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TWO-DIMENSIONAL EXCITONS IN A STRONG PERPENDICULAR MAGNETIC FIELD AND PROPAGATION HANLE EFFECT OF QUADRUPOLE POLARITONS

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We discuss two aspects of the theory of exitons interacting with a magnetic field. The first concerns two-dimensional excitons in a strong perpendicular magnetic field, when only two levels of the Landau quantization of electrons and holes $n_e, n_h = 0; 1$ are taken into account. They give rise to four types of excitons characterized by four combinations of two quantum numbers (n_e, n_h) as follows: (0,0), (1,1), (1,0), and (0,1). Their wave functions and eigen-energies are determined, correspondingly, in the first and second order of perturbation theory in the nondiagonal Coulomb electron-hole interaction. The second aspect is devoted to the propagation Hanle effect with the participation of quadrupole polaritons in crystal Cu_2O in the Faraday geometry. The propagation time of two polariton wave packets with the same group velocity is supposed to be smaller than the dephasing time. The periodic dependence of the intensity of the light arrived to the rear side of the sample on the magnetic field is characterized by the period inversely proportional to the length of the effective propagation way.

Introduction

In recent years, the possibility to produce quantum well structures stimulated the theoretical [1, 2] and experimental studies [3 – 6] of excitons in 2*D* systems including the presence of external electric and magnetic fields. One of the advantages of 2*D* systems is a possibility of much faster cooling of hot photoexcited excitons compared with their bulk counterparts [7]. It permits to achieve sufficiently low temperatures. Another important aspect is the use of a strong magnetic field. It was shown [8, 9] that the properties of atoms and excitons are dramatically changed in a strong magnetic field such that the distance between Landau levels $\hbar\omega_c$ exceeds the Rydberg energies. The binding energy of an exciton, in this case, becomes independent of the masses of its constituent electron and hole, and it is known as a magnetoexciton in 2*D* structures. Electrons and holes in a two-dimensional structure in the presence of a strong perpendicular magnetic field have completely discrete energy spectra. In the Landau gauge, one can introduce, as quantum numbers, the wave vector in one in-plane direction and the number of Landau levels. They appear due to the quantization in another in-plane direction perpendicular to the previous one. While there are the wave vectors in one direction, nevertheless the electrons and holes have no kinetic energies of in-plane motion because their kinetic energy was transformed by the magnetic field into the potential energy of electrons and holes. The wave vectors determine the position of the centers of gyration, around which the Landau quantization takes place. In the past two decades, many papers were devoted to the study of 2*D* systems in a strong magnetic field [10 – 17].

The collective properties of high density $e\text{–}h$ pairs were studied in two aspects. One of them concerns the Bose – Einstein condensation (BEC) of magneto-excitons on a single-particle state with a given wave vector $\vec{k}$, whereas another one deals with the condensation of the type gas-liquid and the formation of metallic-type electron-hole liquid (EHL) and electron-hole drops (EHD). Many calculations were made in the lowest Landau level (LLL) approximation, when only the LLL for electron and the LLL for hole are taken into account. The attempts to avoid these restrictions were proposed. One of them takes into account the simultaneous excitations of two particles from their LLLs, to the excited Landau levels (ELL) of the same number as a result of their Coulomb interactions. Their excitations and subsequent return

