around the total lepton pair momentum P (uncertainty principle). We emphasize as well that the tendency of curves 1, 2 in Fig. 3 is similar to CERES data [1, 2]. In Fig. 4, we show the evaluation of the creation rate $dR^{(\rho)}_{\mu^+\mu^-}/dM$ for $\mu^+\mu^-$-pairs at the same temperature $T = 180$ MeV, as for electron-positron creation. There are also seven curves in this figure which are labelled by numbers from 1 to 7 in the same correspondence to the values of the mean size R and mean lifetime τ of the pion subsystem as we take for the previous figure. Notice that the stronger the deviation of the $\mu^+\mu^-$ creation rate for a finite pion system from that for an infinite pion system, the lesser the spatial size and lifetime of the pion system similarly to the case of e^+e^- creation. Of course, it is a reflection of the uncertainty principle which is formally expressed by the presence of the distribution (pion source formfactor) $|\rho(K - P)|^2$ as a factor of the integrand on the r.h.s. of (15). The presence of the formfactor of the pion subsystem results in a broadening of the rate: the effect is bigger when the parameters R and τ are smaller. Evidently, the biggest broadening was obtained for the pion subsystem with mean radius $R = 1$ fm and lifetime $\tau = 1$ fm/c. In contrast to the standard result, Fig. 4
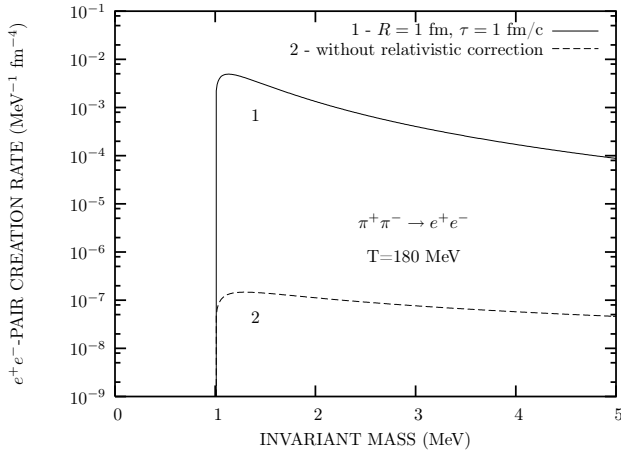
Fig. 5. The rate $dR^{(\rho)}_{e^+e^-}/dM$ of electron-positron pair creation in pion-pion annihilation in the threshold region. Mean spatial size $R = 1$ fm, lifetime $\tau = 1$ fm/c, $T = 180$ MeV. Curve *1* is evaluated in accordance with expression (26). Curve *2* is evaluated for the same integral (26) but without the last factor in square brackets which reflects a relative movement of the pion-pion c.m.s. and the electron-positron c.m.s.

shows that the threshold of $\mu^+\mu^-$ creation is at the value of invariant mass $M^{\mu^+\mu^-}_{\text{thresh}} = 2m_\mu \approx 211.3$ MeV.

An analogous threshold for electron-positron creation, $M^{e^+e^-}_{\text{thresh}} = 2m_e \approx 1.02$ MeV, is not been visible on the scale of mass span which is taken in Fig. 3. In Fig. 5, we show the creation rate $dR^{(\rho)}_{e^+e^-}/dM$ for electron-positron pairs in the region of the threshold for the mean radius $R = 1$ fm and lifetime $\tau = 1$ fm/c of the pion subsystem. The solid curve is a result of the evaluation of expression (26). Dashed curve *2* is a result of the evaluation of expression (26) which is shortened by dropping out the last square brackets on the r.h.s. of the equation. Recall that the expression in these square brackets carries the effect of a relative movement of the pion-pion c.m.s. and the electron-positron c.m.s., which is a consequence of the violation of the energy-momentum conservation in s-channel. We see that, in the region of the electron-positron creation threshold $M = 2m_e$, this effect (Fig. 5, curve *1*) is realized as four additional orders to the creation rate comparing to the rate without taking into account the relative velocity (curve *2*).

