## MULTIPLE-CENTER EIKONAL APPROACH AND SCATTERING OF PROTONS ON NUCLEI WITH A = 3,4 AT ENERGIES OF 600 AND 1000 MeV

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The multiple-center eikonal approach with regard to the scattering of high-energy protons by atomic nuclei is investigated. In contrast to the Glauber—Sitenko theory, a new approach uses the three-dimensional generalized profile function of a nucleon, which allows us to take into account off-shell effects in the intermediate acts of scattering. The developed formalism is applied to the calculations of cross sections of the elastic scattering of protons at energies of 600 and 1000 MeV by <sup>3</sup>H, <sup>3,4</sup>He nuclei, which are considered in the framework of a realistic multiplicative model. The results of calculations are compared with experimental data and calculations based on the common diffraction theory.

## Introduction

A necessity of the use of deformed waves, which describe a motion of interacting nuclear fragments, appears at the theoretical consideration of nuclear processes at high energies with a redistribution of particles in the channels. The deformed waves are usually obtained with the help of optical potentials. However for lownucleon systems (A < 10), it is not evident beforehand that the optical approximation will be justified enough. Moreover, standard methods of partial decomposition at high energies become very complicated, because they need hundreds of partial waves to be taken into account. Therefore, it is desirable to find the deformed waves without use of optical potentials at all, but connect them with free amplitudes of NN-scattering similarly to that of the Glauber–Sitenko (GS) diffraction theory of multi-scattering (DTMS) [1, 2] for a multiparticle Toperator. The basis of such a program was founded in [3]. Having an application of those deformed waves for the theoretical investigation of various nuclear reactions as the ultimate aim, we must be convinced of their capability to describe the simplest process — the elastic scattering of protons by atomic nuclei. The investigation of the elastic scattering of high-energy protons by nuclei with low numbers of nucleons (A = 3, 4) is exactly the aim of the present work. A phenomenological multiplicative model, whose parameters agree with

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the data on charge form-factors obtained from the experiments on the elastic scattering of fast electrons, is used for the wave functions of the ground states of these nuclei. This allows us to fix the parameters of model functions and use them later in the investigations of the elastic scattering of protons by those nuclei without introducing any additional structural parameters. In the case of satisfactory agreement of the calculated cross-sections of the elastic scattering of protons with experimental data, one can come to a conclusion that a technique of construction of the deformed waves works, and it may be used for taking into account an interaction among the nuclear objects in the input and output channels of specific nuclear reactions.

### 1. General Form of Deformed Waves

Consider the elastic pA-scattering in the framework of the high-energy method of deformed waves, which explicitly takes into account the multinucleon structure of a scatterer. The deformed wave function of a scattered proton depends on the coordinates of nuclear nucleons, which play a role of fixed centers, has a form [3]:

$$\Psi_{\mathbf{k}}^{(+)}(\mathbf{r}, \mathbf{r}_{1}...\mathbf{r}_{A}) = \frac{1}{(2\pi)^{3/2}} \exp(i\mathbf{k}\mathbf{r}) \prod_{j=1}^{A} \left[1 - \omega^{(+)}(\mathbf{r} - \mathbf{r}_{j})\right].$$
(1)

The main structure elements of deformed waves are three-dimensional generalized profile functions of a nucleon  $\omega^{(+)}(\mathbf{r})$  which, in contrast to the profile functions of the GS theory, contain a longitudinal part due to the longitudinal component of a transmitted momentum  $q_Z$  in the NN-amplitude.

