PECULIARITIES OF THE ROTATIONAL BANDS OF SUPERDEFORMED NUCLEI

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UDC 539

The Davydov—Chaban model is generalized to the case of superdeformed nuclei. The deformational nuclear motion with the asymmetric potential having two minima is considered quasiclassically. Theory of the decay out of superdeformed rotational levels is built taking into account simultaneously both the residual and electromagnetic interactions. We generalized the two-level model of Stafford and Barrett to the case where a superdeformed level is coupled with the infinite equidistant spectrum of normal states.

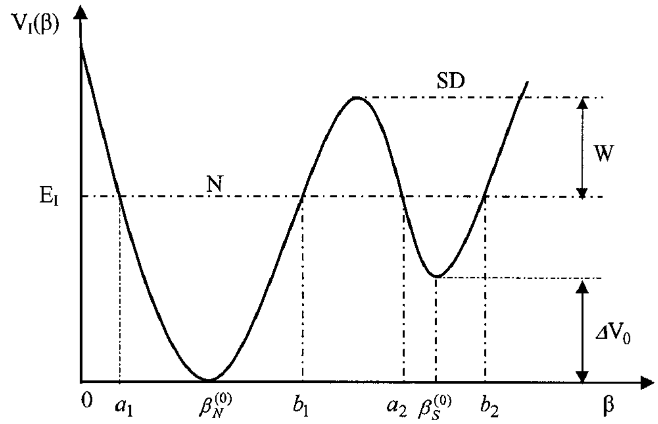
Introduction

Davydov's models [1—4] played an important role in understanding the low-lying collective excitations of deformed nuclei. These models are based on the equation derived by Bohr and Mottelson (see [3, 4]), who treated a nucleus as a liquid drop having a shape slightly deviating from a sphere. In other words, the Bohr—Mottelson equation was derived, assuming that the quadrupole deformation parameter $\beta \ll 1$. This is a good approximation for nuclei with normal (N) deformation, whose equilibrium shape is characterized by $\beta_0 \sim 0.2 \div 0.3$. The calculations by Strutinsky [5] revealed that the nuclear potential energy, as a function of the deformation parameter β, may have the second minimum (see Figure) corresponding to a superdeformed (SD) shape with $\beta \sim 1$. Since then, hundreds of superdeformed rotational bands have been observed (see, e.g., [6—26]). Their energies are calculated usually with the aid of the cranking model [27]. However, the cranking model deals only with static nuclear deformations which may, in principle, change from level to level. It is the Bohr—Mottelson equation that takes into account the relation of nuclear rotation to shape vibrations. Therefore, its generalization to the case of arbitrary deformations seemed to be actual. This task was solved in [28, 29] for axially symmetric nuclei (γ=0), treating the nucleus as an ensemble of nucleons, but not as a liquid drop. We used the exact formula for the kinetic energy operator of the nucleus, expressed in terms of the independent set of collective variables, which was proposed in [30]. A similar derivation of the standard Bohr—Mottelson equation for $\beta \ll 1$ was performed previously in [31]. Starting from the generalized Bohr—Mottelson equation, we shall build a nonadiadatic model like the Davydov—Chaban one, which will be applied further to SD bands. In doing so, we assume the nucleus to be completely located in the SD potential well.

The deformational motion of the nucleus is governed by a one-dimensional Schrödinger equation to determine the deformational wave function $\varphi_I(\beta)$. Usually, the potential energy depending on β is assumed to be a parabola. In the case where the potential has N and SD minima, the motion in both of them may be treated as independent harmonic oscillations only when the barrier separating N and SD potential wells is infinite. In reality, there is always a tunneling through such a barrier, ensuring the mixing of the N and SD wave functions and the repulsion of the corresponding vibrational levels. The wave function $\varphi_I(\beta)$ is then the solution of a one-dimensional Schrödinger equation with asymmetric potential energy having two minima. Previously, this task with symmetric potential has been solved quasiclassically (see, e.g., [31]). Similar results for the asymmetric case will be provided below. The wave function $\varphi_{Is}(\beta)$ is a coherent superposition of the functions $\varphi_s^{(N)}(\beta)$ and $\varphi_s^{(S)}(\beta)$ with amplitudes c_N and c_S, which describe vibrations in N and SD wells. This solution will be used further for the description of the decay out of the SD levels into normal states.

The main feature of the γ spectra corresponding to de-excitation transitions between SD rotational levels is that, at some small spins $I \simeq I_{1/2}$ of nuclei, the intensity of these transitions abruptly falls down and the spectrum quenches [13—26]. These observations were explained by the so-called statistical model [32—35], which assumes that, at spins around $I_{1/2}$, the collective SD levels mix with normal excited configurations decaying into lower-lying normal states. At high spins due to centrifugal barrier, the SD potential well lies lower than the normal one, while it is lifting relative to the N well at low spins. So the SD level lies much higher at low spins than the normal yrast line. Therefore, it turns out to be imbedded into the "sea" of excited configurations $|\alpha\rangle$ of a normally deformed nucleus, so that their mixing becomes essential.

