INTRASUBBAND PLASMONS IN A FINITE ARRAY OF QUANTUM WIRES PLACED INTO AN EXTERNAL MAGNETIC FIELD

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The paper deals with the theoretical investigation of intrasubband plasmons in an array of quantum wires, consisting of a finite number of quantum wires (QWs) arranged at an equal distance one from another and placed into an external magnetic field. Two types of the arrays of QWs are under consideration: an ordered array of QWs with equal electron densities in all QWs and a weakly disordered array of QWs which is characterized by the fact that the density of electrons of one defect QW is different from that of other QWs. For the ordered array of QWs placed into an external magnetic field, the nonmonotone dependence of the plasmon frequency on the $1D$ density of electrons in QWs is predicted. For a high magnetic field, the existence of $1D$ electron density ranges, in which plasmon modes do not exist, is shown. For a weakly disordered array of QWs, the existence of the local plasmon modes, whose properties differ from those of usual modes, is found. At high magnetic fields, the disappearance of the local plasmon modes at certain ranges of the $1D$ electron density in a defect QW is shown.

Introduction

Quasi one-dimensional electron systems (1DESs) or QWs are artificial structures, in which the motion of charge carriers is confined in two transverse directions but is essentially free (in the effective mass sense) in the longitudinal direction [1—3]. Usually, QWs are produced by imposing the one-dimensional confinement to a two-dimensional electron system (2DES). This additional confinement is, in general, weaker than the strong confinement of original 2DES [4]. One of the motivations to study QWs is the fact that the mobility of charge carriers is higher than that in 2DES, on which they are built. The reason for this is that the impurity content and distribution around the QWs can be selectively controlled, thereby producing the enhanced mobility [5].

Collective charge-density excitations or plasmons in QWs are of great interest to physicists. Earlier, plasmons in QWs were investigated both theoretically [5–9] and experimentally [10–13]. It was shown in those papers that plasmons in QWs possess some new unusual dispersion properties. Firstly, the plasmon spectrum strongly depends on the width of QW. Secondly, $1D$ plasmons are free from the Landau damping [6, 9] over the whole range of wavevectors.

From the viewpoint of practical application, the so-called weakly disordered arrays of low-dimensional systems are the objects of interest. Recently the plasmons in a weakly disordered superlattice formed of a finite number of equally spaced two-dimensional electron systems have been theoretically investigated in the cases where the external magnetic field is absent [14–16] or present [17]. The weakly disordered superlattice is characterized by the fact that all of two-dimensional systems possess the equal density of electrons except one ("defect") two-dimensional system, whose density of electrons differs from that of other two-dimensional systems. It was found that the plasmon spectrum of such an array contains the local plasmon mode, whose properties differ from those of other plasmon modes. The existence of a local plasmon mode is completely analogous to the existence of the local phonon mode originally obtained by Lifshitz in 1947 for the problem of phonon modes in a regular crystal containing a single isotope impurity [18]. Notice that practically the entire flux of electromagnetic energy of plasmons, which correspond to the local mode, is concentrated in the vicinity of the defect 2DES. At the same time the opportunity of using the plasmon spectrum peculiarities to determine the parameters of defects in the superlattice was indicated in [17].

Plasmons in a finite weakly disordered array of QWs without an external magnetic field have been investigated theoretically in [19]. It has been supposed that the defect QW can occupy an arbitrary position in the array. It is shown in [19] that the position of a defect QW in the array does not strongly affect the spectrum of the local plasmon mode but it exerts an significant influence on the spectrum of other plasmon modes. At the same time, when the defect QW is arranged inside the array, the plasmon spectrum contains modes, whose dispersion properties do not depend on the electron density in the defect QW.

The external magnetic field is known to cause considerable changes in the plasmon spectrum of low-dimensional structures. So, plasmons in single 2DES placed into the external magnetic field directed perpendicularly to the 2DES were earlier investigated both theoretically [20] and experimentally [21]. It was shown that the dispersion relation for plasmons in 2DES placed into the external magnetic field can be expressed as

$$\omega_H^2 = \omega_c^2 + \omega^2, \tag{1}$$

where ω_H is the frequency of a plasmon in the presence of an external magnetic field, $\omega_c = eB/m^*c$ is the cyclotron resonance frequency, and ω is the frequency of plasmons when the external magnetic field is absent.

Plasmons in a single 1DES was also investigated theoretically [22, 23] and experimentally [11, 12]. As was shown experimentally [11], the dispersion law for a one-dimensional plasmon in the presence of the magnetic field can also be described by (1). Nevertheless, another one-dimensional plasmon mode was found experimentally in [12]. The last possesses the negative magnetic field dispersion. At the same time, it was shown theoretically [22] that the above-mentioned negative magnetic field dispersion for one dimensionality occurs in the high magnetic field only. At a weak magnetic field, the properties of intrasubband plasmons in a single QW depend considerably on the one-dimensional electron density in the QW. Thus, if the density of electrons in the QW exceeds a certain critical value, the intrasubband plasmon frequency increases with the magnetic field. In the opposite case where the density of electrons in the QW is smaller than the critical value, the intrasubband plasmon frequency decreases as the magnetic field increases.

In this paper, we investigate intrasubband plasmons in a finite array of QWs, placed into an external magnetic field. We consider two types of QW array: an array in which the $1D$ elecron densities are equal in all QWs (the ordered array of QWs) and an array in which the $1D$ electron density of one defect QW differs from that of other QWs (weakly disordered array of QWs).

1. Dispersion Relation

We consider the array of QWs consisting of a finite number M of QWs arranged in the planes $z = ld$ ($l = 0, ..., M-1$ is the number of the QW, d is the distance between adjacent QWs). At the same time, we assume that the $1D$ density of electrons in the l-th QW is equal to N_l. The QWs are considered to be placed into the uniform dielectric medium with the dielectric constant ε. We use such a simple model (in which the dielectric constants of the media inside and outside the array are equal) to avoid the appearance of a surface plasmon mode. We reckon the movement of electrons to be free in the x-direction and considerably confined in the directions y and z. We assume that the array of QWs is built on ideal 2DESs by applying an additional confining potential along the y-direction, which is parabolic: $U_{\rm conf} = \frac{1}{2}m^*\omega_0^2 y^2$. Here, m^* is the effective mass of an electron, ω_0 is the classical oscillation frequency of the electron placed in the potential $U_{\rm conf}$. At the same time, we suppose that the width of all QWs is equal to zero in z-direction. The external constant magnetic field is taken to be perpendicular to the plane xy along the axis z.

