BIFURCATION ANALYSIS OF THE DYNAMICS OF LASERS WITH SELF-MODULATION OF A Q-FACTOR

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The bifurcation analysis of the semi-classical model of a single-mode solid-state laser with an inertialess bleachable filter and a quadratically non-linear element has been carried out. The stability criteria of periodic oscillations, the periodic solution in a quadratic approximation, and the analytical expression of a limit cycle in the first approximation have been obtained. The results of calculations of the limit-cycle elements are presented in the tabular and graphic forms.

Introduction

An important task in the framework of investigating the laser models is the theoretical substantiation of how the variation of solid-state laser parameters, characterizing the laser structure or being the control parameters of the Q-switch, affects the laser dynamics. It was repeatedly marked (see, e.g., [1, 2]) that the introducing of non-linear elements into the resonator substantially enriches the laser dynamics and is an effective means to influence it, which has been corroborated experimentally. The methods of control over solid-laser parameters were studied in [3]. To carry out calculations when designing the laser systems and studying their dynamics at various modes of field generation, to formulate and to resolve the inverse problems of laser dynamics, it is necessary to have theoretical relations which describe the influence of variations of laser parameters on its dynamics. To obtain such relations constitutes a problem which cannot be resolved ultimately in principle, since there are no general methods of integrating the non-linear systems of differential equations which, in turn, prohibits writing down the coefficients of the solution sensitivity with respect to the parameters of the model in an explicit form. Although there is a sufficient quantity of local methods of integrating, but not each of them can be applied to the stated problem. In order to solve it, the authors propose to use the Hopf bifurcation algorithm described in [4].

The theoretical basis of the method of bifurcation of the generation of a cycle is the Hopf theorem [4]. The latter states that if the definite relations between model parameters are fulfilled, the phase coordinates will move periodically about the stationary solution. The Hopf bifurcation can be induced by means of the optoelectronic feedback which forms the derivative of an input intensity and affects the modulator parameters, changing the phase portrait of the model at a fixed value of the stationary solution [1]. In the case under investigation, this means the transition from the stable focus to an unstable one, and it is under such conditions that the Hopf bifurcation emerges.

The model of a laser with an inertialess filter, developed in the framework of semiclassical ideas [2], is considered:

$$\begin{cases} \dot{x} = Gx(y - 1 - b(1+\rho x)^{-1} - rx), \\ \dot{y} = A - y(x+1), \end{cases} \tag{1}$$

where x is the photon field density, y is the inversion in the active medium, ρ is the ratio between the saturation density of the active medium and that of the filter, r is the reduced factor of the non-linear interaction between the resonator and a quadratically non-linear element located in it, G is a large parameter in the theory of lasers of the B class [1], A is the pumping parameter, $b = Q_0 l_2/k_a l_1$, l_1 and l_2 are the distances from the filter to the resonator ends, Q_0 is the absorption coefficient of the unbleached filter, $k_a = \eta - \ln r_1 r_2/2l$, l is the resonator length, η determines passive losses, and r_1 and r_2 are the reflection coefficients of the mirrors. All the parameters, the phase coordinates, and time are dimensionless.

The stationary solutions of system (1), its phase-plane portrait, and some features of its dynamics were studied in [2] with the help of methods of the qualitative theory of differential equations. The purpose of the present work is to elucidate the conditions for the periodic oscillations in system (1) to start, to evaluate the bifurcation values of the parameters b and r, to determine the stability criterion and the intervals of stability for some parameters, and to construct a periodic solution of system (1) if the Hopf bifurcation takes place.

1. Elements of the Bifurcation Analysis of the Model

System (1) is studied in the vicinity of the stationary solution obtained from the equations

$$x_c^3\rho r + x_c^2(r\rho + \rho + r) +$$

$$+ x_c(\rho + r + b + 1 - A\rho) + b + 1 - A = 0,$$

$$y_c = \frac{A}{x_c + 1}. \qquad (2)$$

In this case, x_c is considered as a parameter, the variation limits of which will be determined by stability conditions. Another parameter is determined from the first of Eqs. (2) as a function of the others. Such an approach makes the obtained solution more general.