Unfortunately, the original statistical model [32—34] dealt only with nonoverlapping levels. This shortcoming was eliminated by Stafford and Barrett [36], who analyzed the case where the SD level is mixed with a single close-lying configuration and treated both their mixing and radiative decays on the equal footing. We shall generalize such a straightforward approach to the case of an arbitrary number of configurations $|\alpha\rangle$. In the weak-coupling case, we shall give a simple expression for the decay width of the SD level into normal states.



Nuclear potential energy with two minima at low spins versus the deformational parameter β

1. Generalized Nonadiabatical Model

Following [30,31], we shall specify, first, collective nuclear coordinates. As usually, two coordinate frames are introduced with the origins coinciding with the center of mass of the nucleus. One of them, x,y,z, is the laboratory coordinate system and another, ξ,η,ζ, is the moving one with the axes directed along the principal axes of the inertia tensor of the nucleus. Then the projections of the Jacobi vectors of the nucleons $\mathbf{q}_i$ on these axes satisfy the following constraints:

$$\sum_{i=1}^{A-1} q_{i\xi}q_{i\eta} = \sum_{i=1}^{A-1} q_{i\xi}q_{i\zeta} = \sum_{i=1}^{A-1} q_{i\eta}q_{i\zeta} = 0, \tag{1}$$

where A is the number of nucleons in the nucleus.

Rotation of the nucleus is identified with rotation of the coordinate frame ξ,η,ζ , whose orientation with respect to x,y,z is determined by the Euler angles $\theta = \{\theta_1,\theta_2,\theta_3\}$. Equation (1) is formally considered as the orthogonality condition for three vectors $\mathbf{A}_\xi = \{q_{1\xi},q_{2\xi},\dots,q_{A-1,\xi}\}$, $\mathbf{A}_\eta = \{q_{1\eta},q_{2\eta},\dots,q_{A-1,\eta}\}$, and $\mathbf{A}_\zeta = \{q_{1\zeta},q_{2\zeta},\dots,q_{A-1,\zeta}\}$ in an abstract $(A-1)$-dimensional space with basis orts $\mathbf{e}_1,\mathbf{e}_2,\dots,\mathbf{e}_{A-1}$. Such a notion enabled us [30,31] to introduce an independent set of variables. Three of them are defined as the lengths of these vectors:

$$a = \sqrt{\sum_i q_{i\xi}^2}\ , \quad b = \sqrt{\sum_i q_{i\eta}^2}\ , \quad c = \sqrt{\sum_i q_{i\zeta}^2}. \tag{2}$$

Others are generalized Euler angles, which determine the orientation of the vectors $\mathbf{A}_\xi,\mathbf{A}_\eta,\mathbf{A}_\zeta$ in the abstract space.

The kinetic energy operator written in such collective variables reads [30]:

$$\hat{T} = -\frac{\hbar^2}{2m}\left\{\frac{\partial^2}{\partial a^2} + \frac{\partial^2}{\partial b^2} + \frac{\partial^2}{\partial c^2} + \left(\frac{2a}{a^2-b^2} + \frac{2a}{a^2-c^2} + \right.\right.$$
$$\left. + \frac{A-4}{a}\right)\frac{\partial}{\partial a} + \left(\frac{2b}{b^2-a^2} + \frac{2b}{b^2-c^2} + \frac{A-4}{b}\right)\frac{\partial}{\partial b} +$$
$$+ \left(\frac{2c}{c^2-a^2} + \frac{2c}{c^2-b^2} + \frac{A-4}{c}\right)\frac{\partial}{\partial c} -$$
$$-\sum_{k=1}^{A-4}\left(\frac{1}{a^2}\hat{j}^2_{A-3,k} + \frac{1}{b^2}\hat{j}^2_{A-2,k} + \frac{1}{c^2}\hat{j}^2_{A-1,k}\right) -$$
$$-\frac{b^2+c^2}{(b^2-c^2)^2}\left(\hat{I}^2_\xi + \hat{j}^2_{A-2,A-1}\right) -$$
$$-\frac{c^2+a^2}{(c^2-a^2)^2}\left(\hat{I}^2_\eta + \hat{j}^2_{A-1,A-3}\right) -$$
$$-\frac{a^2+b^2}{(a^2-b^2)^2}\left(\hat{I}^2_\zeta + \hat{j}^2_{A-3,A-2}\right) - \frac{4bc}{(b^2-c^2)^2}\hat{I}_\xi\hat{j}_{A-2,A-1} -$$
$$\left.-\frac{4ac}{(a^2-c^2)^2}\hat{I}_\eta\hat{j}_{A-1,A-3} - \frac{4ab}{(a^2-b^2)^2}\hat{I}_\zeta\hat{j}_{A-3,A-2}\right\}, \tag{3}$$

where $\hat{I}_\xi,\hat{I}_\eta,\hat{I}_\zeta$ are the spin projections (in units of $\hbar$) on the axes ξ,η,ζ ; $\hat{j}_{ik}$ are some infinitesimal operators of rotation in the abstract space. Below we shall analyze only the collective motion neglecting both the intrinsic rotation and the Coriolis coupling. The coordinates a,b,c determine the shape of the inertia ellipsoid. For a nucleus with uniform density within the volume, confined by an ellipsoidal surface with half-axes R_ξ,R_η,R_ζ, one has

$$a = R_\xi/\sqrt{5}\ , \quad b = R_\eta/\sqrt{5}\ , \quad c = R_\zeta/\sqrt{5}\ . \tag{4}$$

The volume of such a nucleus equals

$$V=\frac{4\pi}{3}R_\xi R_\eta R_\zeta=\frac{4\pi}{3}5^{3/2}abc=\frac{4\pi}{3}R_0^3, \tag{5}$$

where R_0 is the radius of a sphere with the same volume V. The variables a,b,c should be expressed in terms of the nuclear hyper-radius $\rho=\sqrt{a^2+b^2+c^2}$ and familiar coordinates β and γ to determine the nuclear shape.

