STUDY OF THE EXCITON CONDENSED PHASE IN 2D SYSTEMS

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A theory of exciton condensed phase formation in 2D systems is developed with regard for the temperature of a system and different lifetimes of free excitons and excitons in the condensed state. It is shown that the condensed phase in 2D systems consists of exciton islands. The mean radius of such an island increases with both lifetimes, the exciton creation rate, the temperature of a system, and the surface energy of condensed excitons. The dependence of the mean radius of 2D condensed islands on the lifetime, temperature, exciton creation rate, and mean distance between exciton islands is presented.

Introduction

Low-dimensional electron-hole and exciton liquids are of increasing interest now [1—6]. In particular, there were many attempts to observe the condensed phase in such a system [1, 7]. An interesting system in this respect is a quasi-two-dimensional system of indirect excitons in coupled quantum wells [1, 8, 9]. In such a system, an electron and a hole are spatially separated, so the recombination processes are difficult to occur, the exciton lifetime is large, and one can make a concentration which is adequate to observe the condensed phase. In a series of works, there were attempts to find the Bose—Einstein condensate [1, 8, 10] in such systems.

It is necessary to explore the relation between the level of irradiation and the condensed phase parameters, as well as the role of fluctuations in 2D systems. So far, there is no unambiguous observation of the Bose condensation, though there are many theoretical investigations and experimental indications of it [4, 6, 10].

In the present work, by assuming that the system temperature is larger than the supposed temperature of the Bose condensation and the creation of the condensed phase is a phase transition of the first order, we investigate the dependence of the main parameters of the 2D islands of condensed excitons (similarly to 3D droplets investigated in [11]) on temperature, the exciton creation rate, exciton lifetime, surface energy of exciton islands, and kinetic parameters of the system.

1. Model and Basic Equations

The size of exciton islands (a condensed phase structure inside the domain of free excitons) is determined by four processes: the capture of excitons from the environment, escape of excitons from an island, creation of excitons due to external irradiation, and their decay due to light emission and different destructive processes. Let f_n denote the distribution function of n, the number of excitons in an island. Taking into account the processes mentioned above, a kinetic equation for the distribution function can be presented in the following form:

$$\frac{\partial f_n}{\partial t} = -j_{n+1} + j_n, \tag{1}$$

where j_n is the probability current

$$j_n = (2\pi R_{n-1} c(R_{n-1}) W_{in}(R_{n-1}) + K(n-1)S_0) f_{n-1} - $$
$$-(2\pi R_n c_{in} W_{out}(R_n) + \frac{n}{\tau_1}) f_n, \tag{2}$$

R_n is the radius of the island with n excitons, $c(R_n)$ and c_{in} are the concentrations of excitons on the border of the island and inside it, respectively ($c_{\text{in}} = 1/S_0$, where S_0 is the area occupied by a single exciton inside the island), $W_{\text{in}}(R_n)$ and $W_{\text{out}}(R_n)$ are, respectively, the probabilities of the particle transitions from the outside of the island into it and in the opposite direction per unit length, K is the creation rate, i.e., the number of excitons created in unit area per unit time, and τ_1 is the exciton lifetime inside the island.

As a consequence of the detailed balancing principle, we can obtain the relation for the probabilities $W_{\text{in}}(R_n)$ and $W_{\text{out}}(R_n)$

$$\frac{W_{\text{out}}(R)}{W_{\text{in}}(R)} = \frac{W_{\text{out}}(\infty)}{W_{\text{in}}(\infty)} \exp\left(\frac{\alpha}{R}\right), \tag{3}$$

where $W_{\text{out}}(\infty)$ and $W_{\text{in}}(\infty)$ are the probabilities in the case of a plane boundary between phases,

$W_{\rm out}(\infty)/W_{\rm in}(\infty) = c_{\rm in}/c(\infty)$, $c(\infty)$ is the equilibrium concentration of excitons for a plane boundary between the condensed and gas phases, $c(\infty) = \left(m\kappa T/2\pi\hbar^2\right)\gamma\exp(-\phi/\kappa T)$, ϕ is the condensation energy per exciton, m is the effective exciton mass, γ is the exciton state degeneracy, $\alpha/R = (2S_0\sigma)/(\kappa T R)$ is the change in the surface energy on changing the number of particles by 1, and σ is the surface tension.