An essential element of the bifurcation analysis of the dynamic system is the investigation of the eigenvalues of the Jacobi matrix M of the right-hand sides of system (1), calculated for the stationary solution, which look like

$$\lambda = \frac{1}{2}\left(F \pm (F^2 - 4\det M)^{1/2}\right),$$

$$F = Gx_c\left(\frac{b\rho}{K^2} - r\right) - \alpha,$$

$$\det M = Gx_c\left(\frac{A}{\alpha} - \alpha\left(\frac{b\rho}{K^2} - r\right)\right), \qquad (3)$$

where $\alpha = x_c + 1$ and $K = \rho x_c + 1$. The eigenvalues are considered in the neighborhood of those values of parameters which transform the stationary solution into a stable focus, i.e. provided that the inequalities $F < 0$, $F^2 - 4\det M < 0$, and $\det M > 0$ hold true. In this case, the eigenvalues can be written down as $\lambda = \frac{1}{2}(F \pm i\omega_1)$, where $\omega_1 = (4\det M - F^2)^{\frac{1}{2}}$, $\omega_1/2$ is the frequency of relaxation oscillations, and $F/2$ is the damping factor.

Then, the model parameters or one of them are supposed to vary in such a way that the absolute value of F decreases and ultimately becomes zero

$$Gx_c\left(\frac{b\rho}{K^2} - r\right) - \alpha = 0. \qquad (4)$$

In this case, the matrix eigenvalues are purely imaginary, $\lambda = \pm i\omega_0$, $\omega_0 = \sqrt{\det M}$. A further change of the parameters ensures F to be positive, and the system loses its stability. Under those conditions, in accordance with the Hopf theorem [4], periodic oscillations with the frequency ω_0 emerge in system (1) about the stationary solution.

To obtain the stability criterion for those oscillations and to construct the relevant approximate solution, we use the Hopf bifurcation algorithm [4]. We find the eigenvector of the Jacobi matrix that corresponds to the eigenvalue $\lambda = \frac{1}{2}(F + i\omega_1)$:

$$\mathbf{R} = \left(1; B - \lambda(Gx_c)^{-1}\right)^T, \quad B = b\rho K^{-2}$$

where the superscript T denotes the transposition operation. Its real, Re$\mathbf{R}$, and imaginary, Im$\mathbf{R}$, parts are used to construct the transformation matrix $\mathbf{P} =$ (Re$\mathbf{R}$, −Im$\mathbf{R}$), with the help of which new variables are introduced. Then, system (1) reads

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = P^{-1}\begin{pmatrix} f_1(z_1, z_2) \\ f_2(z_1, z_2) \end{pmatrix} \equiv \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}. \qquad (5)$$

We expand the right-hand sides of system (5) into the Maclauren series and retain summands up to the third order of a joint power of the variables. Since the Hopf bifurcation algorithm uses explicitly only nonlinear summands, the reduced right-hand sides of system (5) can be expressed as follows:

$$\bar{F}_1 = -\left(GA_1 z_1^2 + \frac{\omega_0}{x_c}z_1 z_2 + G\frac{b\rho z_1^3}{K^4}\right),$$

$$\bar{F}_2 = \frac{\alpha}{\omega_0}(GA_1 - 1)z_1^2 + \frac{z_1 z_2}{x_c} + G\frac{\alpha b\rho^2}{\omega_0 K^4}z_1^3;$$

$$A_1 = r - \frac{b\rho}{K^3}. \qquad (6)$$

Then, we compose a quantity Φ making use of the partial derivatives of the second and third orders of the functions $\bar{F}_i$, calculated at zero according to the special formulae quoted in [4]. The real part of Φ, ReΦ, is a principal summand of the Floquet index and the imaginary one, $\mathrm{Im}\Phi$, is used to find a correction to the oscillation period $T = 2\pi/\omega_0$. The availability of the large parameter G, which is of the order of 10^5, in the models of solid-state lasers allows one to select only three terms from the cumbersome expression for $\mathrm{Re}\Phi$ which determine the sign of the Floquet index:

$$\mathrm{Re}\Phi = \frac{G}{8Ax_c}(2A_1^2\alpha^2 + A_1A - 3AD_1x_c),$$

$$D_1 = \frac{\rho^2 b}{K^4}. \tag{7}$$

In so doing, we suppose x_c to be rather distant from zero, the relation $x_c > 0.1$ being sufficient.

Since the stable periodic motion is related to a negative value of the Floquet index, the stability criterion of the periodic oscillations is reduced to the fulfillment of the inequality $\mathrm{Re}\Phi < 0$.