Passing to ρ,β,γ, we should keep in mind that weakly excited nuclei conserve their volume, i.e. V =const at any value of the deformation parameter β, which varies from 0 to ∞. At small β, our definition should correlate with the classical notation [3,4]. The only possibility to satisfy both these conditions is

$$a=\frac{\rho}{\sqrt{3}}\exp\left(\beta\cos(\gamma+\frac{2\pi}{3})\right),$$

$$b=\frac{\rho}{\sqrt{3}}\exp\left(\beta\cos(\gamma-\frac{2\pi}{3})\right),$$

$$c=\frac{\rho}{\sqrt{3}}\exp\left(\beta\cos\gamma\right), \tag{6}$$

where $0\le\rho<\infty$, $0\le\gamma\le\pi/6$. We see that, at $\rho=\rho_0$, the product $abc=(\rho_0/\sqrt{3})^3$, so that the volume V =const. As $\beta\to 0$, definition (6) reduces to that given in [31].

For nuclei, which can be treated as a liquid uniform drop, Eq.(6) corresponds to the following expansion in harmonic functions of the radius-vector directed from the center of the nucleus to its surface:

$$R(\vartheta,\phi)=R_0\exp\left[\sum_\mu \alpha_{2\mu}Y^*_{2\mu}(\vartheta,\phi)\right], \tag{7}$$

where $\alpha_{2\mu}$ are the parameters related by the well-known formulas to β,γ (see [3, 4]). For nuclei with small quadrupole deformation ($\beta\ll 1$), Eq.(7) transforms to the standard expression [3, 4]:

$$R(\vartheta,\phi)=R_0\left[1+\sum_\mu \alpha_{2\mu}Y^*_{2\mu}(\vartheta,\phi)\right]. \tag{8}$$

Substituting relation (6) into Eq.(3), one can rewrite the collective part of the kinetic energy operator in the form

$$\hat{T}=-\frac{\hbar^2}{2B(\beta)}\frac{1}{g(\beta)}\frac{\partial}{\partial\beta}g(\beta)\frac{\partial}{\partial\beta}+\frac{\hbar^2\alpha(\beta)}{6B(\beta)\beta^2}\left(\hat{I}_\xi^2+\hat{I}_\eta^2\right)+$$

$$+\frac{\hbar^2}{2B_\gamma(\beta)\beta^2}\left[-\frac{1}{\gamma}\frac{\partial}{\partial\gamma}\left(\gamma\frac{\partial}{\partial\gamma}\right)+\frac{\hat{I}_\zeta^2}{4\gamma^2}\right], \tag{9}$$

where we used the following notations:

$$B_\gamma(\beta)=B(0)e^{-\beta},\quad B(0)=\frac{m\rho_0^2}{2},$$

$$f(\beta)=\frac{1}{3}\left(e^{-\beta}+2e^{2\beta}\right),\quad B(\beta)=B(0)f(\beta),$$

$$\alpha(\beta)=\frac{3}{2}\beta^2\left(e^{-\beta}+2e^{2\beta}\right)\frac{\left(e^{-\beta}+e^{2\beta}\right)}{\left(e^{-\beta}-e^{2\beta}\right)^2}. \tag{10}$$

The mass parameters $B(\beta)$ and $B_\gamma(\beta)$ for β and γ vibrations depend, respectively, on the deformation parameter β. When $\beta\to 0$, both $B(\beta)$ and $B_\gamma(\beta)$ tend to the same constant value $B(0)$ in correspondence with the Bohr—Mottelson model [3, 4]. Besides, $g(\beta)\to\beta^4$ and $\alpha(\beta)\to 1$ if $\beta\to 0$.

The Hamiltonian of the problem reads

$$\hat{H}=\hat{T}+V(\beta,\gamma), \tag{11}$$

where the potential energy may be written as [3]

$$V(\beta,\gamma)\approx V(\beta)+\frac{\beta_0^4}{\beta^2}V(\gamma), \tag{12}$$

where β_0 determines a minimum of the effective potential $V_0(\beta)$, which is given below by Eq.(17).