Fig. 6 shows the same evaluations as the previous one but in a wider range of invariant masses. Curves *1, 2, 3* are evaluated in accordance with formula (26), whereas, curves *1n, 2n, 3n* are evaluated in accordance with Eq. (26), where the relativistic factor [the last square brackets on the r.h.s. of (26)] is dropped out.
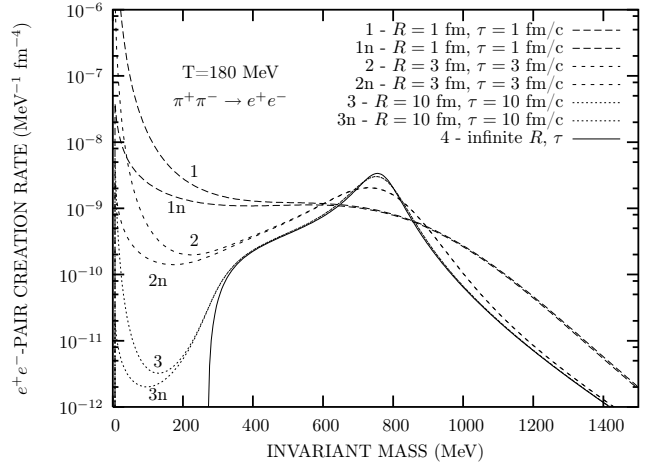

Fig. 6. The rate $dR^{(\rho)}_{e^+e^-}/dM$ of electron-positron pair creation in pion-pion annihilation with (curves *1, 2, 3*) and without (curves *1n, 2n, 3n*) a c.m.s. relativistic correction. Different curves correspond to the different spatial sizes R and different life times τ of the pion system. Curve *4* corresponds to the infinite size of the pion system

Dash-dotted curves *1* and *1n* correspond to $R = 1$ fm, $\tau = 1$ fm/c; solid curves *2* and *2n* correspond to $R = 3$ fm, $\tau = 3$ fm/c; dashed curves *3* and *3n* correspond to $R = 10$ fm, $\tau = 10$ fm/c, and long-dash-dotted curve *4* corresponds to the infinite space-time volume of the reaction region. We see that the relativistic effect under consideration is attributed to the region of invariant masses which is below the invariant mass $M = 2m_\pi$ and increases with decrease in the size of the pion system. The latter is partially explained by the presence of last two terms, $P \cdot K/R^2$ and $1/(2R^4)$, in square brackets on the r.h.s. of (26). Notice that this effect for the muon pair creation rate is practically not visible even for the pion system of the size $R = 1$ fm, $\tau = 1$ fm/c, because the real threshold $2m_\mu$ is close to the $2m_\pi$.

Next, we compare the emission rates obtained in the frame of two models (a) and (b). Model (a) (see Fig. 2,a) reflects the presence of a dense hadron environment which affects the ρ-meson propagation: due to the medium effects, the ρ-meson mean free path and mean lifetime are strongly suppressed inside the dense nuclear matter. As a limit of this suppression, we adopt in model (a) a zero mean free path and zero lifetime keeping vacuum parameters of the ρ-meson formfactor (5). Model (b) (Fig. 2,b) is a contra-pole to model (a): the ρ-meson propagation is not affected at all by the hadron environment. Then, the real picture of the pion annihilation in a dense nuclear matter will be somewhere between these two limit models. Fig. 7 shows the rate
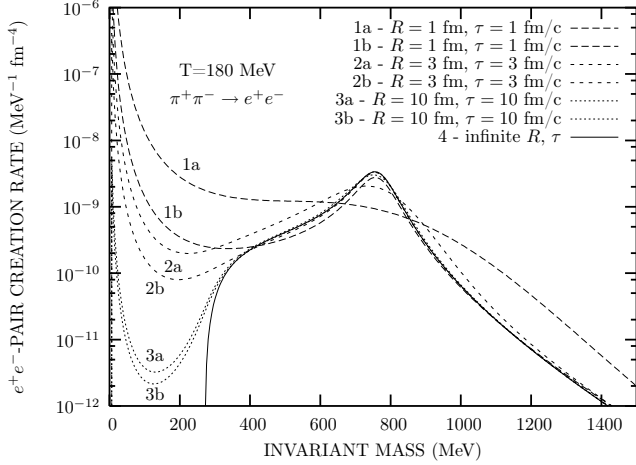
Fig. 7. The rate $dR^{(\rho)}_{e^+e^-}/dM$ of electron-positron pair creation in pion-pion annihilation. All curves which are labelled by letter "a" were evaluated in the model which corresponds to the diagram depicted in Fig. 1,*a*. All curves which are labelled by letter "b" were evaluated in the model which corresponds to the diagram depicted in Fig. 1,*b*. The values of the mean spatial size and life time: dash-dotted curves (*1a* and *1b*) — $R = 1$ fm, $\tau = 1$ fm/c; dashed curves (*2a* and *2b*) — $R = 3$ fm, $\tau = 3$ fm/c; dotted curves (*3a* and *3b*) — $R = 10$ fm, $\tau = 10$ fm/c; solid curve (*4*) — infinite space-time volume. Effective temperature $T = 180$ MeV

