BAND STRUCTURE EFFECTS IN LIGHT SCATTERING BY PLASMONS IN p-TYPE GERMANIUM

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The spectral dependence of the cross section for light scattering by plasmons in germanium has been calculated taking a real valence band structure into account. It is shown that the contribution of virtual transitions from the heavy- to light-hole subbands to the dielectric constant leads to an asymmetric line of light scattered by plasmons and a non-square-root dependence of the plasma frequency on free hole concentration. These effects enable to explain properly experimental spectra of light scattering by plasmons observed for p-type Ge.

Introduction

Light scattering by free carriers in nonpolar semiconductors in the case of strong screening, $qr_s >> 1$ (r_s is the screening length, $\mathbf{q} = \mathbf{k}_i - \mathbf{k}_s$ is the light wave vector variation due to scattering) is caused by collective oscillations of carriers which are referred to as plasmons [1, 2].

For the first time, the light scattering by plasmons was observed in heavily doped n-Ge crystals at 100 K [3]. The line by plasma scattering had a Lorentz shape and its width was equal to the frequency of carrier collisions with lattice defects (phonons and impurities). The plasma frequency of carriers, ω_p, that was determined from the frequency shift of the intensity maximum of the scattered light line relative to the exciting laser line, increased with the free carrier concentration N as $\sqrt{N}$. Similar spectra of light scattered by collective excitations of free carriers and the dependence of the plasma frequency on carrier concentration $\omega_p \propto \sqrt{N}$ were observed earlier for n-type polar semiconductors with carrier concentrations providing ω_p less than the frequency of optical phonon (LO) ω_{ph} [1, 4]. At ω_p close to ω_{ph}, the light scattering was caused by plasmon LO-phonon coupled modes.

Recently, we observed light scattering by plasmons also in the p-type Ge [5]. Unlike the n-type crystals, the line of light scattering by plasmons was strongly asymmetric within the studied range of free hole concentrations ($1 \cdot 10^{17}$ to $7.2 \cdot 10^{17}$ cm^{-3}). Its high frequency side decayed with frequency slower than that at lower frequencies. In addition, the dependence of plasmon frequency on hole concentration was substantially non-square-root.

The theory of light scattering by plasmons in semiconductors was considered in a lot of papers (see, for example, [1, 2]). At the same time, the dielectric constant, which determines the plasma frequency and the shape of the plasmon line, was calculated in the case of a simple band structure. Such a theory describes properly a spectrum of light scattering by plasmons in n-Ge. However, it cannot explain results obtained for light scattering by plasmons in p-Ge.

The dielectric constant of cubic p-type semiconductors was calculated in [6, 7]. Its frequency dependence was shown to differ from that of n-type. This is caused by the existence of two subbands in the valence band (light and heavy holes), and transitions between them. In consequence, the plasma frequency may have a complicated dependence on carrier concentration.

The possibility of optical transitions for holes between valence subbands results in light absorption, light scattering, and other phenomena studied for various p-type semiconductors (see, for example, [8, 9]). Intersubband transitions determine also optical properties of the electron-hole liquid in the infrared spectral range for cubic semiconductors at low temperatures, in particular, resonance plasma absorption of light [10].

In this paper, the spectral dependence of the cross section for light scattering by plasmons in germanium has been calculated taking into account the contribution to the dielectric constant of various transitions (intra- and intersubband) in the valence band. It is shown that a substantially asymmetric shape of the scattering line by plasmons and the unusual, at first sight, dependence of the plasma frequency on the carrier concentration in p-

Ge are caused basically by transitions between subbands of light and heavy holes.

1. Spectra of Light Scattering by Plasmons in Germanium without Taking into Account Intersubband Transitions in the Valence Band

The spectral dependence of the cross section for light scattering by plasmons is given by [1, 2]:

$$\sigma(\omega) \sim \mathrm{Im}(-1/\varepsilon(\omega)), \qquad (1)$$

where $\omega = \omega_i - \omega_S$ ($\omega_{i,S}$ are the frequencies of incident and scattered light), ε is the dielectric constant of a crystal.