In order to find a solution of the Schrödinger equation

$$\hat{H}\Psi(\beta,\gamma,\theta)=E\Psi(\beta,\gamma,\theta), \tag{13}$$

we shall neglect the dependence of the quantities $f(\beta)$, $B_\gamma(\beta)$ and $\alpha(\beta)$ on β, putting $\beta=\beta_0$. Then the wave function is factorized as

$$\Psi(\beta,\gamma,\theta)=g^{-1/2}(\beta)\varphi(\beta)|IMK\rangle\chi(\gamma), \tag{14}$$

where the function

$$|IMK\rangle=\sqrt{\frac{2I+1}{16\pi^2(1+\delta_{0K})}}\Big(D^I_{KM}(\theta)+$$

$$+(-1)^I D^I_{-KM}(\theta)\Big) \tag{15}$$

describes the rotation of an axisymmetric rigid rotator with spin I, its projection M on the axis z, and projection K on the symmetry axis ζ of the rotator. Besides, $D^I_{KM}(\theta)$ are the Wigner functions depending on the Euler angles. The function $\varphi(\beta)$ describing β-vibrations satisfies the equation

$$\left\{-\frac{\hbar^2}{2B}\frac{\partial^2}{\partial\beta^2}+V_I(\beta)-E_I\right\}\varphi_I(\beta)=0, \tag{16}$$

where we put $B = B(\beta_0)$. The effective potential energy is

$$V_I(\beta) = V_0(\beta) + \frac{\hbar^2\alpha}{6B\beta^2} I(I+1)\ , \qquad (17)$$

where

$$\alpha = \alpha(\beta_0)\ , \quad V_0(\beta) = V(\beta) + \frac{1}{4}\left[\left(\frac{1}{g}\frac{\partial g}{\partial\beta}\right)^2 - \frac{2}{g}\frac{\partial^2 g}{\partial\beta^2}\right] . \qquad (18)$$

The difference of Eqs.(16), (17) from the corresponding equations of the Davydov—Chaban model [2, 3] is that $\alpha = 1$ in the latter. One can approximate $V_I(\beta)$ by a parabola, following [2, 3]. It seems to be more useful to take

$$V_0(\beta) = C\beta_0^2\left(\frac{\beta_0^2}{2\beta^2} - \frac{\beta_0}{\beta}\right) + C_0, \qquad (19)$$

where the constant C_0 determines the potential well depth.

Introducing the notations

$$\zeta = \frac{\beta}{\beta_{00}}\ , \quad \beta_{00} = \sqrt[4]{\frac{\hbar^2}{BC}}\ , \quad \omega = \sqrt{\frac{C}{B}}\ , \quad \mu = \frac{\beta_{00}}{\beta_0}\ ,$$

$$Z = \mu^{-3}\ , \quad l = \frac{1}{2}\left[\sqrt{1 + \frac{4}{\mu^4} + \frac{4\alpha}{3}I(I+1)} - 1\right], \qquad (20)$$

one can rewrite equation (16) as

$$\left\{\frac{\partial^2}{\partial\zeta^2} - \frac{l(l+1)}{\zeta^2} + \frac{2Z}{\zeta} + 2\varepsilon\right\}\varphi(\zeta) = 0, \qquad (21)$$

where

$$\varepsilon = (E - C_0)/\hbar\omega. \qquad (22)$$

The parameter β_{00} stands for the amplitude of β vibrations in the ground state of a β oscillator, and μ is the softness parameter.

We see that (21) is formally the equation for the radial part of the wave function of a charged particle bound in the Coulomb potential. This enables one to get the analytic formula for energies:

$$E_{In_\beta} = -\frac{\hbar\omega}{2}\frac{Z^2}{n^2} + C_0, \qquad (23)$$

where $n = n_\beta + l + 1$; and $n_\beta = 0, 1, 2, \ldots$ indicates the number of phonons for β vibrations. Note that n and l are not integers.

The function $\chi(\gamma)$ describing γ vibrations obeys the equation

$$\left\{\frac{\hbar^2}{2B_\gamma\beta_0^2}\left[-\frac{1}{\gamma}\frac{\partial}{\partial\gamma}\left(\gamma\frac{\partial}{\partial\gamma}\right) + \frac{\hat{I}_\zeta^2}{4\gamma^2}\right] + \right.$$

$$\left. + \frac{1}{2}\beta_0^2 C_\gamma\gamma^2 - E_\gamma\right\}\chi(\gamma) = 0. \qquad (24)$$

Excited γ vibrational states are specified by two quantum numbers $n_\gamma = 0, 1, 2, 3, \ldots$ and $K = 0, 2, 4, \ldots$ Their energies are [3]

$$E_\gamma \equiv E_{Kn_\gamma} = \hbar\omega_\gamma\left(2n_\gamma + \frac{1}{2}K + 1\right)\ , \quad \omega_\gamma = \sqrt{\frac{C_\gamma}{B_\gamma}} . \qquad (25)$$

Thus, the nuclear levels are given by

$$E_{IKn_\beta n_\gamma} = E_{In_\beta} + E_{Kn_\gamma} . \qquad (26)$$

Dealing with a superdeformed vibrational-rotational band beginning from the state with spin I_0^+ and $n_\beta = n_\gamma = K = 0$, one must calculate the relative excitation energies

$$\Delta E_{IKn_\beta n_\gamma} = E_{IKn_\beta n_\gamma} - E_{I_0 000}\ . \qquad (27)$$

They are given by

$$\Delta E_{IKn_\beta n_\gamma} = \frac{\hbar\omega}{2\mu^6}\left[\frac{1}{(l_0+1)^2} - \frac{1}{(n_\beta + l + 1)^2}\right] +$$

$$+\hbar\omega_\gamma\left[2n_\gamma + \frac{1}{2}K\right] \qquad (28)$$

with l_0 corresponding to I_0 and l to I. Respectively, for the normal bands, we must take $I_0^+ = 0$ for the ground state of even-even nuclei. Expressions for the wave functions $\varphi(\beta)$ and $\chi(\gamma)$ are provided by [3, 4].