In the following calculations, we will use a Gauss parameterization of the *NN*-amplitude, which satisfies the optical theorem and contains 4 parameters  $\sigma$ ,  $\rho$ ,  $a_t$ ,  $a_l$ , by means of which the experimental

cross sections of  $N\!N\mbox{-}{\rm scattering}$  may be approximated satisfactorily by

$$f(\mathbf{q}) = \frac{k\sigma}{4\pi}(i+\rho)\exp(-a_t q_\perp^2 - a_l q_Z^2),$$
  
$$q_\perp = k\sin\theta, \quad q_Z = 2k\sin^2\frac{\theta}{2}.$$
 (2)

For such an amplitude, the generalized profile function has a form:

$$\omega^{(+)}(\mathbf{r}) = \frac{1}{2}\omega(b) \left[ 1 + \operatorname{erf}\left(\frac{z}{2\sqrt{a_l}}\right) \right],$$
$$\omega(b) = \frac{\sigma(1-i\rho)}{8\pi a_t} \exp\left(-\frac{b^2}{4a_t}\right), \quad \mathbf{r} = (\mathbf{b}, z), \quad (3)$$

where the transversal part of the profile function  $\omega(b)$ corresponds to the standard GS DTMS. The energy dependence of parameters of the pp- and pn-amplitudes was calculated in [4] for a non-local separable potential [5] with Gauss form-factors. Amplitude (2) with  $a_l < a_t$ can describe an increase of cross-sections at back angles and therefore, by its character, may not be necessarily diffractional. This is a principal novelty of the highenergy method of deformed waves and its distinction from DTMS. Exactly the amplitude dependence on  $q_Z$ allows us to take into account off-shell effects in a twoparticle act of scattering. At  $a_l = 0$ , (2) is an ordinary parametrization of the amplitude of NN-scattering in the diffraction approximation. Deformed waves (1) mean a scattering by fixed centers, which corresponds to the adiabatic approximation which works well at high energies. At a limiting case  $q_Z = 0$ , all the belowobtained formulas for the cross-sections of the elastic scattering pass into the well-known expressions of the standard DTMS. The effect from accounting the  $q_Z$ dependence, which appears in the final formulas, turns out to be important enough in the region of diffraction minima.

## 2. Selection of a Model and the Electrical Form-factor of a Nucleus

Let us apply the multiplicative model of non-correlated nucleons with a Gauss coordinate dependence, which factorizes as

$$\Psi_A(\mathbf{r}_1 \cdots \mathbf{r}_A) = N_A \prod_{j=1}^A \varphi(\mathbf{r}_j),$$
  
$$\varphi(r) = \sum_k a_k \exp(-\alpha_k r^2), \ a_1 = 1$$
(4)

to the ground state of a nucleus. Coordinates of nucleons in the nuclear wave function are counted off from the center of mass of the nucleus, which is located at the origin of coordinates. The limitation on the parameter  $a_1$ , which is chosen for convenience, leads to the result that one-particle functions must be normalized by a condition

$$\varphi^{2}(r) = \sum_{k} \rho_{k} \exp(-\sigma_{k} r^{2}),$$

$$\int \varphi^{2}(r) d\mathbf{r} = \sum_{k} \rho_{k} \left(\frac{\pi}{\sigma_{k}}\right)^{3/2} \equiv \rho_{0}.$$
(5)

Condition (5) provides a normalization of the wave function of a nucleus in the form

$$\int |\Psi_A(\mathbf{r}_1 \cdots \mathbf{r}_A)|^2 \prod_{j=1}^A d\mathbf{r}_j =$$
$$= N_A^2 \int \prod_j \varphi^2(r_j) d\mathbf{r}_j = N_A^2 \rho_0^A = 1.$$
(6)

Relation (6) evidently contains non-physical effects of movement of the center of mass of a nucleus. In model (4), the one-particle nuclear density coincides with the normalized one-nucleon density. Indeed, due to the symmetry of the nuclear wave function, we have

$$\rho_A(r) = \int \prod_j d\mathbf{r}_j |\Psi_A(\mathbf{r}_1 \cdots \mathbf{r}_A)|^2 \frac{1}{A} \sum_j \delta(\mathbf{r} - \mathbf{r}_j) =$$
$$= \sum_k \frac{\rho_k}{\rho_0} \exp(-\sigma_k r^2) \equiv \rho(r).$$
(7)

