THEORETICAL DESCRIPTION OF ELASTIC SCATTERING OF 700 MeV DEUTERONS BY ^{40}Ca AND ^{58}Ni

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UDC 539.171
‰ 2003

We propose the method of calculation of the cross section of diffraction deuteron-nucleus scattering at intermediate energies in the quasi-classic approximation. The deuteron wave function was chosen as a Hülten one, the Coulomb interaction and nuclear surface diffuseness of targets were taken into account. The calculated cross sections of elastic scattering of 700 MeV deuterons by ^{40}Ca and ^{58}Ni satisfactorily fit the experimental data.

Introduction

The modern diffraction theory of nuclear reactions is a power instrument of the investigation of various nuclear processes that allows one to research both elastic and nonelastic reactions, excitation and breakup of nuclei, etc. The diffraction phenomena arise when the de Broglie wave length of a projectile become less than the characteristic dimension of the interaction region that corresponds to 10 – 15 MeV per nucleon and more (for medium and heavy targets). The purpose of the given paper is the theoretical description of experiments on the elastic scattering of 700 MeV deuterons by nuclei [1]. At such energies, as is known [2], the movement of a scattered particle in the nuclear field of a target nucleus can be considered as quasi-classic. Earlier, by using the diffraction approach, we performed the preliminary calculations of the differential cross sections of 700 MeV deuteron elastic scattering by ^{40}Ca [3]. However, the deuteron wave function was chosen as Gaussian and the Coulomb interaction was not considered which gave no opportunity to receive enough the good agreement with experiment. In this work, we use Hülten deuteron wave function which has good asymptotic behavior both on small and large proton-neutron distances, and the Coulomb interaction influence also is taken into account.

1. Formalism

Assuming that targets have spherical form, in the non-spin approximation and without taking into account the deuteron D-wave, let's start with the known general formula for the amplitude of deuteron-nucleus elastic scattering [4]

$$F(\vec{q}) = \frac{ik}{2\pi}\int d^{(3)}\vec{r}\int d^{(2)}\vec{\rho}\,\psi_0^2(r)\,e^{i\vec{q}\vec{\rho}}[\omega_1(\rho_1) + \omega_2(\rho_2) - \omega_1(\rho_1)\,\omega_2(\rho_2)],\quad \vec{\rho} = (\vec{\rho}_1 + \vec{\rho}_2)/2, \quad (1)$$

where $\vec{q} = \vec{k} - \vec{k}'$ is the momentum transferred (all our further calculations are carried out for the c.m. system and $\hbar = c = 1$), $\vec{k}(\vec{k}')$ is the incident (scattered) momentum of a deuteron, $\psi_0(r)$ is the intrinsic wave function of relative motion of deuteron clusters. Here $\omega_j(\rho_j)$ are the nucleon-nucleus profile functions ($j = 1$ is the neutron number, $j = 2$ corresponds to a proton) which are expressed, in terms of the impact parameter $\vec{\rho}_j = \{\rho_j, \varphi_j\}$, through scattering phases $\delta(\rho_j)$

$$\omega_j(\rho_j) = 1 - \exp[2i\delta(\rho_j)].$$

From common reasons, it is clear that, as deuterons have relativistic energy, the Coulomb contribution to the differential cross section of the reaction will be insignificant excepting the regions of small scattering angles and diffraction minima. However, as will be seen from the results of our calculations, this contribution may appear essential even for such energies of incident deuterons and leads to an improvement of the experiment fitting. But all over again, let's consider the deuteron-nucleus scattering without taking into account the Coulomb interaction.

As is known [2, 5], the quasi-classic phase $\delta_N(\rho_j)$ of a scattered nucleon can be expressed through the nucleon-nucleus potential $V_N(r)$ as

$$\delta_N(\rho_j) = -\frac{1}{\upsilon}\int_0^\infty ds\,V_N(r),\quad r = \sqrt{\rho_j^2 + s^2}, \quad (2)$$