Energies (28) depend on two fitting parameters ω and μ. The calculated energies for a number of SD bands are compared with experimental data in Tab.1. Everywhere the energy of the lowest observable SD level is taken to be zero.

2. Tunneling between Asymmetric Potential Wells

Let us solve quasiclassically the Schrödinger equation (16) with the potential $V_I(\beta)$ having two minima (see Fig.1). The attenuating WKB solution in the region $\beta < a_1$ is

$$\varphi_I(\beta) = \frac{c_N}{\sqrt{|p|}} \exp\left(-\frac{1}{\hbar}\int_{\beta}^{a_1} |p| d\beta\right), \tag{29}$$

where

$$p(\beta) = \sqrt{2B(E_I - V_I(\beta))}. \tag{30}$$

T a b l e 1. Superdeformed rotational bands

^{158}Yb			^{156}Er		
	$\Delta E_{\rm exp}$ [9]	$\Delta E_{\rm theor}$		$\Delta E_{\rm exp}$ [10]	$\Delta E_{\rm theor}$
I^+		$\omega = 370$ keV $\mu = 0.195$	I^+		$\omega = 417$ keV $\mu = 0.164$
16	0.0	0.0	22	0.0	0.0
18	769.6	769.7	24	802.8	803.1
20	1578.2	1543.4	26	1649.0	1662.3
22	2443.0	2398.7	28	2528.2	2561.7
24	3371.8	3354.0	30	3430.7	3447.2
26	4366.4	4342.5	32	4323.3	4327.4
28	5426.7	5417.3	34	5257.0	5243.2
30	6554.1	6553.2	36	6237.5	6210.7
32	7748.8	7752.1	38	7268.8	7301.2
34	9012.4	9030.7	40	8353.4	8356.1
36	10346.9	10371.4	42	9492.9	9498.3
38	11755.3	11759.8	44	10687.6	10669.0
40	13240.9	13238.2	46	11937.4	11893.2
42	14807.7	14793.1	48	13243.0	13240.8
44	16459.1	16437.2	50	14605.0	14594.7
46	18199.1	18175.4	52	16022.8	16001.3
48	20032.4	20004.7			
^{196}Pb			^{148}Gd		
	$\Delta E_{\rm exp}$ [11]	$\Delta E_{\rm theor}$		$\Delta E_{\rm exp}$ [12]	$\Delta E_{\rm theor}$
I^+		$\omega = 500$ keV $\mu = 0.101$	I^+		$\omega = 180$ keV $\mu = 0.082$
8	0.0	0.0	32	0.0	0.0
10	171.4	172.1	34	830.3	831.4
12	387.0	387.4	36	875.8	875.1
14	646.5	647.7	38	925.7	921.2
16	949.5	951.4	40	976.8	968.7
18	1295.3	1298.0	42	1028.8	1017.6
20	1682.9	1686.4	44	1080.6	1067.8
22	2111.4	2115.7	46	1133.4	1119.4
24	2580.8	2585.0	48	1186.1	1172.4
26	3089.3	3093.1	50	1239.8	1226.7
28	3636.2	3638.9	52	1293.8	1282.4
30	4220.4	4221.3	54	1347.0	1339.5
32	4841.0	4839.1	56	1395.9	1397.9
34	5495.9	5491.1	58	1436.3	1457.6
36	6184.7	6176.0	60	1445.9	1518.3
38	6904.8	6892.7			
40	7656.9	7640.0			

It is matched with the function

$$\varphi_I(\beta) = \frac{2c_N}{\sqrt{p}} \cos\left(\frac{1}{\hbar}\int_{a_1}^{\beta} p d\beta - \frac{\pi}{4}\right) \tag{31}$$

for $a_1 < \beta < b_1$.

Introducing the notations

$$\phi_1 = \frac{1}{\hbar}\int_{a_1}^{b_1} p d\beta, \qquad \phi_2 = \frac{1}{\hbar}\int_{a_2}^{b_2} p d\beta,$$

$$\Delta V_0 = V_I(\beta_S^{(0)}) - V_I(\beta_N^{(0)}), \tag{32}$$

and approximating the potential between the turning points in the N and SD wells ($a_{1(2)} < \beta < b_{1(2)}$) by parabolas

$$V_I(\beta) \approx B\omega_{N(S)}^2 (\beta - \beta_{N(S)}^{(0)})^2/2, \tag{33}$$

one has

$$\phi_1 = \frac{\pi E_I}{\hbar\omega_N}, \quad \phi_2 = \frac{\pi(E_I - \Delta V_0)}{\hbar\omega_S}. \tag{34}$$