A nuclear form-factor is defined as the Fourier transform of a one-particle nuclear density,

$$S_A(\mathbf{Q}) = \int d\mathbf{r} \rho_A(r) \exp(i\mathbf{Q} \cdot \mathbf{r}) =$$
$$= \sum_k \frac{\rho_k}{\rho_0} \left(\frac{\pi}{\sigma_k}\right)^{3/2} \exp(-\frac{Q^2}{4\sigma_k}) \equiv S(\mathbf{Q}), \tag{8}$$

which coincides with a one-nucleon form-factor according to (7). The connection of the electric formfactor of a nucleus with the nuclear and charge formfactors of a proton and neutron [6, 7] is determined by

$$F_{\rm ch}(q) = \left[G_p(q^2) + \frac{A - Z}{Z}G_n(q^2)\right]S_A(q),$$
(9)

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which also does not consider the effects of recoil (translational invariance). The root-mean-square radius of a one-particle nuclear density is

$$\Re_A^2 = \frac{3}{2} \sum_k \frac{\tilde{\rho}_k}{\sigma_k} \left(\frac{\pi}{\sigma_k}\right)^{3/2}, \quad \tilde{\rho}_k = \frac{\rho_k}{\rho_0}.$$
 (10)

Effects of translational invariance for the charge formfactor (9) can be approximately taken into account by introducing the well-known factor of recoil, which automatically appears in the models with translational invariance, into this expression. As a result, we obtain finally

$$F_{\rm ch}(q) = \left[G_p(q^2) + \frac{A - Z}{Z}G_n(q^2)\right]S_A(q)\exp\left(\frac{q^2\Re_A^2}{6A}\right).$$
(11)

Hence, the root-mean-square radius of the distribution of electric charge in a nucleus is

$$r_{\rm ch}^2 = r_p^2 + \frac{A-Z}{Z}r_n^2 + \frac{A-1}{A}\Re_A^2.$$
 (12)

Here  $r_p^2$   $(r_n^2)$  is the root-mean-square radius of a distribution of **charge** in a proton (neutron), and the last term is the correlated root-mean-square radius of a distribution of proton centers in a nucleus.

For processes with large momenta transferred, the parameters of wave functions (4) have to be determined from the condition that electromagnetic form-factors obtained from the data on scattering of high-energy electrons can be described on their base. Therefore, expression (11) was used by us for a search for values of parameters of the wave functions of S-shell nuclei basing on the description of their experimental electrical form-factors (see Table 1). In the last column, the root-mean-square charge radii which correspond to the parameters of a given row are listed. Experimental values of these radii are listed in parentheses. The results of our calculations of electrical form-factors, together with experimental data [10,11], are plotted in Fig. 1, from which it can be seen that, due to a flexible shape of wave functions, a good agreement of theoretical and experimental values can be reached.

T a b l e 1. Parameters of wave functions of the ground states of nuclei in model (4)

Nucleus	$rac{lpha_1}{\mathrm{Fm}^{-2}}$	$a_2$	$a_2, \ { m Fm}^{-2}$	$a_3$	$lpha_3, \ { m Fm}^{-2}$	$r_{ m ch}, \ { m Fm}$
$^{3}\mathrm{H}$	0.188	-0.006	1.624			1.760(1.760)
$^{3}\mathrm{He}$	0.172	-0.287	0.890	2.486	0.279	1.675(1.976)
$^{4}\mathrm{He}$	0.321	-0.138	1.060	-0.395	0.320	1.563(1.671)





Fig. 1. Charge form-factors of nuclei <sup>3</sup>H, <sup>3</sup>He, <sup>4</sup>He in the multiplicative model

## 3. Amplitude of Scattering of Protons by S-nucleon Nuclei in the Multiplicative Model and Calculations of Cross-sections