$dR^{(\rho)}_{e^+e^-}/dM$ of electron-positron pair creation evaluated in the frame of models (a) and (b). Curves *1a, 2a,* 3a are obtained as above by the evaluation of formula (26). Curves *1b, 2b, 3b* are obtained by the evaluation of formula (26), when the argument of the pion formfactor is changed from K^2 to M^2. Then it can be written in front of the integral in the form $\left|F_\pi(M^2)\right|^2$. This change corresponds to the integration of the vertex x'_1 (see Fig. 2,*b*) over infinite space-time volume. Dash-dotted curves *1a* and *1b* correspond to $R = 1$ fm, $\tau = 1$ fm/c; dashed curves *2a* and *2b* correspond to $R = 3$ fm, $\tau = 3$ fm/c; dotted curves *3a* and *3b* correspond to $R = 10$ fm, $\tau = 10$ fm/c, and solid curve *4* corresponds to the infinite space-time volume of the reaction region. The temperature of the locally equilibrated pion system was taken $T = 180$ MeV. The comparison of curves *1a* and *1b* evidently shows that the ρ-meson peak becomes much more pronounced, when we shift from model (a) to model (b). Hence, we can conclude that, due to the small mean size and lifetime of the pion subsystem, we obtain a strong effective smearing of the ρ-meson formfactor $\left|F_\pi(K^2)\right|^2$ in the frame of model (a). But this difference becomes smaller with increase in the mean parameters R and τ. For instance, the behavior of curves *2a* and *2b* is qualitatively the same for $R = 3$ fm, $\tau = 3$ fm/c.
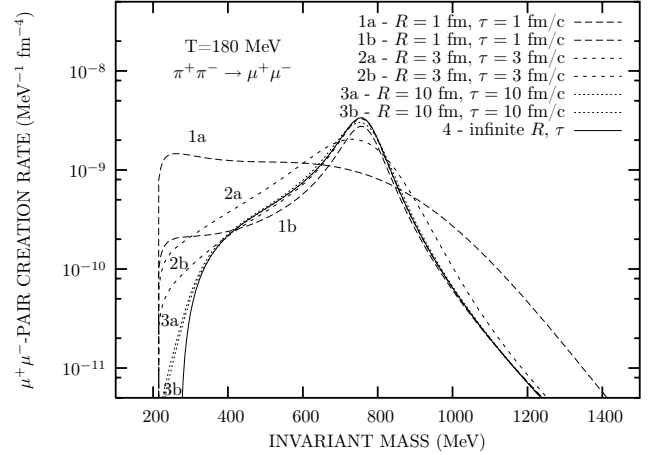

Fig. 8. The rate $dR^{(\rho)}_{\mu^+\mu^-}/dM$ of $\mu^+\mu^-$ pair creation in pion-pion annihilation. All curves are labelled as in Fig. 7. The set of parameters is also taken as in Fig. 7