to LLLs, as well as another virtual quantum transitions considerably influence the magnetoexciton collective properties. These reasons determine the necessity to study the energy spectrum of magnetoexcitons taking into account the ELL. Below we will restrict ourselves only with one, first excited Landau level (FELL) trying to determine its influence on the energy spectra and on the wave functions of magnetoexcitons of four types. They arise due to four possible compositions of two Landau levels for electrons and of two Landau levels for holes. Above we have discussed one limiting case of magnetoexcitons in 2*D* structures in a strong magnetic field $H > H_{cr}$. For 2*D* excitons in GaAs quantum wells, H_{cr} was estimated as 6.25 T. Now another limiting case of 3*D* excitons in bulk crystals in a comparatively small magnetic field $H < H_{cr}$ will be considered. For the 1*S*-type, Γ_5^+ ortho-exciton level of the yellow series of crystal Cu_2O, the critical magnetic field H_{cr} is by two orders of magnitude greater than that in the case of GaAs. Such values are unattainable now under laboratory conditions. The time-integrated quantum beats of two polariton wave packets with the same group velocities belonging to two different Zeeman components of the quadrupole-active Γ_5^+, ortho-exciton level give rise to the propagation Hanle effect [18 – 20]. In the previous paper [19], two quadrupole-polariton branches in the Voigt geometry were considered. Below the case of quadrupole-polaritons in the Faraday geometry will be studied. The dispersion curves of quadrupole-polaritons in the Faraday geometry consist of five branches. One pair of branches is related with the Zeeman component with the magnetic quantum number $m = -1$ and is characterized by a circular polarization σ^+. Another pair of branches originates from the interaction of the σ^-, circularly polarized photons with the upper Zeeman component with $m = +1$ of the ortho-exciton Γ_5^+ level splitted in the presence of the external magnetic field. The fifth curve is a pure quadrupole exciton branch characterized by $m = 0$, which does not interact with photons in the Faraday geometry.

Looking at Fig. 1 of [19], one can see that even in the absence of the spatial dispersion, when the exciton translational mass can be put equal to infinity, there are two polariton branches with opposite circular polarizations $\sigma^\pm$ and with different wave vectors $k^\pm$ at any frequency ω. We recall that the quantum interference effect of two plane waves with circular polarizations in opposite directions and slightly different frequencies $\omega_{or} \pm \omega_L$ in a longitudinal magnetic field parallel to the direction of light propagation gives rise to a resultant wave with linear polarization. Its plane slowly rotates at every point of the propagation way around the axis of the magnetic field with a small Larmor frequency ω_L. In the simplest case when the propagation effects can be neglected, the intensities of the emitted light with two perpendicular linear polarizations, for example $\vec{e_x}$ and $\vec{e_y}$, reveal the quantum beats slowly decreasing in time. The time-integrated intensities I_x and I_y allow one to determine the polarization rate $R = \dfrac{I_x - I_y}{I_x + I_y}$. It happens to have the quasiresonant dependence on the magnetic field or on the Larmor frequency of the type $\dfrac{1}{1 + (2\omega_L \tau_c)^2}$. Here τ_c is the coherence or dephasing time. The similar questions will be disscussed below, taking into account the propagation effects. We will show that, in the Faraday geometry for some polariton branches, as well as in the Voigt geometry considered earlier in [12], the new-type periodic dependence of the polarization rate R on magnetic field appears. The paper is organized as follows. In Sec. 1, the Hamiltonian of a 2*D* electron-hole system is described. In Sec. 2, the energy spectrum of four magnetoexciton bands is determined. In Sec. 3, we study the propagation Hanle effect of quadrupole polaritons in the Faraday geometry. The obtained results are summarized and discussed in Sec. 4.

1. Hamiltonian of a 2*D* electron-hole system in a strong magnetic field

We consider a simple 2*D* model of a semiconductor structure in a strong perpendicular magnetic field, assuming that the Zeeman splitting of Landau levels is large enough and electrons and holes are restricted to the LLL, $n = 0$, and to the FELL, $n = 1$. The typical electron-hole Coulomb interaction, which determines the ionization potential I_1 of a magnetoexciton with $k = 0$, is supposed to be smaller than the cyclotron energies $\omega_{c,e}$ and $\omega_{c,h}$. In turn, I_1 is greater than the exciton Rydberg constant, which means that the magnetic length l is smaller than the 2*D* exciton Bohr radius a_{ex}^{2D} ($l < a_{ex}^{2D}$). The layer plane is chosen as the (x, y) plane, and the magnetic field is directed along the z axis. The vector potential in the Landau gauge looks as $\vec{A} = (-H \cdot y, 0, 0)$. The electron and hole wave functions are characterized by the quantum numbers $n = 0; 1$ of the Landau quantization in the y direction and by one-dimensional wave vector p along x direction.