To obtain the single-particle wave-function of the electron in a QW we write the expression for the vector potential $\mathbf{A}$ in the Landau gauge: $\mathbf{A} = (-By, 0, 0)$. So, in this case, the single-particle Hamiltonian of the electrons is

$$\hat{H} = \frac{1}{2m^*}\left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A}\right)^2 + U_{\rm conf}(x,y), \tag{2}$$

where $\hat{\mathbf{p}} = -i\hbar\nabla$ is the operator of the momentum of an electron. In (2), we neglect the spin splitting in the magnetic field.

We seek an explicit form for the electron wave-function: $\psi(x,y) = \exp(ikx)\phi(y)$. In this case after some algebra, the Schrödinger equation $\hat{H}\psi(x,y) = E\psi(x,y)$ can be written as

$$-\frac{\hbar^2}{2m^*}\frac{\partial^2\phi}{\partial y^2} + \frac{1}{2}m^*\Omega^2\left[y - \alpha k\right]^2\phi(y) =$$

$$= \left\{E - \frac{\hbar^2 k^2}{2m^*}\frac{\omega_0^2}{\Omega^2}\right\}\phi(y), \tag{3}$$

where $\alpha = \hbar\omega_c/m^*\Omega^2$, $\Omega^2 = \omega_c^2 + \omega_0^2$. The solution of (3) is a shifted harmonic oscillator wave function. Therefore, the expression for energy subbands and a single-particle wave function for the electron in l-th QW reads as [24]

$$E_m(k) = E_m + (1/2m^*)(\omega_0/\Omega)^2\hbar^2k^2, \tag{4}$$

$$\psi_{l,m,k}(\mathbf{r}) = (1/2\pi)^{1/2}e^{ikx}\phi_m(y-\alpha k)\left[\delta(z-ld)\right]^{1/2}. \tag{5}$$

Here,

$$E_m = \hbar\Omega(m + 1/2),$$

$$\phi_m(y) = (2^m m!\pi^{1/2}l_\Omega)^{-1/2}\exp\left(-\frac{y^2}{2l_\Omega^2}\right)H_m(y/l_\Omega), \tag{6}$$

m is the number of the energy subband, $H_m(y)$ is an Hermite polynomial, $l_\Omega = (\hbar/m^*\Omega)^{1/2}$ is a typical width of the wave function (which is merely a magnetic length, if $\omega_0 = 0$).

As evident from expression (4), in the presence of a confining potential in the y-direction, the degeneracy of Landau levels is broken and each Landau level forms a subband. At the same time, the wave function (5) in y-direction depends on the wavevector k in x-direction. So, in the presence of confining potential and external magnetic fields the directions x and y are coupled.

To obtain the spectrum of collective excitations, we start with a standard linear-response theory in the random phase approximation. Let us consider $\delta n(\mathbf{r})$ which is a deviation of the electron density from its equilibrium value. On applying the above-mentioned standard linear-response theory and the random phase approximation, the matrix element of the electron density deviation from its equilibrium value $\delta n_{\gamma,\gamma'}=\langle\gamma|\delta n|\gamma'\rangle=\int d\mathbf{r}\psi^*_\gamma(\mathbf{r})\psi_{\gamma'}(\mathbf{r})\delta n(\mathbf{r})$ can be related to the perturbation as

$$\delta n_{\gamma\gamma'} = \frac{f_{\gamma'} - f_\gamma}{E_{\gamma'} - E_\gamma + \hbar\omega} V_{\gamma\gamma'}. \tag{7}$$

Here, $\gamma = (l, m, k)$ is a composite index, f_γ is the Fermi distribution function, $V_{\gamma,\gamma'} = \langle\gamma|V|\gamma'\rangle$ are the matrix elements of the perturbing potential $V = V^{\rm ex} + V^{\rm H}$, $V^{\rm ex}$ and $V^{\rm H}$ are the external and Hartree potentials, respectively.

Note that the matrix elements of the Hartree potential can be expressed in terms of the perturbation [6] as

$$V^{\rm H}_{\gamma\gamma'} = \frac{e^2}{\varepsilon}\int d\mathbf{r}\psi^*_\gamma(\mathbf{r})\psi_{\gamma'}(\mathbf{r})\int \frac{d\mathbf{r_1}}{|\mathbf{r}-\mathbf{r}_1|}\delta n(\mathbf{r}_1). \tag{8}$$

Considering that

$$\delta n(\mathbf{r}_1) = \sum_{\beta\beta'} \delta n_{\beta\beta'}\psi^*_{\beta'}(\mathbf{r}_1)\psi_\beta(\mathbf{r}_1), \quad \beta = (n, s, k_1),$$

we obtain

$$V^{\rm H}_{\gamma\gamma'} = \sum_{\beta\beta'} W_{\gamma\gamma'\beta\beta'}\delta n_{\beta\beta'}. \tag{9}$$

Here,

$$W_{\gamma\gamma'\beta\beta'} = \frac{e^2}{\varepsilon}\int d\mathbf{r}\psi^*_\gamma(\mathbf{r})\psi_{\gamma'}(\mathbf{r})\times$$

$$\times\int \frac{d\mathbf{r_1}}{|\mathbf{r}-\mathbf{r}_1|}\psi^*_{\beta'}(\mathbf{r}_1)\psi_\beta(\mathbf{r}_1) =$$

$$= \frac{\delta(q-q_1)}{2\pi}\frac{2e^2}{\varepsilon}\delta_{n,n'}\delta_{l,l'}W_{l,n|m,m',s,s'}\left(k', k, k'_1, k_1\right), \tag{10}$$

where

$$W_{l,n|m,m',s,s'}\left(k', k, k'_1, k_1\right) = \int \phi_m(y-\alpha k)\times$$

$$\times\phi_{m'}(y-\alpha k')\phi_{s'}(y_1-\alpha k'_1)\phi_s(y_1-\alpha k_1)\times$$

$$\times K_0\left(q((y-y_1)^2 + (l-n)^2 d^2)^{1/2}\right)dy dy_1, \tag{11}$$

$q = k' - k$, $q_1 = k'_1 - k_1$, $K_0(x)$ is the zeroth-order modified Bessel function of the second kind. From (7),(9), and (10), following some algebra, we get

$$\delta n_{lmk,lm'k+q} = \frac{f_{lm'k+q} - f_{lmk}}{E_{lm'k+q} - E_{lmk} + \hbar\omega}\Bigg(V^{\rm ex}_{lmk,lm'k+q} +$$

$$+\frac{2e^2}{\varepsilon}\frac{1}{\pi}\sum_{n,s,s'}\int dk_1\delta n_{nsk_1,ns'k_1+q}\times$$

$$\times W_{l,n|m,m',s,s'}(k+q, k, k_1+q, k_1)\Bigg). \tag{12}$$

The factor of 2 before the summation symbol comes from the spin degeneracy.