Continuing the matching procedure at the turning points a_i, b_i and imposing the evident condition that the wave function $\varphi_I(\beta)$ attenuate as $\beta \to \infty$, we find the following constraint [37]:

$$4\text{ctg}\phi_1 \text{ctg}\phi_2 = \exp(-2A), \tag{35}$$

where A is determined by the integral

$$A = \frac{1}{\hbar}\int_{b_1}^{a_2} |p(\beta)| d\beta. \tag{36}$$

Approximating the barrier by an inverse parabola with frequency ω_B, we come to the well-known formula

$$A = \frac{\pi W_I}{\hbar\omega_B}, \tag{37}$$

where the barrier height is given by

$$W_I = V_I(\beta_B^{(0)}) - E_I. \tag{38}$$

For the wave function inside the SD potential well, one has the expression

$$\varphi_I(\beta) = \frac{2c_S}{\sqrt{p}} \cos\left(\frac{1}{\hbar}\int_{\beta}^{b_2} p d\beta - \frac{\pi}{4}\right), \tag{39}$$

where the amplitude

$$c_S = c_N(\sin\phi_1/\sin\phi_2)e^{-A}. \quad (40)$$

We assume a small transparency of the barrier, $\exp(-2A) \ll 1$. Then condition (35) is fulfilled if

$$\phi_1 \simeq (n_1 + 1/2)\pi \quad (41)$$

or/and

$$\phi_2 \simeq (n_2 + 1/2)\pi, \quad (42)$$

where $n_i = 0, 1, 2, 3, \ldots$. Exact equalities (41), (42) are familiar Bohr–Sommerfeld conditions for binding the particle with mass B in one of the potential wells in the absence of tunneling through the barrier, when $A = \infty$. In view of (34), exact conditions (41), (42) yield the energy levels of harmonic oscillators in the N and SD wells:

$$\epsilon_1 = \hbar\omega_N(n_1 + 1/2), \quad \epsilon_2 = \Delta V_0 + \hbar\omega_S(n_2 + 1/2). \quad (43)$$

If the angles

$$\alpha_{N(S)} = \frac{\pi\Delta\epsilon}{\hbar\omega_{N(S)}} \simeq 0, \quad (44)$$

where $\Delta\epsilon = \epsilon_1 - \epsilon_2$, one can tell about the resonance between the vibrational levels ϵ_1 and ϵ_2. Then, from (35), we get the quadratic equation for the energies E_{In} of the deformational motion in the potential with two minima:

$$E^2 - (\epsilon_1 + \epsilon_2)E + \epsilon_1\epsilon_2 - v^2 = 0, \quad (45)$$

where

$$n = \{n_1, n_2\}, \quad \omega_0^2 = \omega_N\omega_S, \quad v = (\hbar\omega_0/2\pi)\exp(-A). \quad (46)$$

The solution of Eq.(45) is

$$E_{I,n}^{(\pm)} = (\epsilon_1 + \epsilon_2)/2 \pm (1/2)\sqrt{(\Delta\epsilon)^2 + 4v^2}. \quad (47)$$

This formula coincides formally with that for the energies of two levels coupled by the interaction v (see, e.g. [32]). In the case considered, however, v means the tunneling strength.

The wave function may be written in accordance with (31), (39) as

$$\varphi(\beta) = c_N\varphi^{(N)}(\beta) + c_S\varphi^{(S)}(\beta), \quad (48)$$

where the nonoverlapping functions $\varphi^{(N)}$ and $\varphi^{(S)}$ are localized in the N and SD wells, respectively. In the resonance case ($|\alpha_{N(S)}| \ll 1$), the functions $\varphi^{(N)}$ and $\varphi^{(S)}$ approximate the oscillator functions with phonon numbers n_1 and n_2, respectively, while the ratio of the amplitudes is

$$(c_N/c_S)_\pm = (-1)^{n_1+n_2+1}(\omega_N/\omega_S)^{1/2}(E_I^{(\pm)} - \epsilon_2)/v. \quad (49)$$

Since $v \ll \hbar\omega_0$, such a resonance is a very rare event. Nevertheless, it appears to occur in ^{133}Nd, where repulsion and mixing of two couples of states with spins $I = 17/2^+$ and $I = 19/2^+$, which belong to N and SD rotational bands, have been observed [17, 18]. Previously [18], their energy levels were calculated with the aid of the same Eq. (47), but there v meant a matrix element of the interaction between bands. Bazzacco found that v=22 keV for $I = 17/2^+$ and v= 11 keV for $I = 19/2^+$. It would be more natural to explain such effects by the tunneling under the barrier separating the N and SD wells. Using Eq.(46) and taking $\hbar\omega_0 = \hbar\omega_B$= 0.6 MeV, we found the barrier heights: $W_{17/2}$=0.28 MeV and $W_{19/2}$=0.41 MeV.