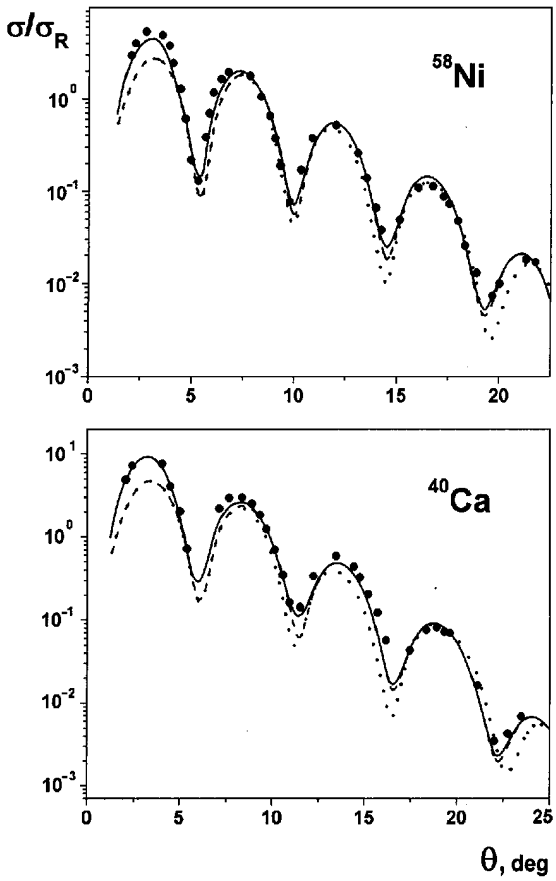

Fig. 1. Relations of differential deuteron-nuclear cross sections σ to the Rutherford ones σ_R. Experimental data were taken from [1]

where υ is the relative speed of the incident nucleon and target nucleus at the infinite distance between them. At relativistic kinetic energies of the nucleon E, the potential $V_N(r)$ can be presented as [4]

$$V_N(r) \equiv V_N(r, E) = -\frac{2\pi}{E+M} f(0)\,\rho_N(r),$$

$$\int d^{(3)}\vec{r}\rho_N(r) = A, \qquad (3)$$

where M is the nucleon mass, $\rho_N(r)$ is the density of nuclear substance of a target which has A nucleons (we supposed that $A >> 1$), $f(0)$ is the amplitude of 0-angle scattering,

$$f(0) = \frac{k_N\,\sigma_{tot}}{4\pi}(i+\gamma),$$

where k_N is the nucleon momentum, $\sigma_{tot} \equiv \sigma_{tot}(E)$ is the total NN cross section, $\gamma \equiv \gamma(E) = \mathrm{Re}\, f(0)\,/\,\mathrm{Im}\, f(0)$ is the real parameter. As a radial distribution $\rho_N(r)$, we used the expression [6, 7]

$$\rho_N(r) = \rho_{N0}\left(1 + \exp\frac{r-R}{\Delta}\right)^{-\xi}, \quad R = r_0 A^{1/3} + \Delta \ln \xi,$$

where ρ_{N0} is the normalization constant, Δ is the diffuseness parameter of the nuclear surface. The potential of a similar kind (3) was found to be useful during the analysis of experimental data on ^{4}He elastic scattering from nuclei both at low [8] and intermediate energies [9]. Thus, the usual geometry of the Woods–Saxon potential was modified in the surface layer of a target nucleus to that with the asymmetry parameter $\xi \neq 1$. The similar potential was used also in [10] for the calculation of some general nuclear characteristics within the framework of a microscopic model. Earlier, within the diffraction model framework [11], we have shown also that the introduction of the additional parameter ξ in the Woods–Saxon potential allows one to improve the agreement with experiments on elastic nucleon-nucleus scattering.

After the integration over the polar angle φ_1 (or φ_2), it is possible to present (1) as

$$F(q) = ik\Big\{\Phi(-q/2)\int_0^\infty d\rho_1\,\rho_1\,\omega_1(\rho_1)\,J_0(q\rho_1) +$$

$$+\,\Phi(q/2)\int_0^\infty d\rho_2\,\rho_2\,\omega_2(\rho_2)\,J_0(q\rho_2) -$$

$$-\int_{-\infty}^{\infty} dz \int_0^\infty d\rho_1\,\rho_1\,\omega_1(\rho_1)\int_0^{2\pi} d\varphi_{12}\int_0^\infty d\rho_2\,\rho_2\,\omega_2(\rho_2)\,\psi_0^2 \times$$

$$\times\left(\sqrt{z^2 + (\vec{\rho}_1 - \vec{\rho}_2)^2}\right) J_0\left(\frac{q}{2}\cdot|\vec{\rho}_1 + \vec{\rho}_2|\right)\Big\}, \qquad (4)$$

where φ_{12} is the angle between $\vec{\rho}_1$ and $\vec{\rho}_2$ and $\Phi(g)$ is the structural formfactor of a deuteron:

$$\Phi(\vec{g}) = \int d^{(3)}\vec{r}\,\psi_0^2(r)\,e^{i\vec{g}\vec{r}}, \quad \Phi(0) = 1$$