Collective excitations of the QW array exist when Eq. (12) has a nonzero solution δn in the case where the external perturbation $V^{\rm ex} = 0$. Since the parameter αk is the small value [22], we can expand the wave function in terms of α as $\phi_m(y - \alpha k) = \phi_m(y) - \alpha k\phi'_m(y) + \frac{1}{2}\alpha^2k^2\phi''_m(y)$. In addition, at $q \to 0$, we can admit $\alpha(k+q) \approx \alpha k$, $\alpha(k_1+q) \approx \alpha k_1$. Under this assumption, we can represent (11) in the form

$$W_{l,n|m,m',s,s'}\left(k+q, k, k_1+q, k_1\right) \approx$$

$$\approx C^{(0)}_{l,n|m,m',s,s'}(q) + \alpha k C^{(1)}_{l,n|m,m',s,s'}(q) +$$

$$+\alpha^2k^2C^{(2)}_{l,n|m,m',s,s'}(q) + \alpha k_1\left\{B^{(1)}_{l,n|m,m',s,s'}(q) +\right.$$

$$\left. +\alpha k B^{(2)}_{l,n|m,m',s,s'}(q)\right\} + \alpha^2 k_1^2 A_{l,n|m,m',s,s'}(q), \tag{13}$$

where

$$C^{(0)}_{l,n|m,m',s,s'}(q) = \int \phi_m(y)\phi_{m'}(y)\phi_s(y_1)\phi_{s'}(y_1)\times$$

$$\times K_0\left(q((y-y_1)^2 + (l-n)^2d^2)^{1/2}\right)dy dy_1,$$

$$C^{(1)}_{l,n|m,m',s,s'}(q) = -\int \phi_s(y_1)\phi_{s'}(y_1)\times$$

$$\times\{\phi'_m(y)\phi_{m'}(y)+\phi'_{m'}(y)\phi_m(y)\}\times$$

$$\times K_0\left(q((y-y_1)^2+(l-n)^2d^2)^{1/2}\right)dydy_1,$$

$$C^{(2)}_{l,n|m,m',s,s'}(q)=\frac{1}{2}\int\phi_s(y_1)\phi_{s'}(y_1)\times$$

$$\times\{\phi''_m(y)\phi_{m'}(y)+\phi''_{m'}(y)\phi_m(y)+$$

$$+\,2\phi'_m(y)\phi'_{m'}(y)\}\times$$

$$\times K_0\left(q((y-y_1)^2+(l-n)^2d^2)^{1/2}\right)dydy_1,$$

$$B^{(1)}_{l,n|m,m',s,s'}(q)=-\int\phi_m(y)\phi_{m'}(y)\times$$

$$\times\{\phi'_s(y_1)\phi_{s'}(y_1)+\phi'_{s'}(y_1)\phi_s(y_1)\}\times$$

$$\times K_0\left(q((y-y_1)^2+(l-n)^2d^2)^{1/2}\right)dydy_1,$$

$$B^{(2)}_{l,n|m,m',s,s'}(q)=\int\{\phi'_m(y)\phi_{m'}(y)+$$

$$+\,\phi'_{m'}(y)\phi_m(y)\}\times$$

$$\times\{\phi'_s(y_1)\phi_{s'}(y_1)+\phi'_{s'}(y_1)\phi_s(y_1)\}\times$$

$$\times K_0\left(q((y-y_1)^2+(l-n)^2d^2)^{1/2}\right)dydy_1,$$

$$A_{l,n|m,m',s,s'}(q)=\frac{1}{2}\int\phi_m(y)\phi_{m'}(y)\times$$

$$\times\{\phi''_s(y_1)\phi_{s'}(y_1)+\phi''_{s'}(y_1)\phi_s(y_1)+$$

$$+\,2\phi'_s(y_1)\phi'_{s'}(y_1)\}\times$$

$$\times K_0\left(q((y-y_1)^2+(l-n)^2d^2)^{1/2}\right)dydy_1.$$

Substituting (13) into (12), we obtain

$$\delta n_{lmk,lm'k+q}=\frac{2e^2}{\varepsilon}\frac{1}{\pi}\frac{f_{lm'k+q}-f_{lmk}}{E_{lm'k+q}-E_{lmk}+\hbar\omega}\times$$

$$\times\sum_{n,s,s'}\int dk_1\delta n_{nsk_1,ns'k_1+q}\left[C^{(0)}_{l,n|m,m',s,s'}(q)+\right.$$

$$+\alpha kC^{(1)}_{l,n|m,m',s,s'}(q)+\alpha^2k^2C^{(2)}_{l,n|m,m',s,s'}(q)+$$

$$+k_1\left\{\alpha B^{(1)}_{l,n|m,m',s,s'}(q)+\alpha^2kB^{(2)}_{l,n|m,m',s,s'}(q)\right\}+$$

$$\left.+\alpha^2k_1^2A_{l,n|m,m',s,s'}(q)\right].\qquad(14)$$

Multiplying both the left- and right-hand side of Eq. (14) by $2k^i$ $(i=0,1,2)$ and integrating, we get

$$\chi^{(0)}_{l|m,m'}=\frac{2e^2}{\varepsilon}\sum_{n,s,s'}\left[\left\{C^{(0)}_{l,n|m,m',s,s'}(q)\Pi^{(0)}_{l|m,m'}+\right.\right.$$

$$+\alpha C^{(1)}_{l,n|m,m',s,s'}(q)\Pi^{(1)}_{l|m,m'}+$$

$$\left.+\,\alpha^2C^{(2)}_{l,n|m,m',s,s'}(q)\Pi^{(2)}_{l|m,m'}\right\}\chi^{(0)}_{n|s,s'}+$$

$$+\left\{\alpha B^{(1)}_{l,n|m,m',s,s'}(q)\Pi^{(0)}_{l|m,m'}+\right.$$

$$\left.+\,\alpha^2B^{(2)}_{l,n|m,m',s,s'}(q)\Pi^{(1)}_{l|m,m'}\right\}\chi^{(1)}_{n|s,s'}+$$

$$\left.+\,\alpha^2A_{l,n|m,m',s,s'}(q)\Pi^{(0)}_{l|m,m'}\chi^{(2)}_{n|s,s'}\right],\qquad(15)$$

$$\chi^{(1)}_{l|m,m'}=\frac{2e^2}{\varepsilon}\sum_{n,s,s'}\left[\left\{C^{(0)}_{l,n|m,m',s,s'}(q)\Pi^{(1)}_{l|m,m'}+\right.\right.$$

