INTERCEPT PARAMETER λ OF TWO-PION (-KAON) CORRELATION FUNCTIONS IN THE q-BOSON MODEL: CHARACTER OF ITS p_T DEPENDENCE

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The observed non-Bose type behavior of the intercept (strength) λ of the two-particle correlation function $C(p,K)$ of identical pions or kaons detected in heavy-ion collisions, can be effectively described using the approach based on the set of q-deformed oscillators and the q-Bose gas picture. For the intercept λ, connected with the deformation parameter q, the model predicts a fully specified dependence of λ on pair mean momentum $\mathbf{K}$. The intercepts λ_π and λ_K for pions and kaons, differing noticeably at small $\mathbf{K}$, should merge at $\mathbf{K}$ large enough, i.e., in the range $|\mathbf{K}| \geq 800$ MeV/c, where the effect of resonance decays is negligible. By fixing q appropriately, we confront the predicted dependence $\lambda_\pi = \lambda_\pi(\mathbf{K})$ with the recent results from STAR/RHIC for $\pi^-\pi^-$ and $\pi^+\pi^+$ pairs, and find a nice agreement. Using the same q, we also predict the behavior of λ for kaons.

Introduction

Two-particle correlations in the momentum space can be used to extract information about the space-time structure of the emitting sources created in heavy ion collisions. In an essential way, the method exploits the quantum mechanical uncertainty relation between coordinates and momenta, and thus any formal treatment of two-particle correlations must be based on a quantum mechanical description. For the so-called "chaotic" sources where two particles are emitted independently, the description can be based on the *single-particle* Wigner density $S(x,K)$ of a source (source function).

In the standard quantum mechanical treatment, the Bose—Einstein correlations are due to the symmetrization of the two-particle (many-particle) wave function (suppose that particles are emitted independently) $\psi_{\gamma_a\gamma_b}(\mathbf{x}_a,\mathbf{x}_b,t) = \frac{1}{\sqrt{2}}\left[\psi_{\gamma_a}(\mathbf{x}_a,t)\,\psi_{\gamma_b}(\mathbf{x}_b,t) + e^{i\alpha}\psi_{\gamma_a}(\mathbf{x}_b,t)\,\psi_{\gamma_b}(\mathbf{x}_a,t)\right]$ with $\alpha = 0$ ($\alpha = \pi$) for identical bosons (fermions). The indices γ_a, γ_b of the 1-particle wave functions label the complete sets of 1-particle quantum numbers. Below, we consider the two-particle correlations of noninteracting zero-spin identical bosons. The correlation function, with $P_1(\mathbf{k})$ and $P_2(\mathbf{k}_a,\mathbf{k}_b)$ being single- and two-particle probabilities to detect particles with given momenta, is defined as

$$C(\mathbf{k}_a,\mathbf{k}_b) = \frac{P_2(\mathbf{k}_a,\mathbf{k}_b)}{P_1(\mathbf{k}_a)\;P_1(\mathbf{k}_b)}. \tag{1}$$

In the absence of final state interactions (FSI, see [1]), for a chaotic source, the correlation function can be expressed as [2]

$$C(\mathbf{k}_a,\mathbf{k}_b) = 1 + \cos\alpha\,\frac{\left|\int d^4x\, e^{ip\cdot x} S(x,K)\right|^2}{\int d^4x\, S(x,k_a)\,\int d^4y\, S(y,k_b)} \tag{2}$$

