MOMENTUM SPREAD IN CHARGED BUNCHES DUE TO THE MOVEMENT OF IONS IN A SELF-COORDINATED ELECTRIC FIELD

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The results of analytical and numerical calculations are presented in order to estimate the space charge effect influence on the momentum spread and beam divergence during carrying out the diagnostic procedure with intense ion beams at RF accelerators.

Introduction

For the beam diagnostic purposes and precise nuclear measurements at particle accelerators, it is important to know the exact information about the ion beam parameters. At the transportation of charge particle beams, it is necessary to take into account influence of the space charge effect on ions in a bunch itself.

In [1—3], the researches of the own electric field of ion beams and its influence on results of the non-destructive beam profile monitoring were performed and the analytical formulas for the potential and electric field strength of bunches with different configurations were given.

Here, the estimation of the momentum spread of particles due to the action of space charge forces as well as the additional transverse divergence arising also from space charge forces will be carried out. For numerical calculations, the derived analytical solutions for the periodical electric field parameters [1—3] are used. Some results of such calculations for changing the momentum spread of the drift space for the beams typical of RF accelerators are presented.

1. Estimation of the Momentum Spread

In general in RF accelerators, the geometric dimensions of ion bunches are very similar in two transverse directions. Therefore, it is expedient to assume the spherical shape for a single bunch. Considering different distributions of the particles within bunches (here, the spherical bunches with homogeneous and parabolic charge distributions are analyzed), the analytical expressions for the representation of electric fields inside and outside of an ion bunch have been derived (see [1—3]).

In the non-relativistic case, a change of the momentum spread can be given by the well-known formula

$$\frac{\Delta p}{p} = \frac{Z_i e \int\limits_0^t \vec{E}(\vec{r}, r')dr'}{M\beta c}, \tag{1}$$

where $\Delta p/p$ — change of the impulse momentum spread, E — electric field, M — ion mass, β — relativistic factor, t — time, c — velocity of light, Z_i — ion charge, e — electron charge.

For the estimation, we consider some ions on the beam axis ($x = 0, y = 0$ — transverse coordinates, $z \neq 0$ — longitudinal coordinate) moving with the same speed βc as the bunch at $t = 0$. Of course, the estimation using our analytical formulas for the description of the space charge effects [1—3] holds only as long as a change of the position with respect to the start position is small compared to the bunch size, which means that there will be no remarkable change of our non-selfconsistent intensity distributions within the bunch. Fig. 1 shows the results of numerical calculations for a β-value of 1.6%, which corresponds to the final energy of the new IH-RFQ of GSI [4] (Darmstadt, Germany).

Fig. 2 shows the same estimations for $\beta = 0.055$ which is the energy in the stripper section of the UNILAC of GSI [4]. These calculations hold if there are no other forces acting on bunches. In both cases, we consider an U^{4+} ion beam with 10^9 particles within one spherical bunch (10 mm in radius) and the parabolic charge distribution within such an ion bunch.

The diagrams on the bottom of Figs.1 and 2 give an estimation of the drift space which may be tolerated considering a markable increase of the momentum spread by a space charge.

For comparison, Fig. 3 shows the results holding for a homogeneous charge distribution within the bunch at $\beta = 0.055$ and the same ion beam parameters.