$$+\alpha C^{(1)}_{l,n|m,m',s,s'}(q)\Pi^{(2)}_{l|m,m'}+$$

$$\left.+\alpha^2C^{(2)}_{l,n|m,m',s,s'}(q)\Pi^{(3)}_{l|m,m'}\right\}\chi^{(0)}_{n|s,s'}+$$

$$+\left\{\alpha B^{(1)}_{l,n|m,m',s,s'}(q)\Pi^{(1)}_{l|m,m'}+\right.$$

$$\left.+\,\alpha^2B^{(2)}_{l,n|m,m',s,s'}(q)\Pi^{(2)}_{l|m,m'}\right\}\chi^{(1)}_{n|s,s'}+$$

$$\left.+\,\alpha^2A_{l,n|m,m',s,s'}(q)\Pi^{(1)}_{l|m,m'}\chi^{(2)}_{n|s,s'}\right],\qquad(16)$$

$$\chi^{(2)}_{l|m,m'}=\frac{2e^2}{\varepsilon}\sum_{n,s,s'}\left[\left\{C^{(0)}_{l,n|m,m',s,s'}(q)\Pi^{(2)}_{l|m,m'}+\right.\right.$$

$$+\alpha C^{(1)}_{l,n|m,m',s,s'}(q)\Pi^{(3)}_{l|m,m'}+$$

$$\left.+\alpha^2C^{(2)}_{l,n|m,m',s,s'}(q)\Pi^{(4)}_{l|m,m'}\right\}\chi^{(0)}_{n|s,s'}+$$

$$+\left\{\alpha B^{(1)}_{l,n|m,m',s,s'}(q)\Pi^{(2)}_{l|m,m'}+\right.$$

$$\left.+\,\alpha^2B^{(2)}_{l,n|m,m',s,s'}(q)\Pi^{(3)}_{l|m,m'}\right\}\chi^{(1)}_{n|s,s'}+$$

$$\left.+\,\alpha^2A_{l,n|m,m',s,s'}(q)\Pi^{(2)}_{l|m,m'}\chi^{(2)}_{n|s,s'}\right],\qquad(17)$$

where

$$\chi^{(i)}_{n|s,s'}=2\int dk_1k_1^i\delta n_{nsk_1,ns'k_1+q},$$

$$\chi^{(i)}_{l|m,m'} = 2\int dk k^i \delta n_{lmk,lm'k+q},$$

$$\Pi^{(i)}_{l|m,m'} = \frac{1}{\pi}\int dk\, k^i \frac{f_{lm'k+q} - f_{lmk}}{E_{lm'k+q} - E_{lmk} + \hbar\omega}.$$

We restrict our consideration to intrasubband plasmons. So, we consider that the intersubband transitions of charge carriers are absent. Then $\Pi^{(i)}_{l|m,m'} = 0$ if $m \neq m'$, and the system of equations (15)–(17) can be rewritten as

$$\chi^{(p)}_{l|m,m} = \frac{2e^2}{\varepsilon}\sum_{n,s,t} U_{l,n,m,s,p,t}\chi^{(t)}_{n|s,s}, \quad (18)$$

where

$$U_{l,n,m,s,p,1} = C^{(0)}_{l,n|m,m,s,s}(q)\Pi^{(p)}_{l|m,m} +$$

$$+\alpha C^{(1)}_{l,n|m,m,s,s}(q)\Pi^{(p+1)}_{l|m,m} + \alpha^2 C^{(2)}_{l,n|m,m,s,s}(q)\Pi^{(p+2)}_{l|m,m},$$

$$U_{l,n,m,s,p,2} = \alpha B^{(1)}_{l,n|m,m,s,s}(q)\Pi^{(p)}_{l|m,m} +$$

$$+\alpha^2 B^{(2)}_{l,n|m,m,s,s}(q)\Pi^{(p+1)}_{l|m,m},$$

$$U_{l,n,m,s,p,3} = \alpha^2 A_{l,n|m,m,s,s}(q)\Pi^{(p)}_{l|m,m}, \quad p = 0, ..., 2.$$

Equation (18) is a set of linear equations, and it has the nonzero solution with its determinant being equal to zero. Thus, the plasmon dispersion relation can be written in the form:

$$\det\left\|\delta_{l,n}\delta_{m,s}\delta_{p,t} - \frac{2e^2}{\varepsilon}U_{l,n,m,s,p,t}\right\| = 0. \quad (19)$$

Note, that as $M = 1$, the dispersion relation (19) coincides with the dispersion relation for plasmons in a single QW in the presence of an external magnetic field obtained in [22].

At a zero temperature and within the long-wavelength limit (where $q \to 0$), the function $\Pi^{(i)}_{l|m,m}$ can be written as

$$\Pi^{(0)}_{l|m,m} = \frac{2g^l_m}{\pi m_r}\frac{q^2}{\omega^2}, \quad \Pi^{(1)}_{l|m,m} = -\frac{a}{b}\Pi^{(0)}_{l|mm},$$

$$\Pi^{(2)}_{l|m,m} = \left(\frac{a}{b}\right)^2\Pi^{(0)}_{l|mm} - \frac{q}{b}\frac{2g^l_m}{\pi},$$

$$\Pi^{(3)}_{l|m,m} = -\left(\frac{a}{b}\right)^3\Pi^{(0)}_{l|mm} + \frac{q}{b}\frac{2g^l_m}{\pi}\left(q + \frac{a}{b}\right),$$

$$\Pi^{(4)}_{l|m,m} = \left(\frac{a}{b}\right)^4\Pi^{(0)}_{l|mm} -$$

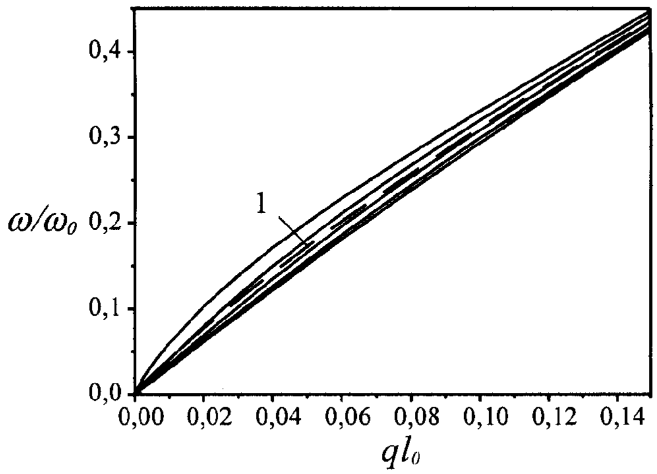

Fig. 1. Dispersion curves of intrasubband plasmons in an ordered array of QWs with parameters: $M = 5$, $d = 15.0 l_0$, $N_l = N = \mathrm{const}$ $(l = 0, ..., M-1)$, $\omega_p = \left(2e^2N/\varepsilon m^* l_0^2\right)^{1/2} = 1.5\omega_0$, $\omega_c = 0.75\omega_0$