with the 4-momenta $K = \frac{1}{2}(k_a + k_b)$ as the pair mean momentum and $p = k_a - k_b$ as the relative momentum. The source function $S(x,K)$ is defined by the single-particle states $\psi_\gamma(x)$ at the freeze-out time and the source density matrix $\rho_{\gamma\gamma'}$ as, e.g., in [2]. Obviously, from (2) at the zero relative momentum $\mathbf{k}_a = \mathbf{k}_b$, one gets $C(\mathbf{k}_a,\mathbf{k}_a) = 1 + \cos\alpha \equiv 1 + \lambda$. Since $\alpha = 0$ for bosons, it follows that $C(\mathbf{k}_a,\mathbf{k}_a) = 2$, i.e., $\lambda = 1$. To fit experimental data, the correlation function of *identical bosons* is usually presented as $C(p,K) = 1 + \lambda f(p,K)$, with $f(p,K)$ commonly taken as Gaussian so that $f(\mathbf{p}=0,K)=1$. From the very first experiments, it was deduced that λ is lesser than one, the typical experimental values being $\lambda = 0.4 \div 0.9$. The second term in (2) is obviously due to quantum-mechanical interference, and a deviation of λ from unity manifests the weakening of interference effects which can occur due to different reasons: the influence of long-lived resonances, coherent emission, etc.

Let us explain the key idea of the model developed in [2, 3] (named the AGI-model in what follows) and further exploited in this letter. In two-boson correlations, a deviation of the intercept λ from unity, besides the contribution due to effects from long-lived resonances, can also be caused by the averaged softening of quantum-statistical effects in the peculiar short-lived many-particle systems formed in relativistic heavy ion collisions. In such a small system, the symmetrization angle α of $\psi_{\gamma_a\gamma_b}(\mathbf{x}_a,\mathbf{x}_b,t)$ can be distorted by an additional phase due to the innhomogeneity of the system at freeze-out times (strong radial and azimuthal flows). These peculiarities can cause the effect analogous to the Aharonov—Bohm one. As a result, a finite value of the averaged symmetrization angle may appear: $\overline{\alpha}>0$ for bosons and $\overline{\alpha}<\pi$ for fermions.

Now, trying to explain experimental data with formula (2), it is natural to relate the parameter λ to the averaged angle $\overline{\alpha}$ to get *the reduction factor* λ by means of $\cos\overline{\alpha}$. That is, a deviation of the intercept λ from unity is viewed to be due to fluctuations of the symmetrization angle α, i.e.,

$$\lambda = \cos\overline{\alpha}\,. \qquad (3)$$

We note that slow bosons (pions, kaons) will experience bigger fluctuations (deviations) of the symmetrization angle α than the particles with high velocities in the fireball frame. That is, a deviation of the intercept λ from unity for slow bosons should be more sizable than for the fast ones.

To implement our key idea, we exploit quantum field theory with q-deformed commutation relations (qDCR) and the techniques of q-boson statistics (see [4] and refs. therein) which reflects a partial suppression of the quantum statistical effects. In [5, 6], it was argued that the algebra of qDCR is connected, *for real q only*, with the so-called nonextensive statistics introduced by Tsallis [7]. This type of a generalized statistics has already found numerous applications in various branches of modern physics (see [8] for refs.). In particular, the nonextensive statistics was applied to the problems of high-energy nuclear collisions ([9] and refs. therein). However, the techniques of q-boson statistics based on qDCR allows the use of complex values, as well as real values, for the deformation parameter q depending on the choice of algebraic realization of qDCR. The physical reasons for the usage of qDCR and the subsequent interpretation of q essentially differ depending on whether q is real or complex. Introducing the deformed statistics with q real enables one to effectively account for interaction effects by means of a non-interacting ideal gas of "modified" particles. On the other hand, the approach based on qDCR provides the ability to model the effects involving the Aharonov—Bohm like phase intimately connected with the symmetrization properties of wave functions.

For the system of pions or kaons produced in heavy ion collisions, we employ the ideal q-Bose gas picture. The physical meaning or explanation of the origin of q-deformation in the considered phenomenon sharply differs in the case of the real deformation parameter q from the case where q is a pure phase factor, as will be seen in what follows.