2. Analysis of the Stability Criterion for Various Bifurcation Parameters

Consider first the parameter b as a bifurcation one. Taking into account the order of the parameter G makes it possible to obtain the bifurcation value $b_0 = rK^2/\rho$ from Eq. (4). Using Eq. (2), the parameter r is determined: $r = (AK - \alpha(K+b))(\alpha K x_c)^{-1}$. If $b = b_0$, r becomes $r_c = (A-\alpha)\rho(\alpha K^2)^{-1}$, and, taking into account the values of r_c and b_0, the elements of criterion (7) are of the form

$$A_1 = A_{1b} = \frac{(A-\alpha)\rho^2 x_c}{\alpha K^3}, \quad D_1 = \frac{\rho^2(A-\alpha)}{\alpha K^4}. \tag{8}$$

A substitution of (8) into (7) brings the Floquet index to the form

$$\mathrm{Re}\Phi_0 = GP_1W_1,$$

$$P_1 = \rho^2(A-\alpha)(8\alpha A K^4)^{-1},$$

$$W_1 \equiv 2\alpha x_c \rho^2 (A-\alpha)K^{-2} + A(\rho x_c - 2). \tag{9}$$

Since $A > \alpha$ in the overwhelming majority of cases, the stability criterion is equivalent to the inequality $W_1 < 0$. If a new parameter $\gamma = \rho x_c$ is introduced, the latter inequality is of the form

$$Q(\gamma) \equiv \gamma^3 + 2\gamma^2 B_1 - 3\gamma - 2 < 0,$$

$$B_1 = \alpha(A-\alpha)(Ax_c)^{-1}.$$

According to the Cartesian rule, the equation $Q(\gamma) = 0$ has a single positive root γ_1, and the criterion $W_1 < 0$ is satisfied if γ is within the interval $(0, \gamma_1)$.

If the parameter r is taken as a bifurcation one, with the bifurcation value $r_0 = b\rho K^{-2}$, then its substitution into Eq. (2) allows one to obtain the value of the parameter b:

$$b = b_c = (A-\alpha)K^2(\alpha(2\rho x_c + 1))^{-1}.$$

Provided those values, the criterion elements and criterion (7) itself are of the form:

$$A_1 = A_{1r} = \frac{(A-\alpha)\rho^2 x_c}{\alpha(2\rho x_c + 1)K}, \quad \mathrm{Re}\Phi_0 = GP_2W_2,$$

$$P_2 = \frac{\rho^2(A-\alpha)}{8\alpha(2\rho x_c+1)AK^2},$$

$$W_2 = \frac{2\rho^2\alpha x_c(A-\alpha)}{2\rho x_c + 1} + A(\rho x_2 - 2). \tag{10}$$

The requirement $A - \alpha > 0$ leads us to the following stability criterion: $W_2 < 0$. With a new parameter $\gamma = \rho x_c$ introduced, the inequality $W_2 < 0$ is reduced to the quadratic inequality $2\gamma^2 B_2 - 3\gamma - 2 < 0$, $B_2 = (A(2x_c+1) - \alpha^2)(Ax_c)^{-1}$. As a result,

$$0 < \gamma < \gamma_2,$$

$$\gamma_2 = (3 + (9 + 16B_2)^{1/2})(4B_2)^{-1}. \tag{11}$$

The criterion $W_2 < 0$ is thus satisfied if the value of γ is within the interval $(0, \gamma_2)$.

3. Approximate Solution

The approximate solution, the functional parameter ε, in a power series of which the solution is expanded, and the period of oscillations are determined with the help of special formulae [4]:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_c \\ y_c \end{pmatrix} + P\vec{z}, \; z_1 = \mathrm{Re}z, \;\; z_2 = \mathrm{Im}z,$$

$$z = \varepsilon e^{2\theta i} + \frac{i\varepsilon^2}{6\omega_0}(g_1 e^{-4\theta i} - 3g_2 e^{4\theta i} + 6g_3) + O(\varepsilon^3),$$

$$\varepsilon^2 = \frac{(\nu - \nu_0)\lambda_0'}{-\mathrm{Re}\Phi}, \quad T = \frac{2\pi}{\omega_0}(1 + \tau\varepsilon^2 + O(\varepsilon^4)),$$

$$\tau = -\frac{1}{\omega_0}\left(\mathrm{Im}\Phi_0 - \frac{\mathrm{Re}\Phi_0 \cdot \omega_0'}{\lambda_0'}\right), \quad \theta = \frac{\pi t}{T},$$

$$\lambda_0' = \frac{\partial\, \mathrm{Re}\lambda}{\partial \nu}, \quad \omega_0' = \frac{\partial\, \mathrm{Im}\lambda}{\partial\nu}, \quad \nu = \nu_0, \tag{12}$$

where ν is any bifurcation parameter and ν_0 is the corresponding bifurcation value.