$$-\frac{q}{b}\frac{2g^l_m}{\pi}\left[q^2 + \left(\frac{2g^l_m}{\pi}\right)^2 + \frac{a}{b}q + \left(\frac{a}{b}\right)^2\right],$$

where

$$a = \frac{\hbar^2 q^2}{2m_r} + \hbar\omega, \quad b = \frac{\hbar^2 q}{m_r}, \quad m_r = m^*(\Omega/\omega_0)^2,$$

$$g^l_m = \frac{1}{\hbar}\sqrt{2m_r(E^l_F - E_m)},$$

E^l_F is a Fermi level in the l-th QW.

2. Intrasubband Plasmons in the Ordered QW Array

Fig. 1 presents the dispersion curves for intrasubband plasmons in a finite ordered array of QWs (in which $1D$ electron densities are equal in all QWs), placed into the external magnetic field. The y-axis gives the dimensionless frequency ω/ω_0, and the x-axis gives the dimensionless wavevector ql_0. As the model of the QW we use a heterostructure GaAs with the effective mass of electrons $m^* = 0.067 m_0$ (m_0 is the mass of a free electron) and the dielectric constant $\varepsilon = 12$. For comparison, the dispersion curve for the plasmons in a single QW with the same parameters is depicted in Fig. 1 by dashed curve *1*. As seen from Fig. 1, the intrasubband plasmon spectrum in the finite ordered array of QWs contains M modes. Thus, the number of modes in the spectrum is equal to that of QWs in the array [14] (it

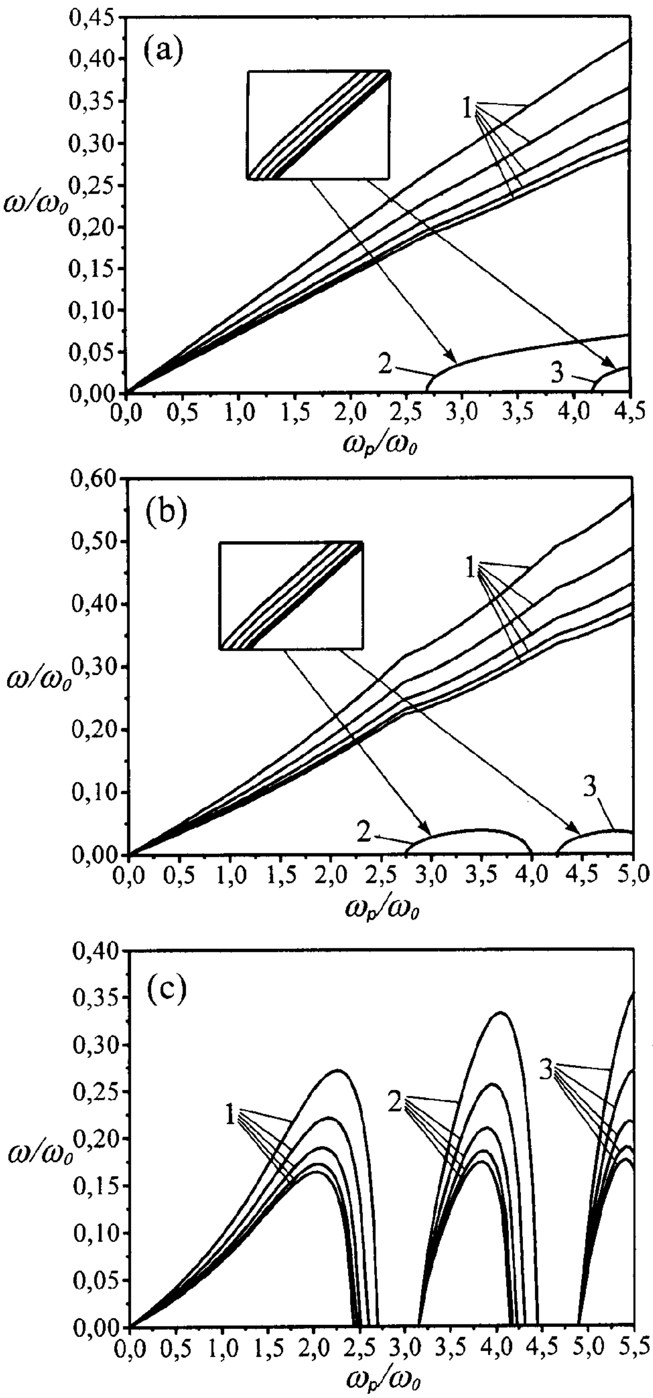

Fig. 2. Dependence of the intrasubband plasmon frequency on the plasma frequency of electrons in QW at $M = 5$, $ql_0 = 0.04$, $d = 15.0l_0$, $N_l = \text{const}$ ($l = 0, ..., M-1$) and for three values of the cyclotron frequency of electrons in QWs: $\omega_c = 0$ (a), $\omega_c = 0.25\omega_0$ (b), $\omega_c = 0.75\omega_0$ (c)

should be mentioned that, in the case under consideration, the value of plasma frequency of the electrons in QWs is chosen in such a way that only the lowest energy subband is occupied by electrons in each QW). Note that, with an increase of the wavenumber q, the plasmon frequency ω increases monotonically likewise. It should be emphasized that in the limit $qd \to \infty$, when the Coulomb interaction between electrons in adjacent QWs is negligible, the dispersion curves for plasmon modes are gradually drawn together and are close to the dispersion curve for the plasmon in the single QW with the same density of electrons (dashed curve *1*).