The AGI-model exploits two different sets of qDCR. The first is a multimode Biedenharn—Macfarlane (BM-type) q-oscillator defined as [10]: $[N_j,b_j] = -b_j$, $[N_j,b_j^\dagger] = b_j^\dagger$, $b_j b_j^\dagger - q^{-1} b_j^\dagger b_j = q^{N_j}$, where different modes ($i \neq j$) commute. Then, $b_i^\dagger b_i = [N_i]_q$ (here, the "q-bracket" means $[r]_q = (q^r - q^{-r})/(q-q^{-1})$) so that $b_i^\dagger b_i = N_i$ is recovered in the "classical" ("no deformation") limit $q \to 1$. Below, for the BM-type of q-oscillators, it is meant that

$$q = \exp(i\theta)\ , \qquad 0 \le \theta < \pi/2\ . \qquad (4)$$

The second multimode q-oscillator used in the AGI-model is the set of Arik—Coon (AC-type) q-oscillators defined by the relations [11] $[\mathcal{N},a] = -a$, $[\mathcal{N},a^\dagger] = a^\dagger$, and $aa^\dagger - qa^\dagger a = 1$ (the subscript is suppressed). Again, at $q \neq 1$, the bilinear $a_i^\dagger a_i$ does not equal to the number operator $\mathcal{N}_i$ (as it is true for ordinary bosonic oscillators, i.e., at $q=1$). Instead, $a_i^\dagger a_i = [[\mathcal{N}_i]]$, where now the notation $[[r]] \equiv (1-q^r)(1-q)$ is used. The q-bracket $[[\hat{A}]]$ for an operator A is understood as a formal series. At $q \to 1$, one recovers $\hat{A}$ from $[[\hat{A}]]$. In what follows, we set

$$-1 \le q \le 1\ . \qquad (5)$$

For each such value of the *deformation parameter* q, the $a_i^\dagger$ and a_i are mutually conjugated. Note that the inverse of the relation $a_i^\dagger a_i = [[\mathcal{N}_i]]$ is given by a formula expressing the operator $\mathcal{N}_i$ as a formal series of creation/annihilation operators.

For a multipion (-kaon) system viewed as the ideal gas of q-bosons, the Hamiltonian is taken as

$$H = \sum_i \omega_i N_i \qquad (6)$$

with i labelling the energy eigenvalues, $\omega_i = \sqrt{m^2 + \mathbf{k}_i^2}$ and N_i defined as above. This is the unique truly noninteracting Hamiltonian with additive spectrum [4]. We assume the discrete 3-momenta of particles (the system is in a box of volume $\sim L^3$). For the set of AC-type q-oscillators, one takes $\mathcal{N}_i$ instead of N_i in (6).

Statistical properties are obtained by evaluating the thermal averages $\langle A \rangle = \mathrm{Sp}(A\rho)/\mathrm{Sp}(\rho)$, $\rho = \mathrm{e}^{-\beta H}$, with Hamiltonian (6) and $\beta = 1/T$.

With $b_i^\dagger b_i = [N_i]_q$ and $q + q^{-1} = [2]_q = 2\cos\theta$, the q-deformed distribution function is obtained as [2,4,12]

$$\langle b_i^\dagger b_i \rangle = \frac{1}{\mathrm{e}^{\beta\omega_i} - 1 + \delta_i}\ , \qquad \delta_i = 2\ \frac{1-\cos\theta}{1-\mathrm{e}^{-\beta\omega_i}}\ . \qquad (7)$$

If $\theta \to 0$, it yields the BE distribution. Note that the q-distribution function (7) is real.

q-distribution (7) deviates from the quantum BE one just in the "right direction" towards the classical Boltzmann distribution, that *reflects a decreasing of quantum statistical effects.* For kaons, whose mass m_K is bigger than m_π, an analogous curve should lie closer, than pion's one, to that of the BE distribution [3].

In the case of AC-type q-bosons with real q from (4), one arrives at the distribution function (cf. [2,4,12])

$$\langle a_i^\dagger a_i \rangle = \frac{1}{\mathrm{e}^{\beta\omega_i} - q}\ . \qquad (8)$$

In the no-deformation limit $q \to 1$, this also reduces to the BE distribution, since, at $q = 1$, we return to the standard system of bosonic commutation relations.