For cubic p-type semiconductors, the dielectric constant may be written as [6, 7]

$$\varepsilon(\omega) = \varepsilon_0 + \Delta\varepsilon_l + \Delta\varepsilon_h + \Delta\varepsilon_{lh}, \qquad (2)$$

where ε_0 is the lattice dielectric constant, $\Delta\varepsilon_l$ and $\Delta\varepsilon_h$ describe the dielectric constant components due to virtual electron transitions within subbands of light and heavy holes, the last term $\Delta\varepsilon_{lh}$ determines the component due to intersubband transitions.

The intrasubband component is described by the Drude formula (see, for example [11]). If the frequency of collisions with lattice defects is assumed to be the same for light and heavy holes (as it is usual made in transport phenomena theory), then

$$\Delta\varepsilon_l + \Delta\varepsilon_h = -4\pi e^2 \frac{N_l/m_l + N_h/m_h}{\omega^2 + i\omega\gamma} =$$

$$= -\frac{4\pi e^2 N}{m_{\mathrm{opt}}(\omega^2 + i\omega\gamma)}, \qquad (3)$$

where $m_{l,h}$ and $N_{l,h}$ are the effective mass and concentration of carriers in a corresponding subband, $N = N_l + N_h$ is the total concentration of holes and γ is the effective frequency of their collisions (collision damping) that depends in general on the light frequency. The optical mass m_{opt} is expressed through effective masses of carriers and effective masses of the density of states in subbands. The expression for it in the cases of isotropic and warped valence subbands is presented in [7].

If one takes into account only the contribution to the dielectric constant of intrasubband transitions, the spectral line shape for light scattering by plasmons is expressed as follows:

$$\sigma(\omega) = \frac{\varepsilon''_{\mathrm{intra}}}{(\varepsilon_0 + \varepsilon'_{\mathrm{intra}})^2 + \varepsilon''^2_{\mathrm{intra}}} \sim \frac{\omega_0^2\omega\gamma}{(\omega^2 - \omega_0^2)^2 + \omega^2\gamma^2}, \qquad (4)$$

where $\varepsilon'_{\mathrm{intra}}$ and $\varepsilon''_{\mathrm{intra}}$ are the real and imaginary parts of (3), $\omega_0^2 = 4\pi e^2 N/m_{\mathrm{opt}}\varepsilon_0$.

The plasma frequency of carriers that is determined by a maximum position of the line of light scattering by plasmons is

$$\omega_p = \omega_{\max} = \sqrt{\frac{4\pi e^2 N}{m_{\mathrm{opt}}\varepsilon_0} - \frac{\gamma^2}{4}}. \qquad (5)$$

As seen from (4), the width of a line by plasmon scattering is determined by the collision frequency of holes γ. The line has a symmetric Lorentz shape when the collision frequency is independent of the light frequency and becomes asymmetric, if the collision frequency considerably depends on the light frequency. It should be noticed that the plasma frequency at small γ is proportional to the square root of hole concentration, as follows from (5).

The line of light scattered by plasmons observed in p-type of germanium is wide (half-width is 60 to 80 cm^{-1} depending on the doping concentration) and strongly asymmetric. In Fig.1, we show the Stokes spectra of light scattered by plasmons in p-Ge with hole concentrations $N = 2.3\cdot10^{17}$ and $N = 7.2\cdot10^{17}$ cm^{-3} at $T = 80$ K [5]. The plasma frequency of carriers ω_p that was determined from the frequency shift of the intensity maximum of the line by plasma scattering relative to the exciting laser line increases with a growing hole concentration slower than $\omega_p \sim \sqrt{N}$. According to (4) and (5), this implies a strong collision damping of plasmons which is enhanced at higher frequencies.

The prevailing collision type in the p-Ge crystals under study is the collisions of holes with ionized impurities, as follows from measurements of hole mobility under dc electric field. The effective frequency of hole collisions, determined from dc mobility $\gamma(0)$, increases with impurity concentration and equals 17 and 34 cm^{-1} for the p-Ge samples presented in Fig.1.