By generalizing the analysis presented in [11] on a $2D$ system, we obtain the radius distribution function in the stationary case as

$$f(\tilde{R}) = = f_0 \exp\left[\int\limits_0^{\tilde{R}} \frac{w[\tilde{c}(\tilde{R}) - \tilde{c}_\infty e^{\alpha/R}] + \frac{\tilde{R}}{2}(\tilde{c}_K - 1)}{w[\tilde{c}(\tilde{R}) + \tilde{c}_\infty e^{\alpha/R}] + \frac{\tilde{R}}{2}(\tilde{c}_K + 1)} 4\pi\tilde{R}d\tilde{R}\right], \quad (4)$$

where we have introduced the following dimensionless variables:

$$\tilde{R} = \frac{R}{S_0^{1/2}}, \quad \tilde{c}(R) = c(R)S_0,$$

$$w = \frac{W_{\rm in}(R)\tau_1}{S_0^{1/2}}, \quad \tilde{c}_{K1} = KS_0\tau_1, \quad \tilde{t} = t/\tau_1. \quad (5)$$

2. Spatial Distribution of Excitons. Concentration of Islands

The diffusion equation for the exciton concentration can be written as

$$\frac{\partial c}{\partial t} = D\Delta c - \frac{c}{\tau_2} + K, \quad (6)$$

where D is the diffusion coefficient, τ_2 is the lifetime of a free exciton (outside an island).

A steady-state solution of Eq. (6) has the form

$$c(\mathbf{r}) = c_{K2} + \sum_i B_i K_0\left(\frac{|\mathbf{r} - \mathbf{r}_i|}{l_D}\right), \quad (7)$$

where $\mathbf{r}_i$ is the position of the i-th island, $l_D = \sqrt{D\tau_2}$ is the diffusion length, $c_{K2} = K\tau_2$, α_i are constants which are determined by the boundary conditions for every island, and K_0 is the zero-order second-kind modified Bessel function. In the vicinity of any island, the exciton concentration field depends on the island under consideration and all other islands. Due to a large distance between islands, the concentration field created by other islands at the place of the island under consideration may be assumed to be uniform. For example, let us consider the concentration field in the vicinity of an island with $\mathbf{r}_i = 0$. We designate this island by $i = 0$. Then we have

$$c(\mathbf{r}) = c_0 + BK_0\left(\frac{\mathbf{r}}{l_D}\right), \quad (8)$$

where

$$c_0 = c_{K2} + \sum_{i\neq 0} B_i K_0\left(\frac{r_i}{l_D}\right). \quad (9)$$

At large values of the parameter l_D, terms of the sum in (9) decrease very slowly with increase in the distance between the i-th island and the island with i=0. This means that, in the vicinity of the island with $i = 0$, the contribution to the concentration distribution is given by many islands. In this situation, we may substitute a_i in the sum by the mean value $\bar{a}$ and replace the sum by an integral. As a result, we have

$$c_0 = c_{K2} + \bar{B}l_D^2 n, \quad (10)$$

where n is the concentration of islands, $n = N/S$, N and S are the total number of islands and the system area, respectively.

The second term in (10) describes the influence of islands on the exciton density. As seen, the value $\bar{B}$ is negative, and the presence of islands results in decreasing the exciton density.