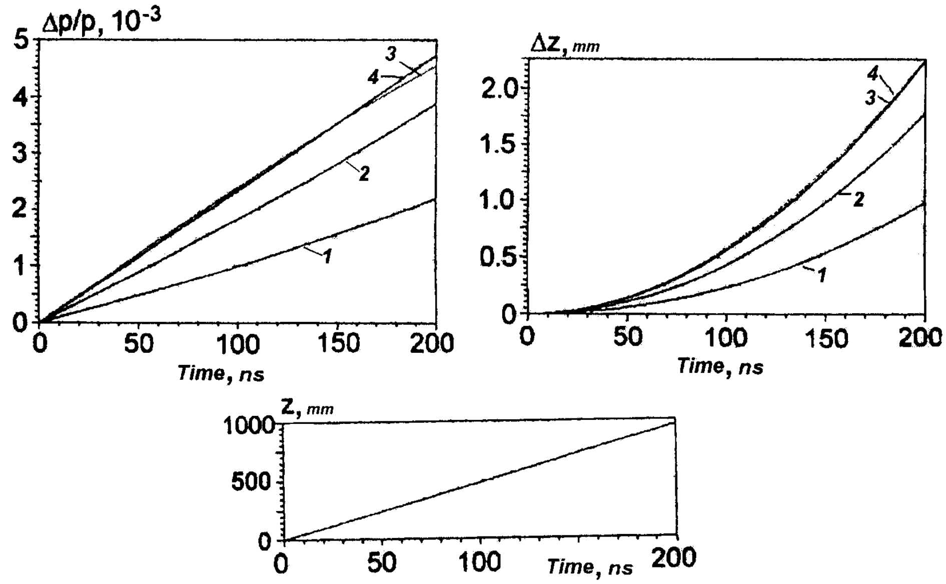

Fig. 1. Estimation of the arising momentum spread $\Delta p/p$ for the U^{4+} beam with 10^9 particle inside one spherical bunch with the parabolic charge distribution and $\beta = 0.016$ due to the space charge force (top left), change of the ion position Δz with respect to the start position z_s at $t = 0$ (*1* — z_s=2 mm, *2* — 4, *3* — 6, *4* — 8), movement of the whole bunch with βc subtracted (top right), distance over which the bunch moves for time t (bottom)

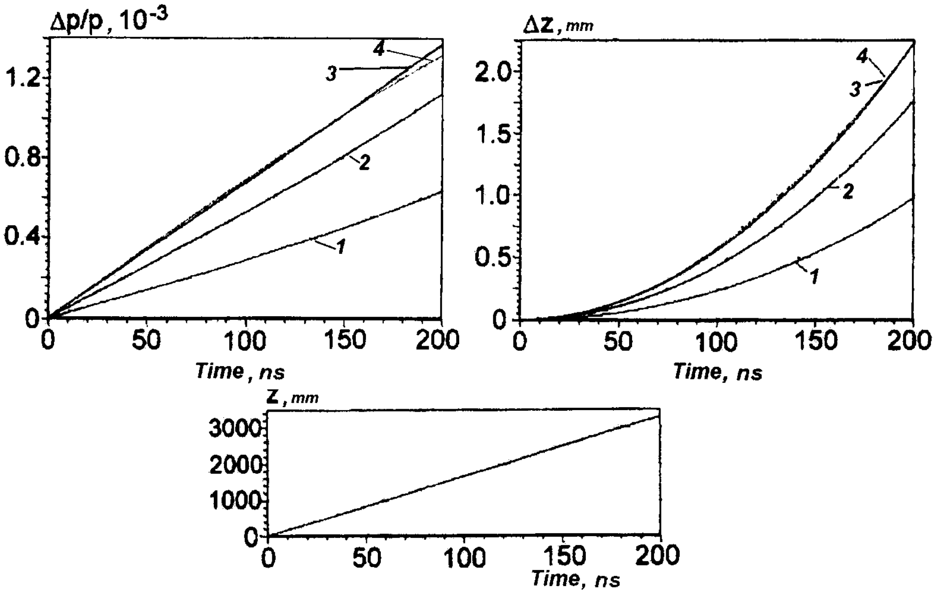

Fig. 2. The same as in Fig. 1, but for $\beta = 0.055$

Here, our figures show the results of calculations for the U^{4+} beam in cases where the space charge influence is greatest. These results theoretically demonstrate the changes in the beam under the influence of its own space charge. We chose several longitudinal coordinates and calculated the particle velocity changes, momentum spread, and partical position. As seen from the figures, the space charge influences mainly the particles located near the bunch border. The maximum momentum spread and change of the ion position are for the particles with coordinates, where the internal electrical field has the maximum value [1—3] (8 mm in our case).