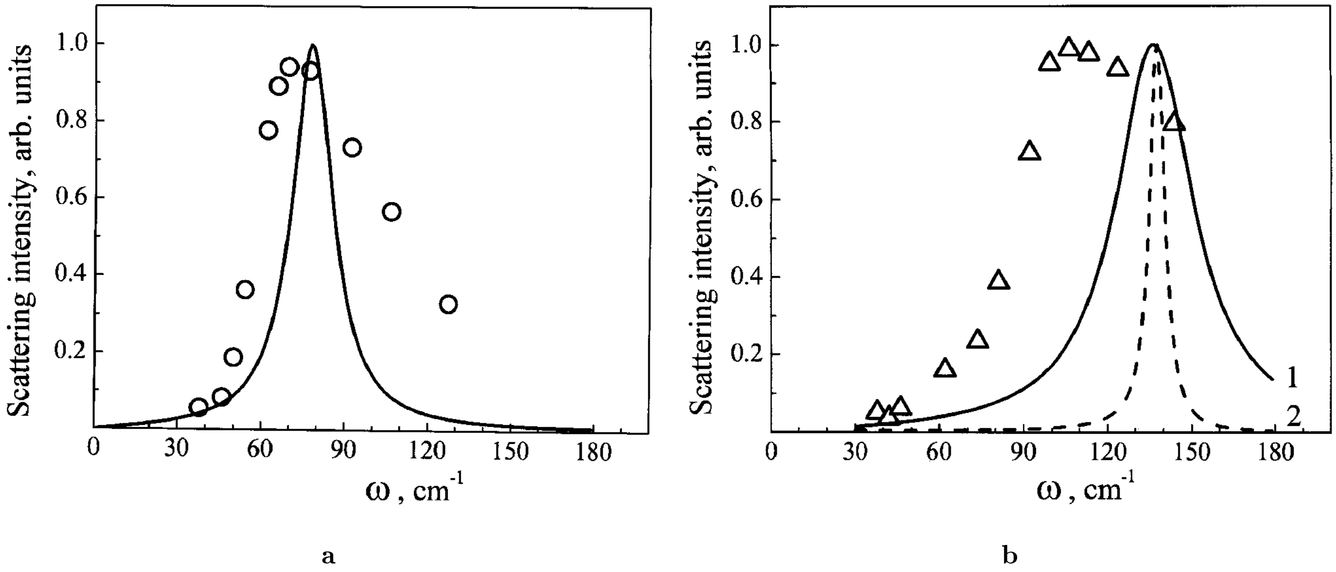
It is known [12] that at the light frequencies satisfying the condition $\hbar\omega >> kT$, the frequency of hole collisions with ionized impurities decreases with growing frequency as

$$\gamma(\omega) = \frac{\gamma(0)(kT/\hbar\omega)^{3/2}}{\ln(Cb)^{-1}}, \qquad (6)$$

where $\ln C = 0.5772$, and b is determined as

$$b \approx \frac{h^2}{4a^2 r_s^2}, \quad a = \frac{\sqrt{2mkT}}{\hbar}. \qquad (7)$$

Note that the line of light scattering by plasmons in p-Ge lies at $T = 80$ K in the spectral range $\hbar\omega >> kT$ for hole concentration $N > 7\cdot10^{17}$ cm^{-3}.



a

b

Fig. 1. The line of light scattering by plasmons in p-Ge at $T = 80$ K for various concentrations of holes N, cm^{-3}: a — $2.3 \cdot 10^{17}$, b — $7.2 \cdot 10^{17}$. Points — experiment, solid line — calculation taking into account intrasubband transitions with $\gamma = \gamma(0)$, dashed line — taking into account the frequency dependence of γ

Fig.1 presents lines of light scattering by plasmons in p-Ge calculated according to (4) with γ equal to $\gamma(0)$ and $\gamma(\omega)$ that takes into account the frequency dependence (6). The following parameters of Ge were used: $\varepsilon_0 = 16$ and $m_{\text{opt}} = 0.23 m_0$ [13].