Far from the vibrational resonance $|\alpha_{N(S)}| \sim 1$, the wave function is mainly localized in one of the wells, having only a weak tail due to the tunneling to another one. In particular, if $|\alpha_{N(S)}| \sim 1$, the wave function $\varphi_s(\beta)$ consists of two components $\varphi_s^{(N)}(\beta)$ and $\varphi_s^{(S)}(\beta)$ with the amplitudes

$$c_S^s \simeq 1, \quad c_N^s \simeq (-1)^{n_1+n_2}\frac{e^{-A}}{2\sin\alpha_N}. \quad (50)$$

The corresponding energy $E_I \simeq \epsilon_2$. In the off-resonance case, the function $\varphi_s^{(S)}(\beta)$ approximates the oscillator wave function, describing vibrations in the SD well with n_2 phonons, while the function $\varphi_s^{(N)}(\beta)$ is represented by a series in terms of the oscillator functions with phonon numbers $n_1 = 0, 1, 2, \ldots$.

3. Decay out of SD Levels

Here we shall analyze the decay out of an SD level, which is formed at the moment $t = 0$, taking into account its mixing with normal states. The Hamiltonian of the system (nucleus + electromagnetic field) may be written as

$$\hat{H} = \hat{H}_0 + \hat{V}, \quad (51)$$

where the unperturbed Hamiltonian $\hat{H}_0$ is the sum of the Hamiltonian of the electromagnetic field $\hat{H}_{\rm rad}$ and the Hamiltonian of the nucleus $\hat{H}_N$, which includes both collective terms and the terms describing the independent motion of nucleons. The perturbation operator is

$$\hat{V} = \hat{V}_r + \hat{V}', \quad (52)$$

where $\hat{V}_r$ is the interaction operator of the nucleus with the electromagnetic field. When the nucleus is in the initial SD state, the system is described by the wave function

$$|s\rangle \equiv \Psi_s(0) = \varphi_{Is}(\beta)|IMO\rangle\Phi_0|0\rangle, \tag{53}$$

where Φ_0 describes the ground state of the nucleons and $|0\rangle$ is the vacuum state of the field. Substituting decomposition (48) into (53), one has

$$|s>= c_N^s|N\rangle + c_S^s|S\rangle, \tag{54}$$

where the function $|N\rangle$ and $|S\rangle$ are determined by Eq.(53) with $\varphi_{Is}(\beta)$ replaced by $\varphi_s^{(N)}(\beta)$ and $\varphi_s^{(S)}(\beta)$, respectively. The eigenvalue of $\hat{H}_N$ corresponding to $|s\rangle$ will be E_s. The SD level with low spin is surrounded by the dense spectrum of excited configurations Φ_α of the normally deformed nucleus. The corresponding states and energies of the system will be denoted by $|\alpha\rangle$ and E_α. The wave function of the system at any subsequent moment $t \geq 0$ is

$$\Psi_s(t) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\varepsilon e^{-i\varepsilon t/\hbar}\hat{G}^+(\varepsilon)\Psi_s(0), \tag{55}$$

where Green's operator

$$\hat{G}^+(\varepsilon) = (\varepsilon + i\eta - \hat{H})^{-1}, \qquad \eta \to +0. \tag{56}$$

So Green's matrix completely determines the evolution of the system. Specifically, the probability of finding the nucleus after its decay in any SD state is given by [36, 37]

$$F_S = \frac{\Gamma_s}{2\pi}\int_{-\infty}^{\infty} d\varepsilon |G_{ss}^+(\varepsilon)|^2. \tag{57}$$

The probability of the decay into N states $F_N = 1 - F_S$.

The Green's matrix is determined by a system of algebraic equations for overlapping resonant levels $|s\rangle$ and $|\alpha\rangle$. They are easily solved [37], when all the normal states $|\alpha\rangle$ have the same radiative width Γ_N and their strength of coupling to the SD level, $v' = \langle\alpha|\hat{V}|s\rangle$, does not depend on α. We assume moreover that $|\alpha\rangle$ form an equidistant spectrum $E_\alpha = E_0 + \alpha D_N$, where $\alpha = 0, \pm1, \pm2, \ldots$ and E_0 denotes the energy nearest to E_s. Then we find the following Green's function:

$$G_{ss}^+(\varepsilon) = \frac{1}{\varepsilon - E_s + i\Gamma_s/2 + i(\Gamma/2)\mathrm{ctg}z}, \tag{58}$$

where

$$z = \pi(\varepsilon - E_0 + i\Gamma_N/2)/D_N, \tag{59}$$

and

$$\Gamma = 2\pi v'^2/D_N \tag{60}$$

is the spreading width. The coupling strength becomes

$$v' = c_N^s\langle\alpha|\hat{V}'|N\rangle, \tag{61}$$

since the normal functions $|\alpha\rangle$ overlap only with $|N\rangle$.