It was found that, for our problem,

$$g_{1,2} = \frac{1}{2}\left(-A_1 G + i\left(\frac{\alpha}{\omega_0} G A_1 \mp \frac{\omega_0}{x_c}\right)\right),$$

$$g_3 = \frac{1}{2}(-A_1 G + i\alpha G A_1 \omega_0^{-1}),$$

$$\mathrm{Im}\Phi_0 = \frac{G}{24\omega_0}\left(\frac{9b\rho^2\alpha}{K^4} - 2GA_1^2\left(2 + 5\left(\frac{\alpha}{\omega_0}\right)^2\right) - \right.$$

$$\left. -\frac{\alpha A_1}{x_c} - \frac{A}{\alpha x_c}\right).$$

Making use of formulae (12), we obtain

$$\mathbf{z} = \varepsilon \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix} + \frac{\varepsilon^2 G}{6\omega_0}\begin{pmatrix} -2\sin 4\theta \\ \cos 4\theta \end{pmatrix} A_1 +$$

$$+\frac{\varepsilon^2 G}{6\omega_0}\left(\frac{\alpha A_1}{\omega_0}\begin{pmatrix} \cos 4\theta \\ 2\sin 4\theta \end{pmatrix} + \frac{\omega_0}{G x_c}\begin{pmatrix} 2\cos 4\theta \\ \sin 4\theta \end{pmatrix} - \right.$$

$$\left. -3A_1 \begin{pmatrix} -\alpha/\omega_0 \\ 1 \end{pmatrix}\right) + O(\varepsilon^3).$$

This expression is common for all the bifurcation parameters, but the values of the coefficient A_1, the functional parameter ε, and the period T change depending on which parameter is assumed as a bifurcation one.

4. Calculation of the Solution Elements and the Limit Cycle

The solution elements are the small functional parameter ε^2, the amplitude factors $b - b_0$ and $r - r_0$ that determine the growth of the bifurcation parameter, the criterion element A_1 that belongs to the quadratic term of the solution, the period of oscillations T, the initial phase, and the criteria and intervals of stability.

In the case of the bifurcation parameter $b = b_0$, the test of the transversability condition results in

$$\lambda_0' = \frac{\partial \mathrm{Re}\lambda}{\partial b} = G x_c \rho (2K^2)^{-1} \neq 0.$$

Then,

$$\varepsilon^2 = \frac{(b - b_0)\lambda_0'}{-\mathrm{Re}\Phi_0} = (b - b_0)L_1.$$

The normalizing factor

$$L_1 = 4Ax_c\alpha K^2(\rho(A - \alpha)(-W_1))^{-1}$$

is used to determine the interval of variation of the expression $b - b_0$: $b - b_0 < 1/L_1$.

In the case of the bifurcation parameter $r = r_0$, we obtain, in the same way,

$$\lambda_0' = -\frac{G x_c}{2}, \quad \varepsilon^2 = (r - r_0)L_2,$$

$$L_2 = 4x_c\alpha(2\rho x_c + 1)AK^2(\rho^2(A - \alpha)(-W_2))^{-1}.$$

In the first case, to ensure ε^2 positive, the inequality $b > b_0$ should be valid, i.e. a supercritical bifurcation takes place. In the second case, the inequality $r_0 > r$ is requested for ε^2 to be positive, which governs an undercritical bifurcation. It can be shown that

$$\mathrm{Re}\Phi_0 \frac{\omega_0'}{\lambda_0'} \sim O(\sqrt{G}), \quad \mathrm{Im}\Phi = -\frac{G^2 A_1^2}{6\omega_0} + O(\sqrt{G}).$$

The first correction to the period T is then of the form:

$$\tau = \frac{1}{6\omega_0^2} G^2 A_1^2 + O(1) \approx \frac{G A_1^2 \alpha}{6 A x_c}.$$

Thus,

$$T = 2\pi\omega_0^{-1}(1 + \varepsilon^2 G A_1^2 \alpha (6Ax_c)^{-1}).$$

The amplitude factors $b - b_0$ and $r - r_0$ allow an interval estimate only, so that a certain ambiguity remains concerning the determination of the period and the solution itself, which is in full agreement with the circumstance that the initial conditions are not specified and the period of non-linear oscillations depends on the amplitude.