As seen from Fig.1, the dielectric constant taking into account only intrasubband transitions does not describe properly the line shape of light scattering by plasmons, plasma frequency of carriers, and its dependence on the hole concentration observed in p-Ge. The calculated plasmon lines are considerably narrower and symmetric. At the same time, the calculated plasma frequencies are higher than experimental ones. This discrepancy increases with the concentration of holes N and it is 30% for $N = 7.2 \cdot 10^{17}$ cm^{-3}.

The second term in (5) is at most 2% of the first term at the used values of γ. Therefore, the plasma frequency increases with the carrier concentration as $\sqrt{N}$, while the dependence of the plasmon frequency on the concentration observed in p-Ge is even weaker. It should be noted that taking into account the frequency dependence of the hole collisions frequency does not change the result substantially.

2. Spectra of Light Scattering by Plasmons in Ge in the Case of Taking into Account Electron Transitions between Valence Subbands

Let us now take into account the contribution to the dielectric constant of carrier transitions between subbands of light and heavy holes $\Delta\varepsilon_{lh}$. In this case, the dependence of cross section on frequency for the light scattering by plasmons is expressed according to (1) as

$$\sigma(\omega) \sim \frac{\varepsilon''_{\text{intra}} + \varepsilon''_{\text{inter}}}{(\varepsilon_0 + \varepsilon'_{\text{intra}} + \varepsilon'_{\text{inter}})^2 + (\varepsilon''_{\text{intra}} + \varepsilon''_{\text{inter}})^2}, \tag{8}$$

where $\varepsilon'_{\text{inter}}$ and $\varepsilon''_{\text{inter}}$ are the real and imaginary parts of $\Delta\varepsilon_{lh}$.

Let us take into account the direct intersubband transitions only. The expression for this component of the dielectric constant in approximation of the isotropic parabolic spectrum of carriers in the subbands was obtained in [7, 14]:

$$\Delta\varepsilon_{lh}(\omega) = \left(\frac{2\sqrt{2}e^2}{\pi\hbar\sqrt{\hbar\omega}}\right)\left(\frac{m_l m_h}{m_h - m_l}\right)^{1/2} \times$$

$$\times \int\limits_0^\infty \frac{E^{1/2}\{f(aE) - f(bE)\}}{(E^2 - 1 - i0)} dE, \tag{9}$$