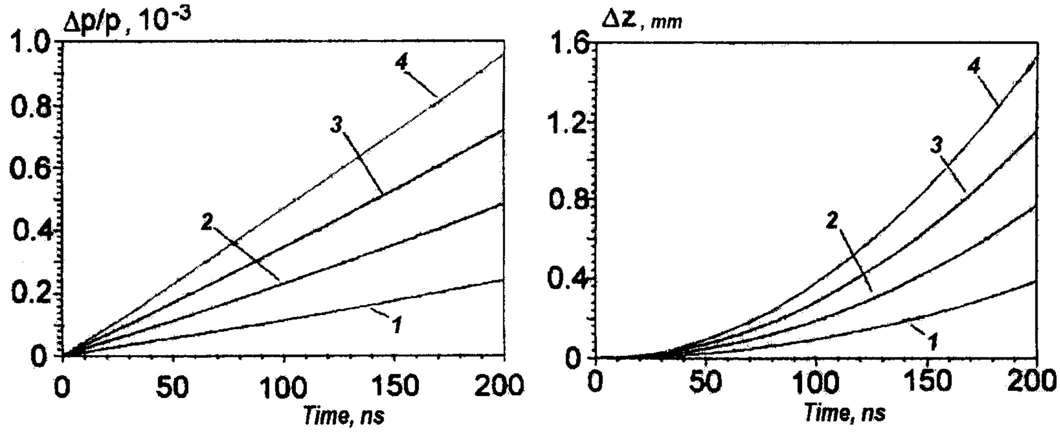

Fig. 3. Estimation of the arising momentum spread $\Delta p/p$ for the U^{4+} beam with 10^9 particle inside one spherical bunch with a homogeneous charge distribution and $\beta = 0.055$ due to the space charge force. The calculated momentum spread (left) and a change of the ion position Δz with respect to the start position z_s at $t = 0$ (right). *1* — $z_s = 2$ mm, *2* — 4, *3* — 6, *4* — 8

In this work, only limiting cases are considered, that is, the worst variants. So there is no need to conduct such a modeling for all particles of the bunch. We make conclusion that, in our case, the momentum spread will increase by $5\cdot10^{-3}$ (+0.5%) and the beam size will grow by 2 mm (on 20%). Such increments are rather essential and, in any case, must be taken into account. Approximately such values were observed on the above-mentioned accelerators (IH-RFQ and UNILAC) [4].

2. Calculation of the Beam Divergence

If we consider ion bunches of spherical shape, the estimation of the arising divergence by the space charge of a bunch can be taken from the estimation of the momentum spread in (1). In the non-relativistic case, we have

$$\frac{\Delta p_{\perp x}}{p} = \frac{\Delta\beta_{\perp x}}{\beta} = \frac{x'}{z'} = \frac{dx/dt}{dz/dt} = x'_z,$$

$$\frac{\Delta p_{\perp y}}{p} = \frac{\Delta\beta_{\perp y}}{\beta} = \frac{y'}{z'} = \frac{dy/dt}{dz/dt} = y'_z, \qquad (2)$$

where $\perp$ holds for the transverse directions x, y. Therefore, the diagrams from Figs. 1—3 show also the estimation of the arising divergence by replacing $[10^{-3}]$ by [mrad]. Similar estimations can be performed for bunches of elliptical shape. In this case, Δp in the longitudinal direction will obviously differ from that in the transverse directions depending on the ellipsoid shape. Referring to our calculations [1] which hold for an ellipsoid with $a > b$, we find $\Delta p_\perp > \Delta p_z$, while $\Delta p_z > \Delta p_\perp$ holds for an ellipsoid with $b > a$. Therefore, in case of very small velocities and rather short bunches, the longitudinal emittance growth will be larger than the transverse one.

Conclusion

Although analytical and numerical methods have to be applied to estimate the effects of space charge on the estimations of the arising momentum spread and beam divergence, the analytical formulas from [1—3] may be needed to describe the bunch parameters at their transportation. The presented results may be useful for the beam diagnostics and the determination of a longitudinal emittance.

Accounting the space charge influence is necessary at the choice of the accelerator parameters such as the voltage on an extending electrode, sizes required for the electrode to seize the whole momentum spread, the collimator parameters, et al. But the algorithms for the calculation of these parameters are a theme of the separate studies and can be presented elsewhere.

Since we have restricted ourselves to small velocities of the moving bunches, no relativistic effects are taken into account in this paper.

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ІМПУЛЬСНИЙ РОЗКИД У ПУЧКАХ ЗАРЯДЖЕНИХ ЧАСТИНОК ВНАСЛІДОК ВПЛИВУ РУХУ ІОНІВ У САМОУЗГОДЖЕНОМУ ЕЛЕКТРИЧНОМУ ПОЛІ

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Резюме

Наведено результати аналітичних та чисельних розрахунків для оцінки впливу ефекту просторового заряду на імпульсний розкид та процес розбухання пучка, які необхідно враховувати при проведенні діагностичних процедур з інтенсивними іонними пучками на ВЧ-прискорювачах.