To obtain the boundary condition for the concentration on the border of an island, we use the conservation particle law

$$2\pi RD\frac{\partial c}{\partial r} = 2\pi R(W_{\rm in}c(R) - W_{\rm out}c_{\rm in}) = = 2\pi RW_{\rm in}(c - c_\infty e^{\alpha/R}). \quad (11)$$

From (3), (8), and (11), we obtain the following condition:

$$B_0 = \frac{W_{\rm in}(c_0 - c_\infty e^{\alpha/R})}{D[\frac{\partial}{\partial R}K_0(R/l_D)] - W_{\rm in}K_0(R/l_D)}. \quad (12)$$

To find $\bar{B}$, we take the mean value of the both sides of Eq. (12). We suggest that the radius distribution function has a sharp maximum at $R = \bar{R}$. Later we will show that this approximation is valid. As a result, we have

$$\bar{B} = \frac{W_{\rm in}(c_{K2} - c_\infty e^{\alpha/\bar{R}})}{\left[-\frac{D}{l_D}K_1(\bar{R}/l_D) - W_{\rm in}K_0(\bar{R}/l_D) - W_{\rm in}l_D^2 n)\right]}. \quad (13)$$

Finally, we get the formula for the exciton concentration on the border of an island as

$$c(R) = c_{K2} + \bar{B} l_D^2 n + B_0 K_0(R/l_D), \qquad (14)$$

where B_0 and $\bar{B}$ are given by Eqs. (12) and (14), respectively.

Further, we substitute $c(R)$ given by (14) in Eq. (4) and obtain the expression for the radius distribution function f at a fixed value of the concentration of islands n. This function is given below in dimensionless units, but the mark "tilde" is omitted:

$$f = f_0 \exp(F(n, R)). \qquad (15)$$

Here,

$$F(n, R) = \\ = \left[\int_0^R \left(\frac{w[c(R) - c_\infty e^{\alpha/R}] + \frac{R}{2}(c_{K1} - 1)}{w[c(R) + c_\infty e^{\alpha/R}] + \frac{R}{2}(c_{K1} + 1)} \right) 4\pi R dR \right]. \qquad (16)$$

The term $c_\infty e^{\alpha/R}$ for a nondegenerate $2D$ exciton gas can be expressed as

$$c_\infty e^{\alpha/R} = \left(\frac{mk}{2\pi\hbar^2} \gamma \right) S_0 T e^{-\frac{1}{T}\frac{1}{k}(\varphi - \frac{\alpha}{R})}. \qquad (17)$$

The probability for the system to have N islands with radii R_1, R_2, ..., R_N is given by

$$W(N, R_1, R_2, \ldots, R_N) = \exp\left(-\sum_i F_n(R_i) \right). \qquad (18)$$

After the integration over the radii of islands, we obtain the probability for the system to have N islands as

$$W(N) = W_0 \exp(-\Phi(N)), \qquad (19)$$

where

$$\Phi(N) = -N \ln z \left(\frac{N}{S} \right),$$

$$z\left(\frac{N}{S} \right) = \int_0^\infty \exp\left(-F\left(\frac{N}{S}, R \right) \right) dR. \qquad (20)$$

The most probable concentration of islands is determined by the condition

$$\frac{d}{dN} \Phi(N) = 0. \qquad (21)$$

In the vicinity of a distribution function maximum, the function $F(n, R)$ can be expanded in a power series in $(R - \bar{R})$ as

$$F(n, R) = F(n, \bar{R}) + b(R - \bar{R})^2 + \ldots \qquad (22)$$

In this case,

$$\Phi(N) = N \left(F\left(\frac{N}{S}, \bar{R} \right) - \frac{1}{2} \ln(\pi/b) \right). \qquad (23)$$

3. Calculations and Discussion

Since islands capture excitons form the environment, two islands cannot be too close each to other (the exciton resources are limited). Also they cannot be too far each from other because, in this case, there would be a high probability of the creation of a new island between them. So the function $\Phi(N)$ has maximum at some value of N. The most probable state is the one corresponding to this maximum. The analysis of properties of the structure of islands was carried out by numerically solving Eq. (21). The main results are presented below; the unit of length is the distance between excitons in the condensed phase ($S_0 \sim 10^{-12}$ cm^2), the temperature is presented in Kelvin degrees.

At the chosen parameters, the Bose condensation in islands does not take place. Really, at an exciton concentration of $10^9 \div 10^{10}$ cm^{-2}, $\gamma = 4$, and $m = 0.12 m_0$, where m_0 is the free electron mass, the temperature of the Bose condensation is less than 1 K according to the Kosterlitz—Thouless formula [12]. We have studied the system at higher temperatures. In islands, the Bose condensation may be realized, but we need only two energy parameters, α and ϕ.