In the first approximation, the periodic solution of system (1) has the form:

$$x - x_c = \varepsilon \cos 2\theta,$$

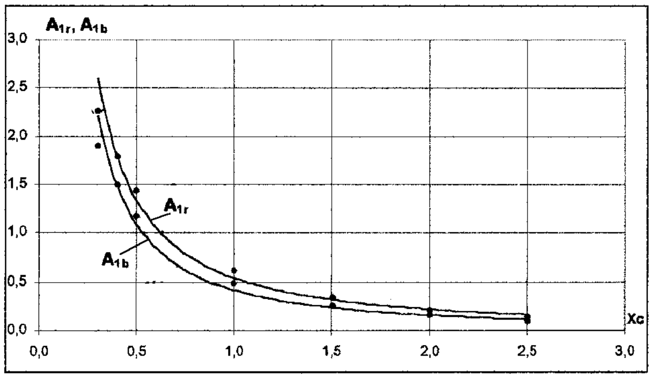
$$y - y_c = -\frac{\varepsilon}{G x_c}(\alpha \cos 2\theta + \omega_0 \sin 2\theta) =$$

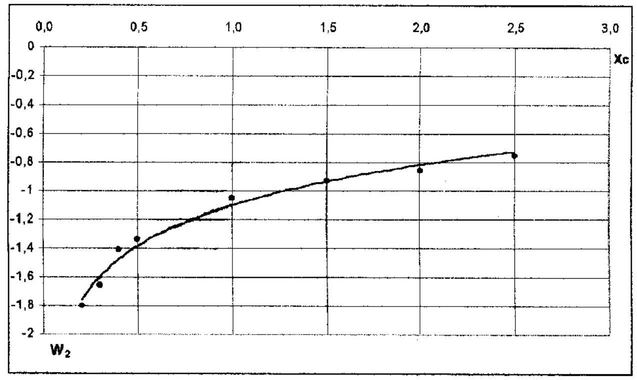
$$= -\varepsilon N (G x_c)^{-1} \cos(2\theta - \varphi),$$

$$\varphi = \mathrm{arctg}\frac{\omega_0}{\alpha}, \;\; N = \sqrt{\alpha^2 + \omega_0^2}. \tag{13}$$

The exclusion of the parameter θ from relations (13) makes it possible to determine the curve describing the limit cycle in the first approximation:

$$\frac{(x - x_c)^2}{a^2} + \frac{(y - y_c)^2}{d^2} = 1 + \Delta,$$





$$\Delta = \frac{2\alpha\omega_0(x-x_c)\sqrt{\varepsilon^2-(x-x_c)^2}}{(\omega_0\varepsilon)^2},$$

$$a = \frac{\omega_0\varepsilon}{\sqrt{\omega_0^2-\alpha^2}},\ d = \frac{\omega_0\varepsilon}{Gx_c}. \qquad (14)$$

Since $\varepsilon > x - x_c$, $(x-x_c)\varepsilon^{-1} = q < 1$, so that $\Delta = 2\alpha q\sqrt{1-q^2}\omega_0^{-1}$. The maximal value of the factor $q\sqrt{1-q^2}$ at $q = \frac{1}{\sqrt{2}} < 1$ and is equal to 0.5. Therefore, $\Delta \le \alpha\omega_0^{-1} = \alpha^{3/2}(GAx_c)^{-1/2}$. For $G = 10^5$, provided that $\alpha^3 < 0.1Ax_c$, Δ becomes less than 0.001. Hence, in the first approximation, the limit cycle is described, within an indicated accuracy, by an ellipse with the half-axes a and d centered at the stationary solution (x_c, y_c).