According to the method of deformed waves [3], the cross-section of the elastic scattering of a proton by a nucleus in the center-of-mass frame is written in the form

$$\frac{d\sigma}{d\Omega} = |F(\mathbf{q})|^2, \quad F(\mathbf{q}) = \frac{ik}{2\pi} \int d\mathbf{r} \exp(i\mathbf{q} \cdot \mathbf{r})\Omega(\mathbf{r}), \quad (13)$$

where k — is the momentum of a proton in the center-ofmass frame,  $\mathbf{q}$  — is the three-dimensional vector of the total transferred momentum. The generalized profile of a nucleus (the T — operator in the eikonal approximation) for the translationally non-invariant model is obtained in the following form:

$$\Omega(\mathbf{r}) = \frac{d}{dz} \int \prod_{j=1}^{A} d\mathbf{r}_{j} |\Psi_{A}(\mathbf{r}_{1}...\mathbf{r}_{A})|^{2} \times \left\{ 1 - \prod_{j=1}^{A} \left[ 1 - \omega^{(+)}(\mathbf{r} - \mathbf{r}_{j}) \right] \right\}.$$
(14)

The longitudinal part of profile (14) brings a principally new dependence of the scattering amplitude on the longitudinal component of the transferred momentum, which is absent in the standard theory and plays a crucial role in filling the deep diffraction minima, which are intrinsic to the ordinary diffraction theory. Due to the factorization of integrand (14) in coordinates of individual nucleons, we have

$$\Omega(\mathbf{r}) = \frac{d}{dz} \int \prod_{j=1}^{A} d\mathbf{r}_{j} \rho(r_{j}) \times \\ \times \left\{ 1 - \prod_{j} \left[ 1 - \omega^{(+)} (\mathbf{r} - \mathbf{r}_{j}) \right] \right\} = \\ = \frac{d}{dz} \left\{ 1 - [1 - G(\mathbf{r})]^{A} \right\},$$
(15)

where the one-particle density  $\rho(r)$  is given by expression (7), and  $G(\mathbf{r})$  is the profile function  $\omega^{(+)}$  averaged over one-particle states:

$$G(\mathbf{r}) = \int \omega^{(+)} (\mathbf{r} - \mathbf{r}_1) \rho(r_1) d\mathbf{r}_1 =$$

$$= \sum_k \frac{\rho_k}{\rho_0} \left(\frac{\pi}{\sigma_k}\right)^{3/2} \frac{\sigma(1 - i\rho)\beta_k}{4\pi} \exp(-\beta_k b^2) \times$$

$$\times \left[1 + erf\left(\sqrt{\zeta_k}z\right)\right], \qquad (16)$$

$$\beta_k = \frac{\sigma_k}{1 + 4a_t \sigma_k}, \quad \zeta_k = \frac{\sigma_k}{1 + 4a_l \sigma_k}.$$

As a result, relation (15) yields the profile function of a nucleus as

$$\Omega(\mathbf{r}) = A[1 - G(\mathbf{r})]^{A-1} \frac{d}{dz} G(\mathbf{r}),$$

$$\frac{d}{dz} G(\mathbf{r}) = \sum_{k} \tilde{\rho}_{k} \left(\frac{\pi}{\sigma_{k}}\right)^{3/2} \left(\frac{\zeta_{k}}{\pi}\right)^{1/2} \times$$

$$\times \frac{\sigma(1 - i\rho)\beta_{k}}{2\pi} \exp(-\beta_{k}b^{2} - \zeta_{k}z^{2}).$$
(17)

After integration in (13) over the azimuthal angle of the transversal vector  $\mathbf{q}_{\perp}$ , we obtain the final expression for the proton-nucleus scattering amplitude in the center-of-mass frame

$$F(\mathbf{q}) = ik \exp\left(\frac{q^2 \Re_A^2}{6A}\right) \int_0^\infty db \, b J_0(q_\perp b) \times \\ \times \int_{-\infty}^\infty dz \exp(iq_Z z) \Omega(b, z),$$
(18)

where, as in the case of electrical form-factor, the approximate factor of recoil is introduced.