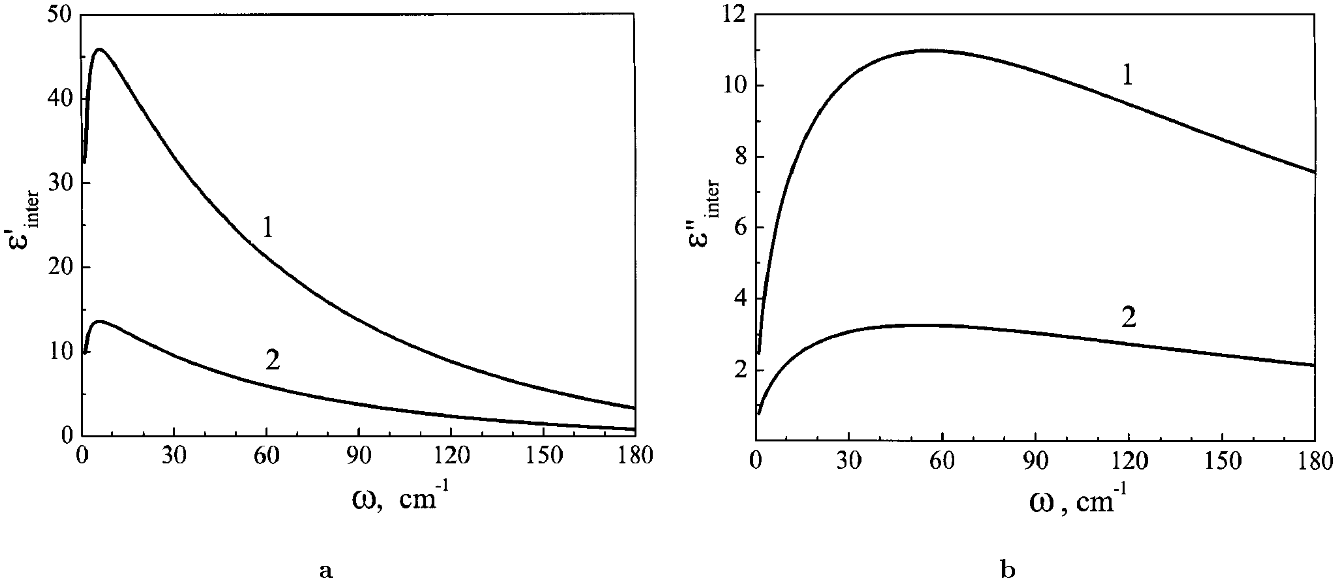


Fig. 2. Intersubband contribution to the real (*a*) and imaginary (*b*) parts of dielectric constant of p-Ge. $T = 80$ K. *1* — $N = 7.2 \cdot 10^{17}$ cm^{-3}; *2* — $N = 2.3 \cdot 10^{17}$ cm^{-3}

where $f(E) = \left[\exp\left(\frac{E-E_{\rm F}}{kT}+1\right)\right]^{-1}$ is the Fermi function, $E_{\rm F}$ is the Fermi energy, $\alpha = \hbar\omega\rho/kT(1-\rho)$, $b = \hbar\omega/kT(1-\rho)$, $\rho = m_h/m_l$.

In the case of complete carrier degeneracy ($T = 0$ K) the integral in (9) may be calculated analytically and corresponding expressions for the real and imaginary parts of dielectric constant $\Delta\varepsilon_{lh}$ are given in [6]. At finite temperatures, only the imaginary part of $\Delta\varepsilon_{lh}$ may be obtained explicitly [7, 8]

$$\varepsilon''_{\rm inter} = \frac{\sqrt{2}e^2}{\hbar\sqrt{\hbar\omega}}\sqrt{\frac{m_l m_h}{m_h - m_l}}[f(a) - f(b)], \qquad (10)$$

while the real part may be obtained only by numerical methods.

Fig. 2 presents the frequency dependences of the real $\varepsilon'_{\rm inter}$ and imaginary $\varepsilon''_{\rm inter}$ parts of the dielectric constant connected with carrier transitions between the subbands of light and heavy holes. They were calculated for p-Ge at $T = 80$ K and hole concentrations $2.3 \cdot 10^{17}$ and $7.2 \cdot 10^{17}$ cm^{-3}. The Fermi energy was determined from the total hole concentration according to the known relationship (see, for example, [15]). At the same time, taking into account the presence of two valence subbands, the effective mass of density of states for holes was taken equal to 0.361 m_0 [13].

As seen from Fig. 2, the intrasubband transitions give a considerable contribution to the dielectric constant in the spectral range containing the line of light scattering by plasmons. This contribution substantially changes the theoretical curve of the cross section spectral dependence for the light scattering by plasmons obtained only in the case of intrasubband transitions. The plasmon line becomes strongly asymmetric and its maximum shifts to a lower frequency region (Fig. 3).

The spectral position of a line maximum is independent of the hole collision frequency γ and its value is in a good agreement with experiment. In addition, the dependence of the plasma frequency on hole concentration deviates from a square-root behavior $\omega_p \sim \sqrt{N}$. Such behavior of plasma frequency is related to the dependence of the contribution to the dielectric constant of the intersubband transitions at the plasma frequency on the carrier concentration.

At the same time, the line shape is sensitive to the hole collision frequency γ. A satisfactory agreement between experiment and theory as for the line shape for the sample with hole concentration $N = 2.3 \cdot 10^{17}$ cm^{-3} is obtained at γ corresponding to that determined from the measured carrier mobility under dc electric field $\gamma(0)$. For the sample with hole concentration $N = 7.2\times 10^{17}$ cm^{-3}, one should take into account the frequency dependence of the hole collision frequency, as seen from Fig.3.