Now we consider the influence of the electron density value on the properties of intrasubband plasmons in the ordered array of QWs. Fig. 2 shows the dependence of the plasmon frequency on the plasma frequency of electrons in QWs for a fixed value of the wavevector q and for different values of the cyclotron frequency of electrons. The y-axis gives the dimensionless frequency ω/ω_0, and the x-axis gives the dimensionless plasma frequency of electrons in QWs ω_p/ω_0. We consider first the case where $\omega_c = 0$ (Fig. 2,a), i.e. when the external magnetic field is absent. Fig. 2,a shows that, in this case, the frequency of intrasubband plasmons increases with the plasma frequency of electrons in QWs ω_p. At the same time at small values of ω_p when the Fermi energy is below the bottom of the first subband ($E_F^l < E_1$) and the lowest (zero) subband in each QW is only occupied by electrons, the intrasubband plasmon spectrum contains M modes (curves *1*). Nevertheless, when the value of plasma frequency of electrons in QWs exceeds the value of order $\omega_p \approx 2.68\omega_0$, the intrasubband plasmon spectrum contains $2M$ modes (curves *1* and *2*). In this case, the Fermi energy is above the bottom of the first subband but below the bottom of the second subband and, consequently, there are already two subbands (zero and first) in each QW which are occupied by electrons. With a further increase of ω_p, new subbands become occupied by electrons, and each occupied subband in each QW supports its own intrasubband plasmon. Hence, the general number of intrasubband plasmon modes in a finite ordered array of QWs without external magnetic field is equal to nM (n is the quantity of subbands in each QW occupied by electrons).

The properties of intrasubband plasmons change to a certain extent, when the ordered array of QWs is placed into an external magnetic field. So, at a weak magnetic field (Fig. 2,b), the frequency of intrasubband plasmons supported by the lowest subband (curves *1*) is increased monotonically with ω_p. At the same time, when the Fermi energy exceeds the bottom of the first subband (and it becomes to be populated by electrons), the intrasubband plasmons supported by the first subbands in each QW arise in the spectrum (curves *2*). The frequency of these plasmons (as distinct from the case of the zero magnetic field, see Fig. 2,*a*) increases nonmonotonically with ω_p. Therefore, starting with some value of ω_p (in our case starting with $\omega_p \approx 3.55\omega_0$) the frequency of intrasubband plasmons supported by the first subbands is decreased with an increase of ω_p, and these plasmons disappear at $\omega_p \approx 4.0\omega_0$. Note that intrasubband plasmons supported by the second subbands (curves *3*) possess the same properties.

In the case of a higher magnetic field (Fig. 2,*c*), the dependence of the intrasubband plasmon frequency on the value of plasma frequency of electrons in QWs offers the following properties. So, the frequency of intrasubband plasmons supported by zero (curves *1*), first (curves *2*), and second (curves *3*) subbands depends nonmonotonically on the value of ω_p. At the same time, there are certain intervals of values of ω_p (in this case, $2.7\omega_0 < \omega_p < 3.2\omega_0$, $4.45\omega_0 < \omega_p < 4.9\omega_0$), in which the intrasubband plasmons do not exist.

3. Intrasubband Plasmons in a Weakly Disordered Array of Quantum Wires

Now we consider the spectrum of intrasubband plasmons in a weakly disordered array of QWs, in which all QWs have the equal $1D$ density of electrons N except one defect QW whose density of electrons is equal to N_d. Hence, the density of electrons in the l-th QW can be expressed as $N_l = (N_d - N)\delta_{pl} + N$. Here, p is the number of defect QWs arranged in the plane $z = pd$, δ_{pl} is the Kronecker delta.

Fig. 3 presents the spectrum of intrasubband plasmons (solid curves) in a weakly disordered array of QWs for the zero external magnetic field. For comparison the dispersion curves for the intrasubband plasmons in a single QW with the electron densities N and N_d are shown by dashed curves *1* and *2*, correspondingly. As seen from Fig. 3, the propagation of intrasubband plasmons in a weakly disordered array of QWs is characterized by the presence of the local plasmon mode (LPM). In the zero external magnetic field when the density of electrons in the defect QW is less than the density of electrons in other QW ($N_\mathrm{d} < N$), the LPM lies in the lower-frequency region in comparison with the usual plasmon modes (Fig. 3,*a*). Accordingly, if $N_d > N$, the LPM lies in the higher-frequency region in comparison with the usual ones (Fig. 3,*b*) [14]. It should be poined out that, in the limit $qd \to \infty$ as the Coulomb interaction between electrons in adjacent QWs is negligible, the LPM dispersion curve is close to the dispersion curve for the plasmons in a single QW with the density of electrons N_d (curve *2*). Meanwhile, the dispersion curves for usual plasmon modes in the limit $qd \to \infty$ are gradually drawn together and are close to the dispersion curve for the plasmon in the single QW with the density of electrons N (curve *1*).

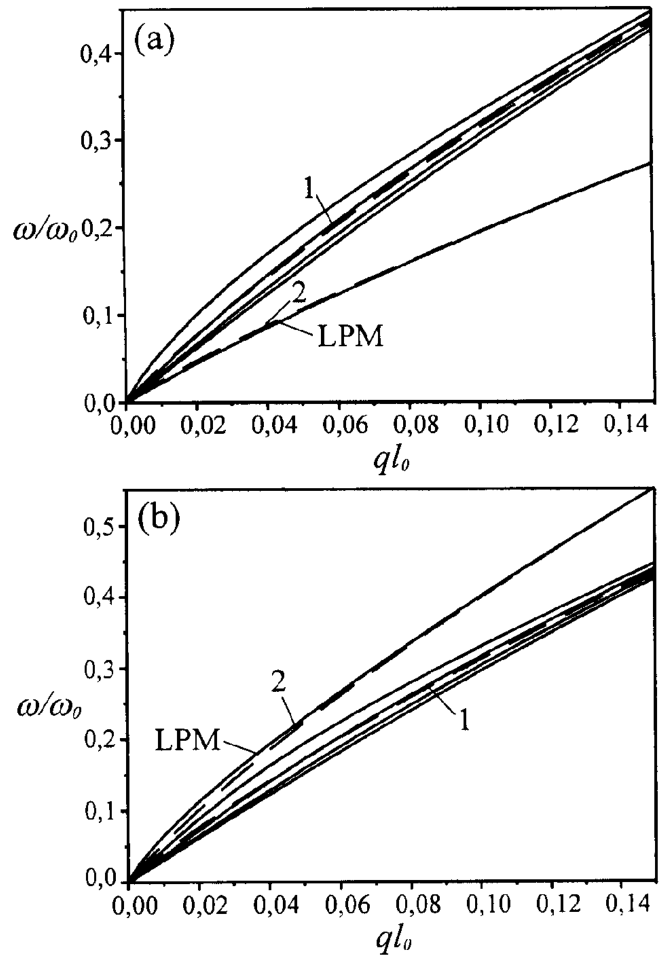

Fig. 3. Disperison curves of intrasubband plasmons in a weakly disordered array of QWs for parameters $M = 5$, $\omega_p = 1.5\omega_0$, $\omega_c = 0$, $d = 15.0l_0$, $p = 0$ and for two values of the $1D$ density of electrons in defect QW: $N_d/N = 0,5$ (*a*), 1,5 (*b*). The values of parameters are chosen in a manner that one (zeroth) subband in all QWs is only occupied by electrons