Fig. 8 shows the emission rate $dR^{(\rho)}_{\mu^+\mu^-}/dM$ of the $\mu^+\mu^-$ pair creation for models (a) and (b). We see that model (a) again reveals indeed a very strong smearing of the ρ-meson peak because of the presence of the pion system formfactor $\left|\rho(K - P)\right|^2$, when it characterized by the smallest parameters under consideration: $R = 1$ fm, $\tau = 1$ fm/c. Meanwhile, for the same set of parameters, the $\mu^+\mu^-$ emission rate evaluated in the frame of model (b) (dash-dotted curve *1b*) practically coincides with the standard result (solid curve *4*) in the region $M \geqslant 450$ MeV. But, for invariant masses $M \leqslant 450$ MeV, the emission rate reveals a strongly pronounced flat shoulder with a cut off at the threshold $M = 2m_\mu = 211.3$ MeV. For the parameters $R = 3$ fm, $\tau = 3$ fm/c, the emission rates for models (a) and (b) are not so distinguishable for invariant masses above the ρ-peak (dotted curves *2a* and *2b*). But when the invariant mass is below the ρ-peak, these emission rates manifest a well visible corridor where, as we discussed before, the real rate should be found.

At last, when the mean size of the pion subsystem is $R = 10$ fm, i.e. is comparable to the size of the fireball, and the mean lifetime is $\tau = 10$ fm/c which is also comparable with the lifetime of the fireball, the emission rates evaluated in the frame of models (a) and (b) are not distinguishable (dense dotted curves *3a* and *3b*), but a cut off of the rates is at the threshold $M = 2m_\mu$.

For comparison, we present in Figs. 9 and 10 the results of the evaluation of the rate $dR^{(\rho)}_{e^+e^-}/dM$ of electron-positron pair creation in a QGP (quark-gluon plasma) drop from quark-antiquark annihilation. The
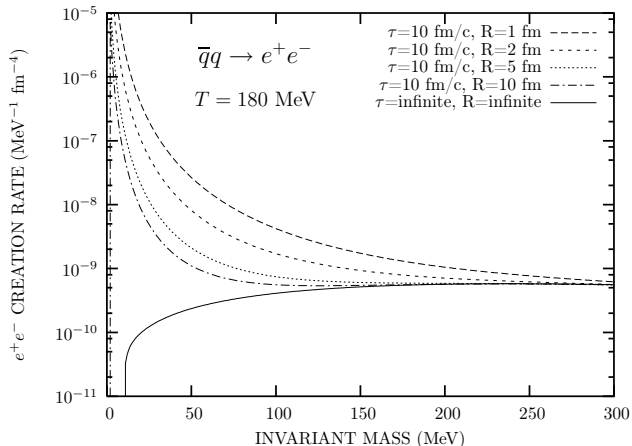
Fig. 9. The rate $dR^{(\rho)}_{e^+e^-}/dM$ of electron-positron pair creation in quark-antiquark annihilation. Different curves correspond to the different spatial sizes R at the lifetime $\tau = 10\,\mathrm{fm}/c$ of a QGP drop, $T = 180$ MeV

Fig. 10. The rate $dR^{(\rho)}_{e^+e^-}/dM$ of electron-positron pair creation in quark-antiquark annihilation. Different curves correspond to the spatial size $R = 2$ fm and the different life times τ of QGP drop, $T = 180$ MeV