The deviation from standard BE statistics is a natural thing if one considers the system of interacting particles versus that of non-interacting particles (ideal gas). For instance, the natural type of interaction is the hard-core repulsion of particles, which assumes the finite self-volume of a particle. This type of interaction, as was shown in [13], results in the same kind (7) of the modified statistics. At the microscopical level, a finite self-volume arising due to a composite structure of particles results in q-deformed commutation relations [14] and subsequently results in certain q-deformed statistics of the gas of such particles.

The two-particle distribution corresponding to the BM-type q-oscillators is

$$\langle b_i^\dagger b_i^\dagger b_i b_i \rangle = \frac{2\cos\theta}{\mathrm{e}^{2\beta\omega_i} - 2\cos(2\theta)\mathrm{e}^{\beta\omega_i} + 1}\ . \qquad (9)$$

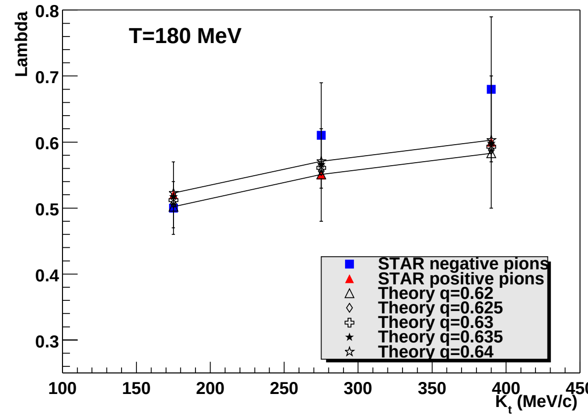

Fig. 1. Intercept λ of two-pion correlation vs the transverse momentum $|\mathbf{K}_t|$. The deformation parameter q is a real quantity, $0 \le q \le 1$

From this and Eq. (6), one obtains the intercept $\tilde\lambda_i \equiv \lambda_i + 1 = \langle b_i^\dagger b_i^\dagger b_i b_i \rangle / (\langle b_i^\dagger b_i \rangle)^2$ of two-particle correlations (omitting the subscript) as

$$\lambda = -1 + \frac{2\cos\theta\ (\cosh(\beta\omega) - \cos\theta)^2}{(\cosh(\beta\omega) - 2\cos^2\theta + 1)(\cosh(\beta\omega) - 1)}. \qquad (10)$$

As $\beta\omega \to \infty$ (i.e., at low temperatures and fixed momenta or large momenta and a fixed temperature), the asymptotics of the intercept is given merely by the deformation angle θ (recall that $q = \exp(\mathrm{i}\theta)$):

$$\lambda = \lambda^{\mathrm{asymp}} = 2\cos\theta - 1 \qquad (T \to 0\ \text{ or }\ |\mathbf{K}| \to \infty). \qquad (11)$$

From this and Eq. (3), we have the (asymptotical) relation $\cos\theta = \cos^2\frac{\alpha}{2}$. Note that, if the unique cause forcing the intercept to be lesser than one is the decays of resonances (the conventional viewpoint), all the curves would tend to the value $\lambda = 1$ in the large $|\mathbf{K}|$ limit. In contrast, we predict a constant $\lambda < 1$, as in (11).

In the case of AC-type q-oscillators, the formula $\langle a_i^\dagger a_i^\dagger a_i a_i \rangle = (1+q)(\mathrm{e}^{\beta\omega_i} - q)^{-1}(\mathrm{e}^{\beta\omega_i} - q^2)^{-1}$ for the two-particle distribution combined with (7) leads to

$$\lambda = -1 + \langle a^\dagger a^\dagger a a \rangle / \langle a^\dagger a \rangle^2 = q - \frac{q\ (1-q^2)}{\mathrm{e}^{\beta\omega} - q^2}\ . \qquad (12)$$

In this case, as $T \to 0$ or $|\mathbf{K}| \to \infty$, we have $\lambda^{\mathrm{asymp}} = q$.