Expression (18) generalizes the known result of GS DTMS and is reduced to it in the single case where  $q_Z = 0$ . Thus, if we neglect the transversal component of the transferred momentum  $q_Z$  in (18) and perform an integration over z, then we obtain the known result of the GS diffraction theory using representation (15) and replacing  $q_{\perp}$  by q in a final result:

$$F(q) = ik \exp\left(\frac{q^2 \Re_A^2}{6A}\right) \int_0^\infty db \, b \times \\ \times J_0(qb) \left\{ 1 - [1 - G(b)]^A \right\}, \\ G(b) = \int \omega(\mathbf{b} - \mathbf{b}_1) \rho(r_1) d\mathbf{r}_1 = \\ = \sum_k \tilde{\rho}_k \left(\frac{\pi}{\sigma_k}\right)^{3/2} \frac{\sigma(1 - i\rho)\beta_k}{2\pi} \exp(-\beta_k b^2).$$
(19)

As an illustration of capability of the developed theory, the cross-sections of the elastic scattering of protons with energies of 600 and 1000 MeV by nuclei <sup>3,4</sup>He and <sup>3</sup>H are calculated. The results of these calculations are depicted in Fig. 2. Calculations of the cross-sections of the elastic scattering of protons by these nuclei at energies around 1 GeV carried out earlier on the base of DTMS, describe experimental data well enough with exception of the regions of the minima of cross-sections, where this theory gives the deep dips (dashed lines in Fig. 2.), which in no way match the experiment. A number of unsuccessful efforts have been made for the liquidation of the mentioned discrepancies. But all of them turned out to be futile. In our calculations according to the theory presented above (see Fig. 2), taking the longitudinal component of a transferred momentum into account provides the necessary degree of filling the minima with a good stability over all the parameters of the theory and agrees well with the experimental data [12, 13]. The used values of parameters of the NN-amplitudes [8, 9, 14] are listed in Table 2.

While carrying out the calculations, the parameters from Table 2 were averaged in according to the numbers of protons Z and neutrons N (this does not influence the results essentially, but substantially simplifies the

T a b l e 2. Values of parameters of NN-amplitudes

$E,  \mathrm{MeV}$	$\sigma_{pp}$ , mb	$\sigma_{np}$ , mb	$ ho_{pp}$	$\rho_{np}$	$a_{pp},  \mathrm{Fm}^2$	$a_{np}$ , Fm <sup>2</sup>
600	39.6	36.6	0.38	-0.21	0.05	0.05
1000	47.2	39.2	-0.09	-0.46	0.09	0.12

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Fig. 2. Differential cross-sections of the elastic scattering of protons by nuclei at energies of 600 and 1000 MeV in the multiplicative model. Dots — experiment, solid line — calculation by our theory, dashed line — calculation by Glauber—Sitenko DTMS

calculations):

$$A\sigma = Z\sigma_{pp} + N\sigma_{np}, \quad \rho = \frac{1}{A\sigma} \left( Z\sigma_{pp}\rho_{pp} + N\sigma_{np}\rho_{np} \right),$$
$$a_t = a_l = \frac{1}{A\sigma} \left( Z\sigma_{pp}a_{pp} + N\sigma_{np}a_{np} \right).$$

As seen from the shown results, by using the wave functions of the multiplicative model which reconstruct the electrical form-factor well in the whole interval of transferred momenta, we succeeded in almost an ideal description of experimental data on the cross-sections of the elastic scattering.

Let us note that, from the point of view of the eikonal approximation, the accounting of  $q_Z$  in the amplitudes of *NN*-scattering means a removal from the energy surface. In other words, the quantitative description of the regions of minima of cross-sections is completely obliged, in the considered approach, to accounting of the off-shell effects which are not taken into account in GS DTMS.