Let us assume that

$$\Gamma_N \gg \Gamma_s, \quad v' \ll \sqrt{\Delta^2 + (\Gamma_N/2)^2}, \tag{62}$$

where Γ_s is the radiative width of the SD level, $\Delta = E_0 - E_s$. These inequalities hold for nuclei having mass numbers $\sim$ 190 [36]. In such a weak-coupling limit, Green's function (58) reduces to a single resonant term:

$$G_{ss}^+(\varepsilon) = \frac{1}{\varepsilon - E_s + i(\Gamma_s + \Gamma^\downarrow)/2}, \tag{63}$$

where the width $\Gamma^\downarrow$ determines the decay rate of the SD level into normal states. For it, we derived the following expression:

$$\Gamma^\downarrow = \frac{v'^2\Gamma_N}{(\Gamma_N/2)^2 + (D_N/\pi)^2\sin^2(\pi\Delta/D_N)}, \tag{64}$$

Table 2. Parameters characterizing the decay of nuclei with mass ~ 190

	I $\hbar$	F_S	Γ_S (meV)	Γ_N (meV)	D_N (eV)	$\overline{v}'$ (eV)	$v'_{\min}$ (meV)	$v'_{\max}$ (eV)	$\overline{W}$ (MeV)	$W_{\min}$ (MeV)	$W_{\max}$ (MeV)
^{192}Hg-1 [36]	12^+	0.87	0.116	10.3	34	0.16	0.21	0.45	2.86	2.65	4.12
^{192}Hg-1 [36]	10^+	0.09	0.054	10.3	30	1.51	1.18	2.19	2.43	2.36	3.79
^{194}Hg-1 [26]	12^+	0.60	0.108	21	344	3.41	0.62	6.41	2.27	2.15	3.92
^{194}Hg-1 [26]	10^+	0.03	0.046	20	493	30.04	2.73	42.83	1.85	1.79	3.64
^{194}Hg-3 [35]	15^+	0.90	0.230	4.0	26.5	0.20	0.16	0.68	2.81	2.57	4.17
^{194}Hg-3 [35]	13^+	0.84	0.110	4.5	19.9	0.32	0.15	0.43	2.74	2.66	4.18
^{194}Hg-3 [35]	11^+	< 0.07	0.048	6.4	7.2	0.41	1.02	0.72	2.78	2.57	3.83
^{194}Pb-1 [35]	8^+	0.62	0.014	0.50	2200	64.12	0.01	95.79	1.73	1.62	4.46
^{194}Pb-1 [35]	6^+	< 0.09	0.003	0.65	1400	66.45	0.04	106.21	1.74	1.63	4.33

which transforms to the result in [36], when $|\Delta| \ll D_N$ and only one nearest configuration plays a significant role. Substituting (63) into (57), one has the branching ratios

$$F_S = \frac{\Gamma_s}{\Gamma_s + \Gamma^{\downarrow}}, \quad F_N = \frac{\Gamma^{\downarrow}}{\Gamma_s + \Gamma^{\downarrow}}. \tag{65}$$

We obtain the same simple expressions (63) and (65) with $\Gamma^{\downarrow}$ replaced by a standard spreading width Γ in the case of greatly overlapping normal levels with $\Gamma_N \gg D_N$. In both these cases, the decay of the SD state becomes exponential, while, generally, there are Rabi oscillations of the probabilities of finding the excited nucleus in the SD or $|\alpha\rangle$ state.

The ratio F_S, which determines the relative intensity of in-band electromagnetic transitions, is measured experimentally. Using experimental data on F_S, one can find v'. Since Δ lies between $-D/2$ and $D/2$, we get the interval $(v'_{\min}, v'_{\max})$ of possible values for v'. Then, substituting (37), (50) into product (61), we can estimate barrier heights W_I. For this aim, we accept $\alpha_N = \pi/2$ and $\hbar\omega_0 = \hbar\omega_B = 0.6$ MeV. Besides, following [38], we put $|\langle\alpha|\hat{V}'|N\rangle| \sim 1$ MeV. The values of parameters F_S, Γ_S, D_N are taken from [26, 35, 36]. Averaging F_N over Δ, we obtained also the most probable magnitudes of v' and the barrier heights designated by $\bar{v}'$ and $\bar{W}_I$, respectively. All these estimations are presented in Tab.2.

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ОСОБЛИВОСТІ ОБЕРТАЛЬНИХ РІВНІВ СУПЕРДЕФОРМОВАНИХ ЯДЕР

А.Я. Дзюблик, В.В. Утюж

Р е з ю м е

Модель Давидова — Чабана узагальнено на випадок супердеформованих ядер. Ядерний рух супердеформованих ядер із асиметричним потенціалом, що має два мінімуми, розглянутий у квазікласичному наближенні. Теорію розпаду супердеформованих обертальних рівнів побудовано на основі одночасно двох взаємодій — залишкової й електромагнітної. Ми узагальнили дворівневу модель Стаффорда і Барретта на випадок, коли супердеформований рівень, змішується з великою кількістю нормальних станів еквідистантного спектра.

ОСОБЕННОСТИ ВРАЩАТЕЛЬНЫХ УРОВНЕЙ СУПЕРДЕФОРМИРОВАННЫХ ЯДЕР

А.Я. Дзюблик, В.В. Утюж

Р е з ю м е

Модель Давыдова — Чабана обобщена на случай супердеформированных ядер. Ядерное движение супердеформированных ядер с асимметричным потенциалом, имеющим два минимума, рассмотрено в квазиклассическом приближении. Теорию распада супердеформированных вращательных уровней построили, взяв за основу одновременно два взаимодействия — остаточное и электромагнитное. Мы обобщили двухуровневую модель Стаффорда и Барретта на случай, когда супердеформированный уровень связывается с большим количеством нормальных состояний эквидистантного спектра.