Below, two versions (10) and (12) corresponding to the BM- and AC-types of q-deformation are compared to the recent STAR/RHIC data. The experimental values for the intercept parameter λ in Figs. 1 and 2 are taken from [15]. The theoretical values are obtained by averaging over the given rapidity y and transverse

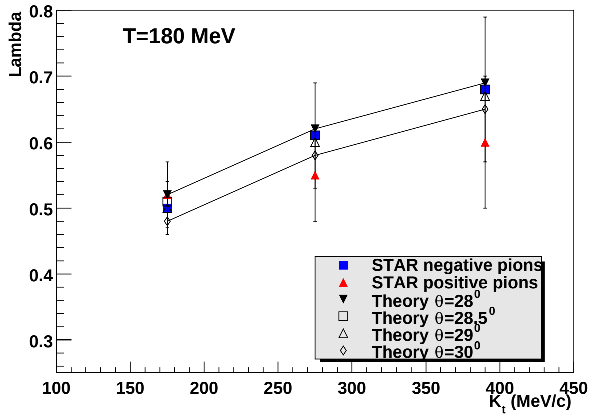

Fig. 2. The same dependence as in Fig. 1. The deformation parameter q is taken in the form $q = e^{i\theta}$

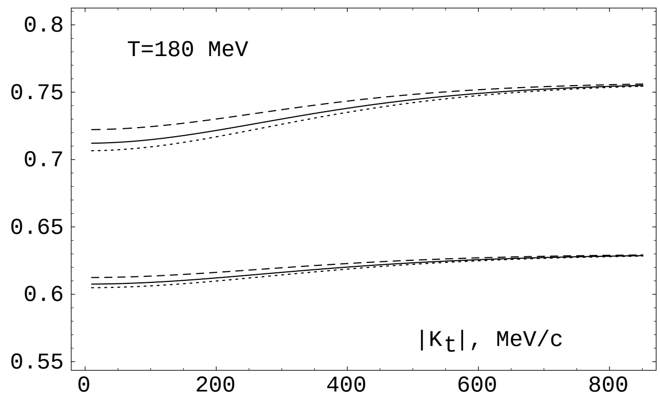

Fig. 3. Intercept λ of the two-kaon correlation vs transverse momentum $|\mathbf{K}_t|$. Both the case of real $q = 0.63$ (lower triple of curves) and the case of $q = e^{i\theta}$ (upper triple of curves) with $\theta = 28.5°$ are shown

momentum K_t intervals $\Delta_j \equiv K_t^{j,\max} - K_t^{j,\min}$, $j = 1, 2, 3$:

$$\lambda_j = \frac{1}{\Delta_y} \int\limits_{-\Delta_y/2}^{\Delta_y/2} \mathrm{d}y \, \frac{1}{\Delta_j} \int\limits_{K_t^{j,\min}}^{K_t^{j,\max}} \mathrm{d}K_t \; \lambda(q, m, T, y, K_t)\,, \quad (13)$$

where m is the particle mass.

Expressions (12) and (10) for $\lambda(q, m, T, y, K_t)$ were used in (13) for obtaining the theoretical points shown in Figs. 1 and 2, respectively. One can see from these figures that the agreement of experimentally measured values of the intercept parameter λ with the theoretically calculated ones is very good.

The detailed comparison with the experiment [15] gives: the values λ_i obtained from (13) at real q, see (12), fit better three experimental values for the intercept of $\pi^+\pi^+$ correlations. On the other hand, the values calculated by (13) with q as a pure phase factor, see (10), agree better with three experimental values for the intercept of $\pi^-\pi^-$ correlations. A possible explanation of the observed difference between experimental values of the intercept for $\pi^-\pi^-$-pairs and $\pi^+\pi^+$-pairs could be the influence of the Coulomb FSI of these charged pions with the positive charge of fireball protons. The AGI-model predicts that the parameter λ will asymptotically reach a constant value $\lambda^{\mathrm{asymp}} < 1$ determined by q only, at sufficiently large $(500-600$ MeV/c) pion pair mean momentum $|\mathbf{K}|$. In order to check this prediction, measurements at higher K_t are necessary. Such measurements should be available in the near future at RHIC.