Note that a small discrepancy between the calculated and experimental spectra may be related to the fact

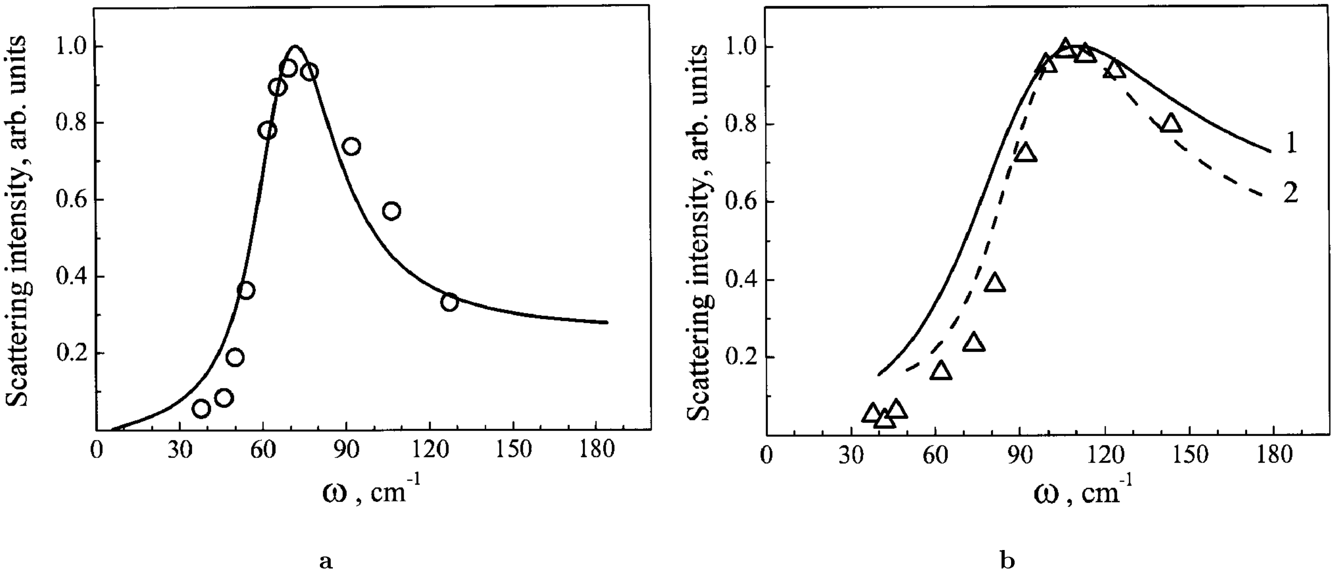


Fig. 3. The line of light scattering by plasmons in p-Ge at $T = 80$ K for various concentrations of holes N, cm^{-3}: a — $2.3 \cdot 10^{17}$, b — $7.2 \cdot 10^{17}$. Points — experiment, solid line — calculation taking into account intrasubband and intersubband transitions with $\gamma = \gamma(0)$, dashed line — taking into account the frequency dependence of γ

that the collision frequency was taken the same both for light and heavy holes. This approximation is valid for hole collisions with phonons [16]. However, in the case of hole collisions with ionized impurities, the frequencies of collisions for light and heavy holes may differ from each other. In addition, the calculations do not take into account the impact of hole collisions on intersubband transitions. Meanwhile, in the case of collisions of holes with ionized impurities, the indirect transitions from the heavy- to light-hole subbands are possible. We observed the absorption of IR radiation in p-Ge crystals connected with such transitions in [17].

Thus, the performed calculations indicate that the presence of upper subbands in the valence band of germanium and carrier transitions between them play an important role in the description of properties of free holes, plasma oscillations, and light scattering by plasmons.

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ПРОЯВ ОСОБЛИВОСТЕЙ СТРУКТУРИ ВАЛЕНТНОЇ ЗОНИ ГЕРМАНІЮ В РОЗСІЯННІ СВІТЛА ПЛАЗМОНАМИ

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Резюме

З урахуванням структури валентної зони проведено розрахунок спектральної залежності перерізу розсіяння світла плазмонами в кристалах германію. Показано, що наявність підзон легких та важких дірок та внесок переходів носіїв між ними у діелектричну проникність кристала приводить до асиметрії лінії плазмонного розсіяння світла та відмінної від кореневої залежності частоти плазмонних коливань дірок від їх концентрації і що врахування цього дозволяє пояснити спектр плазмонного розсіяння світла, який спостерігається у кристалах *p*-Ge.