## 4. The Optical Limit of Elastic Scattering

Having denoted all the quantities by the index "opt" in this case, we write the amplitude of scattering in the optical approximation  $(A \gg 1)$  in the form (13),

$$F_{\rm opt}(\mathbf{q}) = \frac{ik}{2\pi} \int d\mathbf{r} \, \exp\left(i\mathbf{q}\cdot\mathbf{r}\right) \Omega_{\rm opt}(\mathbf{r}). \tag{20}$$

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Using expression (15) for the profile function of a nucleus and passing on to the limit  $A \gg 1$  in it, we obtain the optical nuclear profile

$$\Omega_{\text{opt}}(\mathbf{r}) = \frac{d}{dz} \{1 - \exp\left[-AG(\mathbf{r})\right]\} =$$
$$= A \exp\left[-AG(\mathbf{r})\right] \frac{d}{dz} G(\mathbf{r}). \tag{21}$$

In this case, the amplitude of scattering takes a form (we drop temporarily the recoil factor)

$$F_{\text{opt}}(\mathbf{q}) = ikA \int_{0}^{\infty} db \, b \, J_{0}(q_{\perp}b) \times$$
$$\times \int_{-\infty}^{\infty} dz \exp\left[iq_{Z}z - AG(\mathbf{r})\right] \frac{d}{dz}G(\mathbf{r}).$$
(22)

Let us obtain now an expression for the effective optical potential, which corresponds to the optical profile (21). For that, we use the following representation of the amplitude of elastic scattering in the eikonal approximation

$$F_{\rm opt}(\mathbf{q}) = \frac{ik}{2\pi} \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{d}{dz} \times$$

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$$\times \left\{ 1 - \exp\left[ -\frac{i}{v} \int_{-\infty}^{z} V_{\text{opt}}(b, z') dz' \right] \right\} \equiv$$
$$\equiv \frac{ik}{2\pi} \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{d}{dz} \{ 1 - \exp\left[ -AG(\mathbf{r}) \right] \}.$$

An integral equation for the optical potential, that has been obtained from this, is solved by a simple differentiation. As a result, we find (v is a relative velocity):

$$\frac{i}{v}V_{\rm opt}(\mathbf{r}) = A\frac{d}{dz}G(\mathbf{r}) = A\frac{d}{dz}\int\omega^{(+)}(\mathbf{r}-\mathbf{r}_1)\rho(r_1)d\mathbf{r}_1.$$
(23)

Let us express this optical potential in terms of the amplitude of NN-scattering. For this, we use a connection of the elementary profile with the amplitude of NN-scattering [3]:

$$\frac{d}{dz}\omega^{(+)}(\mathbf{r}) = \frac{1}{(2\pi)^2 i p_0} \int d\mathbf{Q} \exp(-i\mathbf{Q} \cdot \mathbf{r}) f(\mathbf{q}), \qquad (24)$$

where  $p_0$  is the momentum of a nucleon in the centerof-mass frame of two nucleons. As a result, we get

$$\frac{i}{v}V_{\rm opt}(\mathbf{r}) = \frac{A}{(2\pi)^2} ip_0 \int d\mathbf{Q} \exp(-i\mathbf{Q}\cdot\mathbf{r}) f(\mathbf{Q})S(\mathbf{Q}).$$
(25)

It is seen from last expression that this potential differs from those optical potentials which are used in calculations of the elastic scattering of nucleons in the optical model of a nucleus. Its distinctive feature is that it depends on the angle between  $\mathbf{r}$  and the direction of the momentum of an incident proton  $\mathbf{k}$ . That is, it is not spherically symmetric, because  $\beta_k \neq \zeta_k$  in (16). The flattening of a potential  $(a_l \leq a_t)$  takes place along the direction of a phase trajectory. In the case where  $a_l = a_t$ , formula (25) passes into the known expression for optical potential in the Kerman-McManus-Thaler theory [15].