If $\psi_0(r)$ is choosed as Gaussian, i.e.,

$$\psi_0(r) \equiv \psi_0^{G}(r) = \left(\frac{2\lambda}{\pi}\right)^{3/4} \exp[-\lambda r^2], \qquad (5)$$

then the quadruple integral in (4) can be reduced to a triple one; the corresponding formfactor is

$$\Phi^{G}(g) = \exp(-g^2/8\lambda).$$

Nevertheless, by choosing $\psi_0(r)$ as a Hülten function,

$$\psi_0(r) \equiv \psi_0^{H}(r) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\beta-\alpha)^2}}\,\frac{e^{-\alpha r}-e^{-\beta r}}{r},$$

$$\alpha = \sqrt{M\varepsilon},\quad \beta \approx 7\alpha,\quad \varepsilon = 2.23\ \text{MeV}, \qquad (6)$$

we carry out the quadruple integration, and the form-factor is calculated in explicit form:

$$\Phi^{H}(g) = \frac{\beta(\beta+\alpha)}{(\beta-\alpha)^2\zeta}\times$$

$$\times\ \text{arctg}\,\frac{(\beta-\alpha)^2(\beta+\alpha)\zeta}{\beta(\beta+\alpha)^2+\alpha(3\alpha^2+2\alpha\beta+3\beta^2)\zeta^2+4\alpha^3\zeta^4},$$

$$\zeta = \frac{g}{2\alpha}.$$

2. Results of Calculation and Discussion

The computed relations σ/σ_R for the elastic scattering of 700 MeV deuterons by ^{40}Ca and ^{58}Ni are shown in the figure. There, $\sigma = |F(q)|^2$ is the differential cross section and $\sigma_R = (2kn)^2/q^4$ is the Rutherford one, where $n = Ze^2/\upsilon$ is the Coulomb parameter, Ze is the charge of a target nucleus, $q = 2k\sin(\theta/2)$ is the modulus of momentum transfer, θ is the scattering angle. The dotted curves are calculated by using the deuteron wave function (5), the dashed ones correspond to (6). The chi square method was used as a criterion of the fitting, thus the optimal values of parameters of potential (3) were found as

$$\gamma = 0.25,\ \xi = 2.27,\ r_0 = 1.11\ \text{fm},\ \Delta = 0.42\ \text{fm}$$

for ^{40}Ca (dotted),

$$\gamma = 0.25,\ \xi = 2.27,\ r_0 = 1.04\ \text{fm},\ \Delta = 0.69\ \text{fm}$$

for ^{40}Ca (dashed),

$$\gamma = 0.24,\ \xi = 2.30,\ r_0 = 1.11\ \text{fm},\ \Delta = 0.40\ \text{fm}$$

for ^{58}Ni (dotted),

$$\gamma = 0.24,\ \xi = 2.30,\ r_0 = 1.07\ \text{fm},\ \Delta = 0.65\ \text{fm}$$

for ^{58}Ni (dashed). (7)

The values of σ_{tot} in (3) were taken from [12 – 14], the structural parameter in (5) is defined as $\lambda = (3/16)\langle r_d^2\rangle^{-1}$, where $\langle r_d^2\rangle^{1/2}$ is the root mean square radius of a deuteron [15, 16].

The Coulomb interaction can be taken into account if we add the Coulomb phase [4]

$$\delta_C(\rho_j) = \frac{Ze}{\upsilon_j}\ln\left(\frac{1}{2}k\rho_j\right),\quad kR >> 1,$$

to $\delta_N(\rho_j)$ of a proton ($j = 2$). It is necessary to note that as is known [17], the resulting phase for two potentials of interaction (the nuclear potential and the Coulomb one) does not equal in general to the sum of phases for each of the potentials separately. But for intermediate energies of projectiles, when it is possible to use the quasi-classic approximation, the specified procedure of simple algebraic addition of phases justifies itself, as the phase in the given approximation depends linearly on the potential of interaction. The proposed approach leads to substantial simplifications of computation as compared with the methods used in [18].