Fig. 1 shows the dependence of the mean radius on temperature for different values of pumping. It is seen that islands become larger with increase in temperature.

It is seen from Fig. 2 that the critical pumping value exists. If the pumping is less than this value there is no exciton condensation. We note that this critical value grows with increase in the temperature.

With the increase in the pumping, the fraction of the condensed phase (the ratio of the area occupied by islands to the whole area) becomes larger (Fig. 3).

Fig. 4 shows that the mean radius of islands grows with increase in the lifetime of excitons in islands, and their concentration goes down (Fig. 5). We note that, as compared to a $3D$ system ([11]), the radius distribution functions are broader, and the distance between disks is smaller.

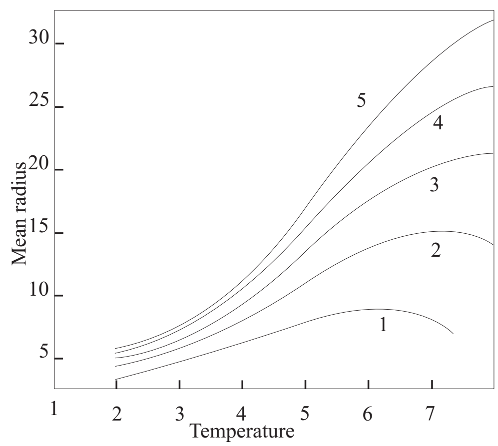

Fig. 1. Dependence of the mean radius $\bar{R}$ on temperature T for different values of pumping: $\tilde{C}_K = 0.01$ (*1*), 0.02 (*2*), 0.03 (*3*), 0.04 (*4*), 0.05 (*5*)

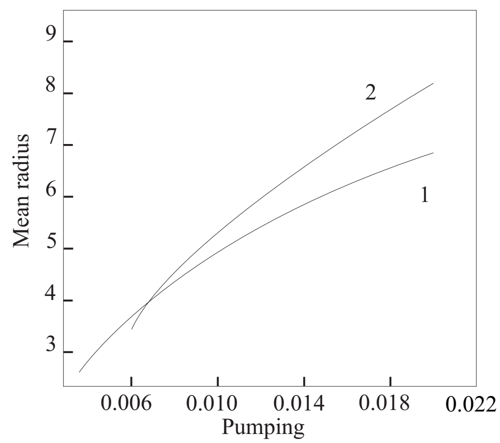

Fig. 2. Dependence of the mean radius $\bar{R}$ on pumping $\tilde{K}$ for different temperatures: $T = 5$ (*1*), $T = 6$ (*2*)

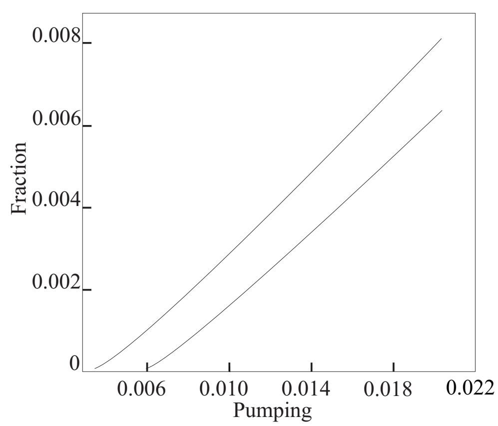

Fig. 3. Dependence of the fraction of the condensed phase in the system on the pumping