For the prediction of the intercept of kaons, we use the values of q which provide the best fit of experimental data for pions (see Figs. 1, 2): $q = 0.63$ or $\theta = 28.5°$, assuming a universality of the deformation parameter for the description of excited hot hadronic matter. The result of the averaging over rapidity $-0.5 \leq y \leq 0.5$, given by first integral in (13), is shown in Fig. 3 as a solid curve in each of the triples of curves. The other two curves in each triple correspond to a fixed value of rapidity: $y = 0$ (dotted curve) and $y = 0.5$ (dashed curve). Note that the $y = 0$ curve and the solid curve almost coincide. As is clearly seen, the cases of real q and q as a phase factor supply significantly different values for the kaon intercept λ_K. It is tempting to use just this feature for making preference of a particular version of the deformation parameter q — real or pure phase. The choice is important because different physics is behind these two versions: real q may reflect, for instance, particle finite size effects [13] or particle composite structure [14], and complex-valued q may refer to deformed symmetrization properties of wave functions (like in the Aharonov—Bohm effect) relevant for short-lived systems occurred in heavy-ion collisions. It is also possible that a phase-type q encodes [16] the effects from mixing at the composite (quark) level. Recent data from NA44 [17], i.e. $\lambda = 0.84 \pm 0.13$ and $\lambda = 0.61 \pm 0.36$ resp. for $\langle K_t \rangle \approx 0.25$ GeV/c and $\langle K_t \rangle \approx 0.91$ GeV/c do not yet help in making choice of optimal version for q.

In summary, we have presented a comparison of the AGI model with experimental data on two-particle correlations at RHIC and found a remarkable agreement. We used the parameters extracted from the comparison with pion's data to predict the behavior of the intercept of kaon correlation functions. We stress again the crucial importance of correlation measurements at high transverse momenta in order *to check the predicted asymptotical "saturation" of intercept parameters.* Measurements of $|\mathbf{K}|$ in the range up to 500−600 MeV/c for pions (up to 700−800 MeV/c for kaons) should be possible by RHIC detectors such as STAR and PHENIX. The asymptotical behavior of the intercept parameter λ within the proposed model, see (11) for phase-type q, should determine the actual value of the deformation parameter q supposed to be a universal quantity for relativistic heavy ion collisions.

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ПАРАМЕТР ІНТЕРСЕПТА λ ДВОПІОННИХ (КАОННИХ) КОРЕЛЯЦІЙНИХ ФУНКЦІЙ У q-БОЗОННІЙ МОДЕЛІ: ХАРАКТЕР p_T-ЗАЛЕЖНОСТІ

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Р е з ю м е

Розглянуто відхилення від бозеподібної поведінки інтерсепта λ двочастинкових кореляційних функцій $C(p,K)$, які отримуються при детектуванні тотожних π- та K-мезонів у зіткненнях релятивістських ядер. Для опису цих особливостей запропоновано підхід, який базується на використанні властивостей системи q-деформованих осциляторів та моделі q-бозевського газу. Оскільки інтерсепт λ пов'язаний із параметром деформації q, то одним із висновків моделі є цілком визначена залежність λ від середнього імпульсу $\mathbf{K}$ пари частинок, які детектуються. Отримано, що інтерсепти λ_π та λ_K для піонів та каонів, які, як відомо з експериментів, значно відрізняються у випадку малих імпульсів $\mathbf{K}$, мусять прямувати до одного і того ж значення при достатньо великих $\mathbf{K}$, а саме в діапазоні $|\mathbf{K}| \geq 800$ MeV/c, де можна знехтувати впливом розпадів резонансів на інтерсепт. Зафіксувавши q відповідним чином, ми порівняли передбачену нами залежність інтерсепта від середнього імпульсу пари, $\lambda_\pi = \lambda_\pi(\mathbf{K})$, з експериментальними результатами, недавно отриманими колаборацією STAR/RHIC для $\pi^-\pi^-$- та $\pi^+\pi^+$-пар, і виявили добре узгодження теоретичного результату з екпериментальним. Використовуючи те саме значення параметра q, ми також передбачили поведінку інтерсепта λ для каонів.