The solid curves on the figure, which correspond to taking the Coulomb interaction into account, are calculated for the same set of parameters (7) by using the Hülten wave function of a deuteron. The features of these curves consist in increasing the cross section, especially for regions of the first diffraction maximum $\theta \le 7°$, and in filling the diffraction minima. Thus, the satisfactory description of the experimental differential cross sections of deuteron-nuclei elastic scattering at intermediate energies can be reached if one takes into account

1) correct radial asymptotics of the intrinsic wave function of a deuteron;

2) real part of the nucleon-nucleus potential together with its imagine one (with the assumption that $\gamma \ne 0$);

3) diffuseness of the target nuclear surface;

4) Coulomb interaction of deuterons with nuclei.

1. *Nguyen van Sen et al.*//Phys. Lett. B.¯ 1985.¯ **156**.¯ P.185.
2. ⁄° .., ¤¸˘¤˝ .. ¢·Ė¢Ö ‡¥Ì·¤– (·¥•¥†Œ¢¤-,Ę–Ö Ė¥µ¤Ö.¯ .: °–, 1989.
3. µ¢†Q°– .Ü, •É–µ¢,–¤° .., °•, ¥¢ ..// Ø. ¤¨¢. °·-É. ¥•. ¸Ø.-‡É ·°–¤.¯ 2000.¯ ”4.¯ .445.
4. ¤Ė¥·–µ .. ¥µ¤Ö Ø¥•·Ò •¥–˝¤°.¯ .: ·¥•£µĖ‡¤§-⁄Ę 1983.
5. •˘¥·–µ¢ .., µ¥¥¢ .. §¤‡µ¥°,Ė¤Ö ¢Q‡µÔ¥•£¥-Ė¸¥,–¤Ì ¸,Ė˝ ¤ Ė‡·Ò Ø¥• , Ø‡¤.¯ .: Ė‡¤§⁄Ę 1972.
6. µ†µ‡¤¥˝ ..// µ††¥–Ė¢·Ö Ø¥••Ö ⁄¤·‡¤–: ¡. ·°¸- ·Ò É°⁄µ¢.¯ .: °–, 1990.¯ .67.
7. ¤Ė¥·–µ .., •É–µ¢,–¤° .. ¥µØÖ Ø: ¢¸†Ó¤° ¶µØ¤–.¯ ¤¨¢: ¤¡ØÓ 2000.
8. *Michel F., Vanderpoorten R.*//Phys. Rev. C.¯ 1977.¯ **16**.¯ P.142.
9. *Goldberg D.A.*//Phys. Lett. B.¯ 1975.¯ **55**.¯ P.59.
10. ••¥É ., ƒ¥–,µ . §‡¥•Ò ¤ ,É°–É• Ø¥•.¯ ¤¥¢: °–. ⁄‡–, 1981.
11. •É–µ¢,Ó¤° .., °•, Û¢ .., µ¢†Q°– .Ü/ .¯ 2001.¯ 46, ”1.¯ .28.

12. *Saloner D., Teopffer C.*//Nucl. Phys. A.¯ 1977.¯ **283.**¯ P.108.
13. *Hess W.N.*//Rev. Mod. Phys.¯ 1957.¯ **30.**¯ P.368.
14. *Karol P.J.*//Phys. Rev. C.¯ 1975.¯ **11**.¯ P.1213.
15. *Varga K., Suzuki Y.*//Ibid.¯ 1995.¯ **52**.¯ P.2885.
16. µ¡ É¢ . . ¤ ⁄•.// .¯ 2000.¯ **63.**¯ .501.
17. † É *ƒ*., °‚–µ¶¸ . . ¥µ¥É¸¥‚– Ö Ø¥• Ö ¸¤§¤– .¯ .: §⁄-¢µ ¤·µÉ. †¤É, 1954.
18. *¢† ·µ¢* . ., µ–µ†µ¢ . ., •É–µ¢‚–¤° . .// .¯ 1996.¯ **59**, ¢Ô.4.¯ .679.

Received 14.11.02