If we neglect the longitudinal component of the transferred momentum  $q_Z$  in (22), make an integration over z, and replace  $q_{\perp}$  by q in the final result, we come into the known expression of GS DTMS,

$$F_{\text{opt}}(q) = ik \int_{0}^{\infty} db \, b \, J_0(qb) \{1 - \exp\left[-AG(b)\right]\},$$

$$G(b) = \frac{\sigma(1-i\rho)\beta}{2\pi} \exp(-\beta b^2).$$
(26)

It is evident from a general consideration that the optical limit must works well for nuclei with  $A \gg 1$ . But the calculations of the cross-sections of elastic scattering of protons in the optical approximation for nuclei <sup>3,4</sup>He and <sup>3</sup>H, that are carried out by us, are not distinguished practically from the exact ones.

#### Conclusions

A new approach of construction of deformed wave functions, which describe the motion of nuclear particles and take into account the microscopic structure of scatterers, was developed with the use of the multicenter eikonal approximation. In the case of the lightest nuclei, this allows us to take into account the effects of multiple collisions more correctly in contrast to the opticalmodel approach, in which this microscopic structure is not explicitly reflected. Those wave functions are necessary for the calculations of the cross-sections of various nuclear processes with reconstruction at middle and intermediate energies.

In the offered method, the optical potential was not used at all, but the deformed waves were connected with the amplitude of free NN-scattering, whose parameters are usually calculated with the help of a realistic potential or are found from experiment. The main constituents of these waves are generalized profile functions which, in contrast to the standard diffraction theory, contain the longitudinal part conditioned by the explicit accounting of the longitudinal component of the transferred momentum in the NN-amplitude of scattering. This corresponds to the accounting of the off-shell effects in this scattering. The constructed multicenter waves correspond, by their physical meaning, to the scattering of nucleons by a system of fixed centers.

Thus, we have derived new formulas for calculations of amplitudes of the elastic scattering of nucleons by atomic nuclei, which are considered in the multiplicative model of independent particles with a Gauss coordinate dependence of their spatial functions. The developed formalism was applied to the description of the elastic scattering of protons with energies of 600 and 1000 MeV by nuclei <sup>3</sup>H, <sup>3,4</sup>He. The accuracy of the optical limit ( $A \gg 1$ ) for S-shell nuclei was analyzed, and it was found that the results of calculations in this approximation have practically not been distinguished from the exact ones. It was shown that, without additional parameters, the offered generalization of the diffraction theory allows us, particularly, to solve quantitatively the long-standing problem of the deep

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diffraction minima of differential cross-sections, which is intrinsic to the ordinary approach, and describe the experimental angular distributions of the elastic scattering of protons by nuclei much better than GS DTMS.

Finally, we emphasize that the high-energy method of deformed waves will be especially useful in the description of nuclear reactions with a redistribution of particles in channels, reactions of fission of nuclei at high energies, and other processes which are accompanied by the transfer of large longitudinal momenta.

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# БАГАТОЦЕНТРОВЕ ЕЙКОНАЛЬНЕ НАБЛИЖЕННЯ I РОЗСІЯННЯ ПРОТОНІВ ЯДРАМИ З A = 3, 4 ПРИ ЕНЕРГІЯХ 600 ТА 1000 МеВ

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Резюме

Проведено дослідження багатоцентрового ейконального наближення стосовно розсіяння протонів високих енергій атомними ядрами. На відміну від теорії Глаубера—Ситенка новий підхід використовує тривимірну узагальнену профільну функцію нуклона, яка дозволяє врахувати позаенергетичні ефекти в проміжних актах розсіяння. Побудований формалізм застосовано для розрахунків перерізів пружного розсіяння протонів з енергіями 600 та 1000 МеВ ядрами <sup>3</sup> Н, <sup>3,4</sup> Не, які розглядаються в реалістичній мультиплікативній моделі. Результати розрахунків порівнюються з експериментальними даними і з розрахунками за звичайною дифракційною теорією.