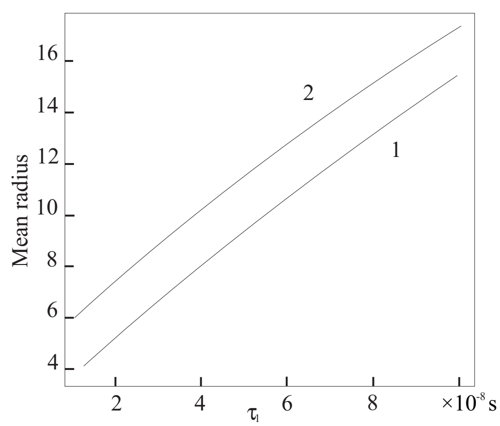

Fig. 4. Dependence of the mean radius $\bar{R}$ on the lifetime of excitons in an island τ_1 for a fixed value of $\tilde{\tau_2}$: $\tilde{\tau_2} = 1$ (*1*), $\tilde{\tau_2} = 0.5$ (*2*)

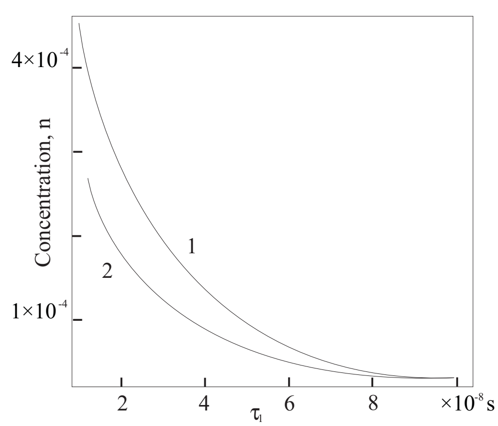

Fig. 5. Dependence of the concentration of islands n on the lifetime of excitons in an island τ_1 for a fixed value of $\tilde{\tau_2}$: $\tilde{\tau_2} = 1$ (*1*), $\tilde{\tau_2} = 0.5$ (*2*)

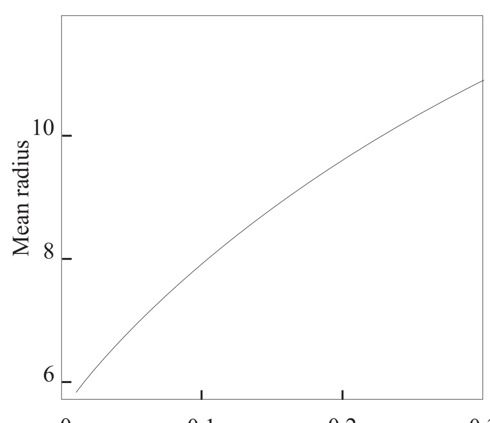

Fig. 6. Dependence of the mean radius $\bar{R}$ on the surface energy α/ϕ

It is seen from Fig. 6 that the dependence of the mean radius on the surface energy is a non-linear monotone increasing function.

Conclusion

The paper presents the dependences of main parameters of the exciton condensed phase (the distance between islands of the condensed exciton phase, their mean radius, the fraction occupied by excitons in the condensed phase) in a $2D$ system on the pumping, temperature, lifetime of excitons in and outside islands, and energy parameters. It is shown that the mean radius becomes larger with increase in temperature, the pumping, and lifetime of excitons, and the concentration of free excitons goes down with increase in their lifetime.

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ДОСЛІДЖЕННЯ КОНДЕНСОВАНОЇ ЕКСИТОННОЇ ФАЗИ У ДВОВИМІРНИХ СИСТЕМАХ

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Р е з ю м е

Виконано розрахунки формування конденсованої екситонної фази у двовимірних системах при різних температурах і часах життя вільних екситонів і екситонів у конденсованій фазі. Досліджено область співіснування конденсованої і газової екситонних фаз. У цьому випадку двовимірна конденсована фаза являє собою сукупність острівців у вигляді дисків. Розглянуто випадок, коли щільність екситонів у газовій фазі (поза острівцями) нижча за щільність, необхідну для бозе-конденсації. Знайдено залежність середнього радіуса конденсованих острівців від часу життя екситонів (як вільних, так і екситонів у конденсованій фазі), швидкості утворення екситонів, температури системи і поверхневої енергії екситонів конденсованої фази. Визначено також концентрацію екситонних острівців як функцію часу життя екситонів і частку екситонів, що перебувають у конденсованій фазі, як функцію накачки.