Here only the envelope wave functions of the Bloch wave functions for electrons and holes with $n = 0; 1$ are listed:

$$\psi_{n=0,p}^{e}(x, y) = \frac{1}{\sqrt{L_x}\, l \sqrt{\pi}}\, e^{ipx} \exp\left[\frac{-(y - pl^2)^2}{2l^2}\right],$$

$$\psi^{e}_{n=1,p}(x,y) =$$

$$= \frac{\sqrt{2}}{\sqrt{L_x\, l\sqrt{\pi}}}\, e^{ipx} \exp\left[\frac{-(y-pl^2)^2}{2l^2}\right] \frac{(y-pl^2)}{l},$$

$$\psi^{h}_{n=0,q}(x,y) = \frac{1}{\sqrt{L_x\, l\sqrt{\pi}}}\, e^{iqx} \exp\left[\frac{-(y+ql^2)^2}{2l^2}\right],$$

$$\psi^{h}_{n=1,q}(x,y) =$$

$$= \frac{\sqrt{2}}{\sqrt{L_x\, l\sqrt{\pi}}}\, e^{iqx} \exp\left[\frac{-(y+ql^2)^2}{2l^2}\right] \frac{(y+ql^2)}{l}, \qquad (1)$$

where l is the magnetic length. The wave vectors of electrons and holes p and q are restricted by the size of the layer surface in the y direction $-\frac{L_y}{2} < pl^2$, $ql^2 < \frac{L_y}{2}$. The total number N of possible states is equal to L_y divided by the discreteness pass $\frac{2\pi}{L_x}$, which gives rise to the value

$$N = \frac{S}{2\pi l^2}; \quad S = L_x L_y; \quad l^2 = \frac{\hbar c}{eH}, \qquad (2)$$

where S is the surface area of the sample. Each Landau level of electrons and holes is N-fold degenerated. They are determined by the cyclotron frequencies $\omega_{0,i} = \frac{eH}{m_i c}$, where $i = e, h$. The condition of a strong magnetic field is

$$H > H_{\rm cr}, \text{ where } H_{\rm cr} = \frac{4c\, e^3 \mu^2}{\hbar^3 \varepsilon_0^2}; \quad \mu = \frac{m_e m_h}{m_e + m_h}. \qquad (3)$$

The parameters of GaAs mean the reduced exciton mass $\mu = 0.1 m_0$, the dielectric constant $\varepsilon_0 = 11$, and the $2D$ exciton radius $a^{2D}_{\rm ex} = 100$ Å without magnetic field H. The magnetic length l at $H = H_{\rm cr} = 6.25$ T equals also to 100 Å.

The Hamiltonian of the electron-hole system on the surface of an ideal $2D$ layer in a strong magnetic field is written in the second quantization representation. It contains the creation and annihilation operators $a^{+}_{n,p}$, $a_{n,p}$ with $n = 0; 1$ for the electrons on the LLL and FELL, as well as the similar operators $b^{+}_{n,p}$, $b_{n,p}$ for the holes. The full Hamiltonian $\hat{H}$ consists of the zero order Hamiltonian $\hat{H}_0$ and the Coulomb interaction term $\hat{H}_{\rm Coul}$:

$$\hat{H} = \hat{H}_0 + \hat{H}_{\rm Coul}, \qquad (4)$$

where

$$\hat{H}_0 = \sum_{n=0}^{1} \sum_{p} [(n-1)\hbar\omega_{c,e}\, a^{+}_{n,p} a_{n,p} +$$

$$+ (n-1)\hbar\omega_{c,h}\, b^{+}_{n,p} b_{n,p}. \qquad (5)$$

The factor $(n-1)$ means that the energies of electrons and holes are accounted from the corresponding LLL. The Coulomb interaction terms describe only the scattering processes with the conservation of the total number of electrons separately and of the total number of holes apart. The terms describing the simultaneous creation or annihilation of two electron-hole pairs in semiconductors with the considerable lifetimes of charge carriers can be neglected.