evaluation was carried out under the same assumptions as for pion-pion annihilation. As in the previous case, we see the increase of the rate with decrease in the invariant mass up to the two-quark mass threshold, i.e. $M = 2m_q = 10$ MeV. The different behavior of the rate (compared to the case of infinite space-time) for small invariant masses $M \leqslant 300$ MeV is due to quantum fluctuations which occurs because of a finite size of the QGP drop. It should be pointed out the absence of the dependence of the rate on the lifetime of a quark-antiquark system when the size of the system is comparatively small, for instance $R = 2$ fm like in Fig. 10. This fact can be explained with the use of the quark (or pion) density taken in particular form (21). Indeed, for the system which is confined in three spatial dimensions, every of three spatial coordinate densities in (21), for instance, $\exp(-x_i^2/R^2)$ with $i = 1, 2, 3$, corresponds to one integration in (15) over a component of the pion pair total momentum K_i. Hence, there are three integrations on the r.h.s. of (15) every of each carries a factor which is responsible for quantum fluctuations in its own spatial dimension, for instance, one of three factors is $(2\pi)^{1/2}R\exp(-R^2 K_i^2)$. Meanwhile, the spectral function, which is responsible for the finiteness of the system lifetime, for instance $(2\pi)^{1/2}\tau\exp(-\tau^2 K_0^2)$ as in our case, emerges as an integrant factor just once during integration over K_0. That is why the variations of the spatial size of a many-particle system influence the shape of the dilepton production rate much stronger than the variations of the system lifetime.
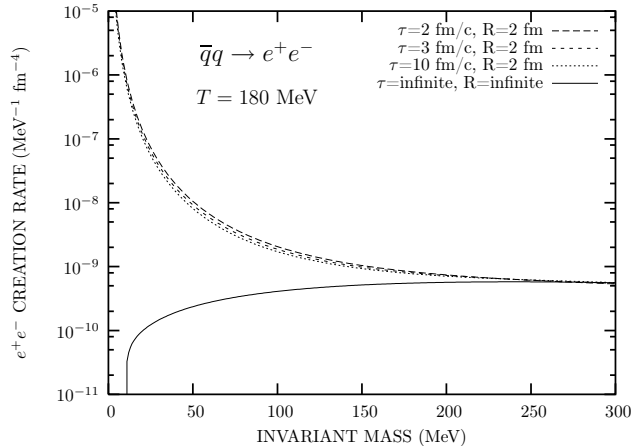
2. Discussion and Conclusions

In the present work, we have studied the effect of pion-pion and quark-quark annihilation in a finite space-time volume on the dilepton yield. Due to the uncertainty principle, a restriction of the volume and time of these reactions results in breaking the detailed energy-momentum conservation in the s-channel of the reactions. Formally, this expresses in the alteration of $(2\pi)^4\delta^4(K - P)$ to the distribution $\rho(K - P)$ in the amplitude of the reaction, where $|\rho(K - P)|^2$ appears as the formfactor of a multipion subsystem (quark-antiquark system), $K = k_1 + k_2$ is the total 4-momentum of an annihilating pion (quark) pair, and $P = p_+ + p_-$ is the total 4-momentum of the created lepton pair. For this kinematics, the energy-momentum conservation is valid as an integral law which is guaranteed by the relation $\int [d^4K/(2\pi)^4]\rho(K - P) = 1$, where the registered 4-momentum P plays the role of the center (or the mean value) of the distribution $\rho(K - P)$ with respect to the total 4-momentum K of an annihilating pair as a fluctuating quantity. As a result of the above mechanism, the rate of lepton pair yield is a finite quantity below the "threshold" $M = 2m_\pi$. Actually, the real threshold is determined as the total mass of the registered particles, i.e., $M^{e^+e^-}_{\mathrm{thresh}} = 2m_e$ for e^+e^- production (see Fig. 5), where m_e is the electron mass, and $M^{\mu^+\mu^-}_{\mathrm{thresh}} = 2m_\mu$ for $\mu^+\mu^-$ production (see Fig. 4), where m_μ is the mass of a μ-meson. In the calculations made, we kept the invariant mass of a pion-pion (quark-quark) pair not lesser than $2m_\pi$ ($2m_q$), or the range

of integration over the total 4-momentum K of a pion (quark) pair was determined as $K^2 \geqslant 4m_\pi^2$ ($K^2 \geqslant 4m_{\rm q}^2$).