Now we consider the dependence of the intrasubband plasmon spectrum on the $1D$ electron density in a defect QW. Fig. 4 depicts the dependence of the intrasubband plasmon frequency on the ratio N_d/N for a fixed value

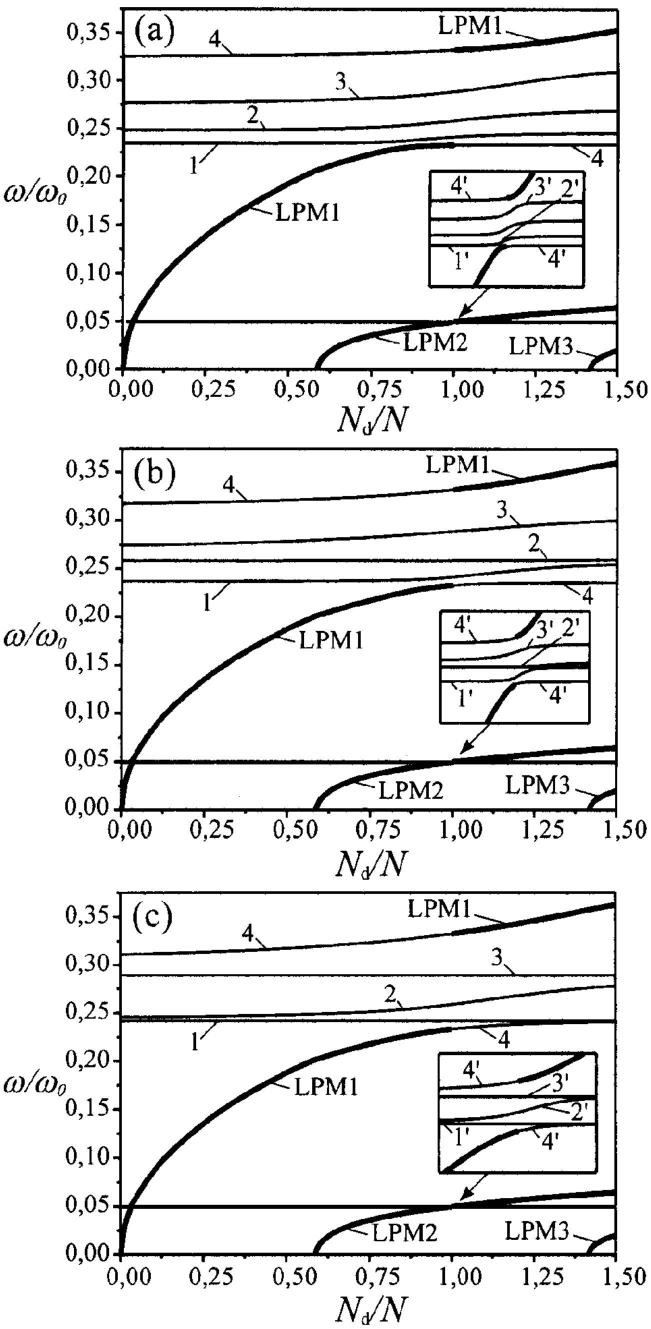

Fig. 4. Dependence of the intrasubband plasmon frequency on the ratio N_d/N at $M = 5$, $ql_0 = 0.04$, $d = 15.0l_0$, $\omega_p = 3.0\omega_0$, $\omega_c = 0$ and for three different positions of defect QW in the array: $p = 0$ (a), 1 (b) and 2 (c). At these values of parameters there are two subbands in all QWs (except the defect QW), filled by electrons. Meanwhile, in the defect QW, the number of filled subbands is determined by the value of N_d

of the wavevector q and for different positions of the defect QW in the array. As seen from Fig. 4, in the zero external magnetic field the number of LPM (depicted by bold solid curves) in the intrasubband plasmon spectrum is equal to the number of subbands in the defect QW occupied by electrons. As seen from the comparison of Fig. 4a—c, the LPM spectrum is weakly dependent on the position of the defect QW in a weakly disordered array of QWs. That phenomenon can be explained by the fact that almost the entire flux of the LPM electromagnetic energy is localized in the vicinity of the defect QW [19]. However, the spectrum of usual plasmon modes is more sensitive to the position of the defect QW in the array. Note that the frequency of LPM increases with the ratio N_d/N. At the same time, the usual spectrum of plasmon modes is characterized by these features. As $p = 0$ (Fig. 4,a) with an increase of the value N_d/N, the frequency of all usual plasmon modes increases as well. It should be noted that the frequencies of intrasubband plasmons supported by the first subbands in QWs (curves $1'$—$4'$) are less sensitive to the value of ratio N_d/N as compared to the frequencies of intrasubband plasmons supported by zero subbands of QWs (curves 1—4). However, at $p = 1$ (Fig. 4,b), the frequencies of two of the usual plasmon modes (curves 2 and $2'$) do not practically depend on the value of ratio N_d/N. For $p = 2$ (Fig. 4,c), there are already four intrasubband plasmon modes (curves $1,1',3,3'$) which possess such a distinctive feature. The spatial distribution of the Hartree potential for those modes is distinguished by the fact, that the absolute value of the Hartree potential in the vicinity of the defect QW is negligible. Therefore, the defect QW does not exert a significant influence on the dispersion properties of plasmon modes [19].

The properties of intrasubband plasmons change if a weakly disordered array of QWs is placed into an external magnetic field. Fig. 5 presents the dependence of the plasmon frequency on the ratio N_d/N for a fixed value of the wavenumber q and for different positions of the defect QW in the array. As seen from Fig. 5, the dependence of LPM frequency on ratio N_d/N is nonmonotonic in the external magnetic field. So, the frequency of LPM supported by the defect QW zero subband (curve LMP1) increases with the ratio N_d/N in the range $0 < N_d/N < 0.39$. At the same time, as N_d/N increases in the range $0.39 < N_d/N < 0.51$, the frequency of LMP supported by the defect QW zero subband is decreased. Meanwhile, the frequencies of LPM supported by the defect QW first and second subbands (curves LPM2 and LPM3, correspondingly) also depend nonmonotonically on the N_d/N. Notice that when a weakly disordered array of QWs is placed into an external magnetic field, there are certain ranges of the $1D$ electron density in the defect QW (in Fig. 5, e.g., $0.51 < N_d/N < 0.82$ and $1.46 < N_d/N < 1.98$), in which the LPMs do not exist. As evident from Fig. 5, in the external magnetic field (as in the case with the zero external magnetic field) at $p = 1$ or $p = 2$, the spectrum of usual plasmon modes contains intrasubband modes (curve *2* in Fig. 5,*b*, curves *1* and *3* in Fig. 5,*c*), whose frequencies do not practically depend on the ratio N_d/N.