The procedure of the solution element calculation begins with the evaluation of the roots γ_1 and γ_2 taken as the right ends of the intervals of stability of the Q-factor self-modulation. The signs of the stability criteria W_1 and W_2 are checked for the values of ρx_c somewhat less than those obtained for the roots. Due to the non-linear dependence of the stability criteria on their parameters, the left shifts from the right end are not uniform for different values γ_1 and γ_2. Then we calculate the normalizing factor L_2 and the criterion elements A_{1r} and A_{1b}, which belong to the solution. The values of the root γ_2 for several values of x_c, the values of ρx_c and the corresponding values of the criterion W_2, as well as the values of the normalizing factor L_2 and of the criterion elements A_{1r} and A_{1b} at $\gamma_2 = \rho x_c$, are quoted in the table. The calculations were carried out for $A = 10$.

x_c	γ_2	ρx_c	W_2	$\mathrm{Re}\Phi_0$	L_2	A_{1r}	A_{1b}
0.2	0.536	0.5	–1.8	–22860	0.408	3.0555	2.716
0.3	0.641	0.6	–1.66	–9843.8	1.416	2.2614	1.9006
0.4	0.720	0.68	–1.405	–4678.6	3.954	1,7910	1.4976
0.5	0.782	0.74	–1.339	–2765.9	9.038	1.4382	1.1781
1.0	0.972	0.929	–1.047	–510.29	117.983	0.6209	0.4783
1.5	1.078	1.035	–0.926	–130.01	576.876	0,3430	0.2543
2.0	1.154	1.11	–0.860	–53.896	1855.41	0.2116	0.1530
2.5	1.224	1.175	–0.749	–21.273	4629.16	0.1412	0.0998

The plots of the functions $A_{1r}(x_c)$, $A_{1b}(x_c)$, and $W_2(x_c)$ are shown in the figure.

Conclusions

1. With the help of the Hopf bifurcation algorithm, the stability criteria W_k, the intervals of stability $(0, \gamma_k)$ of the photon irradiation self-modulation, and the approximate periodic solution for the dynamic system, provided that the Hopf bifurcation takes place, have been obtained. It should be noted that to obtain those results by methods of the qualitative theory is impossible, which was corroborated when considering a similar problem in [2].

2. The selection of a specific parameter as a bifurcation one may lead to different consequences, namely, the selection of the other equation to obtain the stationary solution, the appearance of additional limitations to the parameters' values, the changes of the stability interval and of the stability criterion elements. This is one of the reasons why each parameter of the laser model is worth being considered as a potentially bifurcation one. This point of view was stated, in particular, in [1].

3. Considering x_c as an independent parameter allows one to construct solutions in the vicinity of any physically eligible and technologically expedient stationary solution, which compensates, to some extent, the locality of the method.

4. The more the absolute value of the stability criterion, the more the interval of the bifurcation parameter variation. For example, $r - r_0 < 1/L_2 = 2.45$ if $x_c = 0.2$ and $|W_2| = 1.8$, but $r - r_0 < 0.11$ if $x_c = 0.5$ and $|W_2| = 1.339$, i.e. a larger amplitude of oscillations corresponds to a smaller value of x_c. The previous conclusion becomes more sound due to the fact that a smaller value of x_c is related to larger values of A_{1r} and A_{1b} which

are the factors of the solution summand quadratic in ε.

5. Since $A_{1r}(A_{1b})^{-1} = 1 + \frac{\gamma^2}{2\gamma+1}$ and $\gamma = \rho x_c$, $A_{1r} > A_{1b}$ for all γ's, with a discrepancy between A_{1r} and A_{1b} being the largest at the right end of the interval. At the same time, the absolute values of A_{1r} and A_{1b} decrease here.

The Hopf bifurcation algorithm has wide perspectives when being applied to the investigation of the laser model dynamics, including the retardation phenomena, and when formulating and resolving the inverse problems of laser dynamics.

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БІФУРКАЦІЙНИЙ АНАЛІЗ ДИНАМІКИ ЛАЗЕРІВ З АВТОМОДУЛЯЦІЄЮ ДОБРОТНОСТІ

Г.П. Коваленко, С.В. Коломієць

Р е з ю м е

Проведено біфуркаційний аналіз моделі одномодового твердотільного лазера з безінерційним фільтром і квадратично-нелінійним елементом. Одержано критерії стійкості періодичних коливань, періодичний розв'язок у квадратичному наближенні і аналітичний вигляд граничного циклу в першому наближенні. Результати розрахунків елементів граничного циклу наведено у табличному і графічному вигляді.