At first sight, the appearance of finite values of the rate below the invariant mass $M = 2m_\pi$ can look paradoxical. Meanwhile, we recall that, in the calculation of the rate (see (17) and (20)), replacing a δ-function in the integrand by a distribution $|\rho(K-P)|^2$ results in the additional integration where the total momentum of a registered pair P or its invariant mass $M = \sqrt{P^2}$ plays the role of an external parameter. Then, if the distribution $|\rho(K-P)|^2$ is wide enough, it can lead to a finite result of integration for those values of P or M which gave zero value of the integral in the presence of the δ-function $\delta^4(K-P)$.

To give the intuitive feeling how it is going on, let us consider the toy averaging of a "good" function $R(M')$ over the fluctuating variable M' by using the distribution function $\rho(M' - M) = (\tau/\sqrt{\pi}) \exp\left[-(M'-M)^2\tau^2\right]$ which has the mean value M (measurable quantity) and the width of the distribution $1/\tau$. Then, this averaging reads

$$\langle R \rangle(M) = \int_{2m_\pi}^{\infty} dM'\, R(M')\, \frac{\tau}{\sqrt{\pi}}\, e^{-\left(M'-M\right)^2\tau^2}\ , \qquad (32)$$

where the variable M' (an analog of the two-pion invariant mass) varies in the region $2m_\pi < M' < \infty$. For long enough lifetimes ($\tau \to \infty$), we immediately obtain, as a result of averaging the function R taken at the value $M' = M$, $\langle R \rangle(M) = R(M)\theta(M - 2m_\pi)$. The presence of the θ-function in this expression stresses the fact that, for the infinite lifetime, the invariant mass M of a lepton pair should be bigger than the total mass of annihilating particles, it is $2m_\pi$ in the case under consideration. Meanwhile, for a finite lifetime, for instance $\tau = 1 \div 10\,{\rm fm}/c$, the integral is not zero for mean values of the distribution M which are lesser than $2m_\pi$. For example, we can consider even the extremely small value $M = 2m_e \approx 1$ MeV. Then, though the distribution function $\rho(M' - M)$ is centered around the mean value $M = 2m_e$, i.e. beyond the region of integration $2m_\pi \leqslant M' < \infty$, the right wing of the distribution $\rho(M' - M)$ is valued enough to make the result of integration on the r.h.s. of (32) quite significant to be taken into account. The illustration of this evaluation is depicted in Fig. 11, where the integrand in (32) is shown schematically as a solid line. The value of integral (32) $\langle R \rangle(M)$ equals the shaded area which starts from the value $M' = 2m_\pi$.
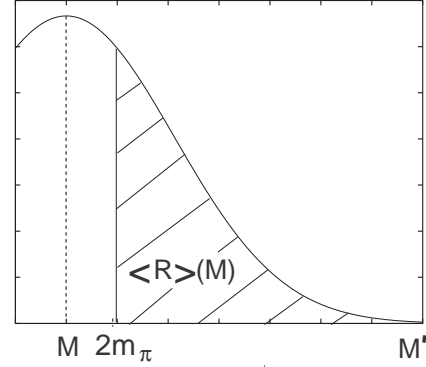

Fig. 11. Sketch of the integrand in Eq. (32). The integral is evaluated when the value of the parameter M is lesser than the lower limit of integration, $2m_\pi$

In a similar way, if the lifetime or/and size of the multipion system is small enough, we obtain a nonvanishing rate for dilepton invariant masses which are less than $2m_\pi$, whereas the invariant masses of the two-pion system (it is M' in our toy example) is certainly bigger than $2m_\pi$.