Conclusion

We have calculated the intrasubband plasmon spectrum of a finite array of QWs placed into an external magnetic field. Two types of QW arrays have been considered: an ordered array of QWs (in which all the QWs possess the same $1D$ density of electrons) and a weakly disordered array of QWs (in which the $1D$ densities of electrons are equal in all QWs except one defect QW). It is found that, in the ordered array of QWs in the zero magnetic field, each subband filled by electrons in each QW supports its own intrasubband plasmon. Hence, the total quantity of intrasubband plasmon modes in the ordered QW array is equal to the number of QWs in the array multiplied by that of filled subbands in QW. Nevertheless, in nonzero external magnetic fields, the quantity of intrasubband plasmon modes depends on the magnetic field and the $1D$ electron density of QWs. In particular, in a high enough external magnetic field, there are certain ranges of $1D$ electron densities, in which none of intrasubband plasmon modes exists in the spectrum.

In the case of a weakly disordered array of QWs, the LPMs whose properties differ from those of other modes exist in the plasmon spectrum. We point out that, in contrast to the case of the zero magnetic field, the dependence of the LPM frequency on the defect QW $1D$ density of electrons for a sufficiently high magnetic field is of nonmonotone character. Moreover, for high magnetic fields, there are certain ranges of the $1D$ electron density in the defect QW, in which LMPs do not exist. In addition, it is found that the intrasubband plasmon modes, whose spectrum does not depend on the density of electrons of the defect QW [19], also exist in the case of a nonzero external magnetic field.

To conclude, it should be emphasized that the above-mentioned features of plasmon spectra can be used for the diagnostics of defects in QW structures. Hence, the LPM properties can be applied to determine the electron density in the defect QW. At the same time, the properties of usual plasmon modes can be used to define the position of the defect QW in the array.

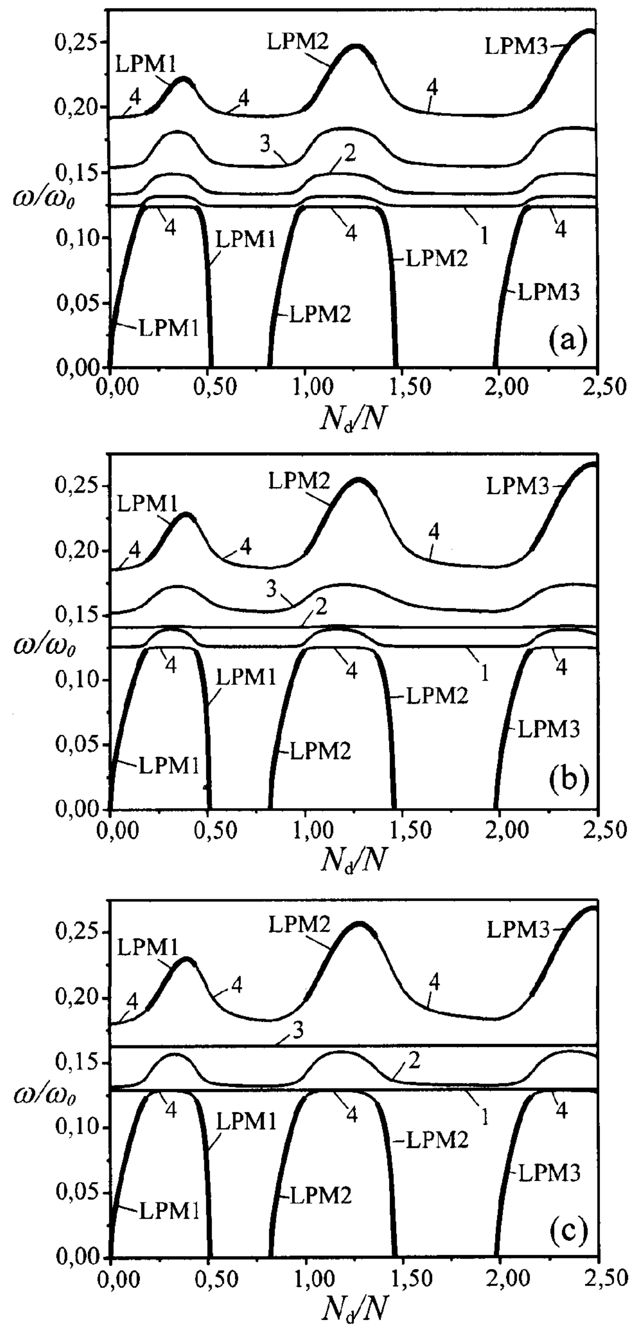

Fig. 5. Dependence of the intrasubband plasmon frequency on the ratio N_d/N at $M = 5$, $ql_0 = 0.04$, $d = 15.0l_0$, $\omega_p = 3.2\omega_0$, $\omega_c = 0.75$ and for different positions of defect QW in the array: $p = 0$ (*a*), 1 (*b*) and 2 (*c*). These values of parameters correspond to the case where, in all QWs (except the defect one), two subbands (zeroth and first) are occupied by electrons. The number of filled subbands in the defect QW is determined by the value of N_d

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ВНУТРІШНЬОПІДЗОННІ ПЛАЗМОНИ У СКІНЧЕННОМУ МАСИВІ КВАНТОВИХ ДРОТІВ, РОЗМІЩЕНИХ У ЗОВНІШНЬОМУ МАГНІТНОМУ ПОЛІ

Ю.В. Блудов

Р е з ю м е

Теоретично досліджено внутрішньопідзонні плазмони у масиві квантових дротів (КД), який складається зі скінченної кількості КД, розташованих на однаковій відстані один від одного та розміщених у зовнішньому магнітному полі. Було розглянуто два види масивів: упорядкований масив з однаковою одновимірною концентрацією електронів в усіх КД та слабкорозупорядкований масив, в якому одновимірна концентрація електронів в одному дефектному КД відрізнялась від концентрації електронів у решті КД. Для упорядкованого масиву КД, розміщених у зовнішньому магнітному полі, передбачено немонотонний характер залежності частоти плазмонів від величини одновимірної концентрації електронів у КД. У сильному магнітному полі показано існування певних діапазонів одновимірної концентрації електронів у КД, в яких плазмонні моди не існують. Для слабкорозупорядкованого масиву КД виявлено існування локальних плазмонних мод, властивості яких відрізняються від властивостей звичайних плазмонних мод. У сильному магнітному полі показано ефект зникнення локальних плазмонних мод у певних діапазонах значень одновимірної концентрації електронів у дефектному КД.