One of the consequences of breaking the exact equality between the total 4-momentum K of an incoming pion pair and the total 4-momentum P of an outgoing lepton pair is a noncoincidence of two center-of-mass systems: velocity of the $\pi^+ \pi^-$ c.m.s., for instance in lab system, differs from velocity of the $l^+ l^-$ c.m.s. This results in the following: if some quantity was obtained through calculations in the $l^+ l^-$ c.m.s. for its use in the $\pi^+ \pi^-$ c.m.s. this quantity should be Lorentz-transformed. The Lorentz transformation brings an additional deviation of the dilepton yield from the standard one. As seen from Figs. 5, 6, the manifestation of this effect for $e^+ e^-$ production is essential in the region which is below $M = 2m_\pi$ and the effect is washed out practically for invariant masses which are bigger than two pion masses. Note that, this effect is not visible practically for $\mu^+ \mu^-$ production (see Fig. 4) because the threshold invariant mass is big enough, $M_{\rm thresh}^{\mu^+ \mu^-} = 2m_\mu \approx 211.3$ MeV, and lies very close to $M = 2m_\pi$. We use the concept that a dense hadron medium decreases the ρ-meson mean free path and lifetime. As a limit of this diminution, one can assume the zero mean free path and the zero mean lifetime of a ρ-meson. We considered two models of pion-pion annihilation in a hadron medium. The first model (a) (Fig. 2,a) mimics an extreme influence of the dense hadron environment through adopting a ρ-meson mean free path and mean lifetime to be equal to zero (a ρ-meson does not propagate in the hadron environment), whereas the parameters of the ρ-meson formfactor is

taken as the vacuum one. In accordance with the second model (b) (Fig. 2,*b*), a ρ-meson propagates in the hadron environment without any suppression due to the environment (the vacuum parameters of the ρ-meson propagator are adopted as well). The models under consideration reflect two limiting cases: an extreme influence of the dense hadron medium [model (a)] and the absence of influence of the hadron environment [model (b)] on the ρ-meson mean free path and mean lifetime. Then, the evaluations of dilepton rates in the frame of both models mean the determination of the corridor which should absorb a real rate. We compared the rates evaluated in the frame of both models and found that the difference between them is considerable just for small sizes of the multipion systems (see Figs. 7, 8). So, if annihilating (interacting) particles are confined to a finite space-time region, this inspires several effects which give rise to the new features of the dilepton production rate in comparison to the standard rate, which is attributed to the infinite space-time. In the present paper, we have considered just two of these effects: 1) Direct contribution from the multiparticle system formfactor and 2) the relativistic correction which is due to the difference of velocities of the c.m.s. of outgoing particles ($l^+ l^-$ pair) and the c.m.s. of incoming particles ($\pi^+ \pi^-$ or $q\bar{q}$ pair). It is found that even these two contributions give a significant effect on the dilepton yield. Meanwhile, the third contribution which comes from the nonstationarity of a confined multiparticle (multipion, multiquark) system is still left beyond the scope of the paper. We reserve a consideration of this problem to the next paper which is in progress as well as the application of the approach elaborated to other reactions which take place in quark-gluon and hadron plasmas.

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АНІГІЛЯЦІЙНИЙ МЕХАНІЗМ ВИПРОМІНЮВАННЯ ДИЛЕПТОНІВ З ФАЄРБОЛА СКІНЧЕННИХ РОЗМІРІВ

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Р е з ю м е

Розглянуто зміни в середовищі процесів анігіляції $\pi^+ \pi^-$ та $q\bar{q}$ у дилептони при релятивістських зіткненнях важких іонів. Внаслідок щільного адронного оточення піони, які утворилися під час зіткнення, ефективно утримуються в скінченному об'ємі, в якому вони існують скінченний час, що за масштабом збігається з часом життя фаєрбола. Припускаючи вакуумні значення маси та ширини ρ-мезонного форм-фактора, ми провели порівняння двох підходів, які описують поведінку ρ-мезонів у середовищі. У першому підході припускається нульова довжина вільного пробігу ρ-мезона у середовищі, тоді як у другому підході він поширюється, як у вакуумі. Наші результати показують, що завдяки просторово-часовій скінченності пі-мезонної системи, що генерує відповідну квантову хаотизацію, дилептонні спектри відмінні від нуля в області малих інваріантних мас, менших за дві маси пі-мезона. Знайдено, що просторова обмеженість хвильових функцій кварків, а також скінченність часу життя станів веде до аналогічного ефекту при $q\bar{q}$-анігіляції у дилептони. Обговорюється порушення детального закону збереження енергії-імпульсу, пов'язане з порушенням трансляційної інваріантності у середовищі.