In this approximation, we have

$$H_{\rm Coul} = \frac{1}{2} \sum_{n,m,n',m'=0}^{1} \sum_{p,q,S} [F_{e-c}(p,n;q,m;p-s,n';$$

$$q+s, m')\, a^{+}_{n,p}\, a^{+}_{m,q}\, a_{m',q+s}\, a_{n',p-s} +$$

$$+ F_{h-h}(p,n;q,m;p-s,n';q+s,m') \times$$

$$\times\, b^{+}_{n,p}\, b^{+}_{m,q}\, b_{m',q+s}\, b_{n',p-s} - 2F_{e-h}(p,n;q,m;$$

$$p-s,n';q+s,m')\, a^{+}_{n,p}\, b^{+}_{m,q}\, b_{m',q+s}\, a_{n',p-s}]. \qquad (6)$$

Here the Coulomb matrix elements are determined as follows:

$$F_{i-j}(p,n;q,m;p-s,n';q+s,m') = \int d\vec{\rho}_1 \int d\vec{\rho}_2 \times$$

$$\times\, \psi^{i*}_{n,p}(\vec{\rho}_1)\, \psi^{j*}_{m,q}(\vec{\rho}_2)\, V_{12}\, \psi^{i}_{n',p-s}(\vec{\rho}_1)\, \psi^{j}_{m',q+s}(\vec{\rho}_2), \qquad (7)$$

where

$$V_{12} = \frac{e^2}{\varepsilon_0 |\vec{\rho}_1 - \vec{\rho}_2|}. \qquad (8)$$

The operators of the total number of electrons $\hat{N}_e$ and holes $\hat{N}_h$ are

$$\hat{N}_e = \sum_{n=0}^{1} \sum_{p} a^{+}_{n,p} a_{n,p}; \quad \hat{N}_h = \sum_{n=0}^{1} \sum_{p} b^{+}_{n,p} b_{n,p}. \qquad (9)$$

They commute with Hamiltonians (4), (5), and (6). This means that we could introduce two independent chemical potentials μ_e and μ_h for electrons and holes,

correspondingly, when the quantum statistical properties will be studied. But below we will determine only the energy levels of four magnetoexciton states in the frame of the written Hamiltonian.

Now we will determine, in the zero order approximation, the magnetoexciton creation operators $d^{+}_{n,m,\vec{k}}$ characterized by the number n of the electron Landau level, by the similar number m for the hole, and by the two-dimensional wave vector $\vec{k}$ with two components κ_x and κ_y. The first determines the center-of-mass wave vector, whereas the second does the quantum number of the continuous spectrum of the relative electron-hole motion in the case of Landau gauge. The zero order operators can be constructed in the same way as was proposed in [14, 15] for the particular case $n = m = 0$. Below, the values $n, m = 0,1$ will be allowed:

$$d^{+}_{n,m,\vec{k}} = \frac{1}{\sqrt{N}} \sum_{t} e^{-ik_y t l^2} a^{+}_{n,\frac{k_x}{2}+t} b^{+}_{m,\frac{k_x}{2}-t}. \qquad (10)$$

The zero order exciton wave functions are determined by the action of the exciton creation operators on the vacuum state $|0\rangle$ as follows:

$$|\psi^{0}_{n,m,\vec{k}}\rangle = d^{+}_{n,m,\vec{k}} |0\rangle; \quad a_{n,p}|0\rangle = b_{n,p}|0\rangle = 0. \quad (11)$$

To simplify the notations, the combinations (n, m) describing two starting Landau levels will be denoted as

$$(0,0) = 1; \quad (1,1) = 2; \quad (1,0) = 3; \quad (0,1) = 4. \qquad (12)$$

The free electron-hole pair in state 1 has the zero energy; in state 2, the energy equals $\hbar\omega_{c,e} + \hbar\omega_{c,h}$, in state 3, it equals $\hbar\omega_{c,e}$, and in state 4, it equals $\hbar\omega_{c,h}$. Four types of excitons are determined by the Landau levels combination of the electron-hole pair. The matrix elements of Hamiltonian (4) between the states $|\psi^{0}_{i,\vec{k}}\rangle$ will be named as $H_{i,j}(\vec{k})$:

$$\langle\psi^{0}_{j,\vec{k}}|\hat{H}|\psi^{0}_{i,\vec{k}}\rangle = H_{i,j}(\vec{k}); \quad i, j = 1, 2, 3, 4. \qquad (13)$$

They are different from zero only for the same wave vectors $\vec{k}$ in the left and in the right exciton wave functions.

The calculations of the matrix elements $H_{i,j}(\vec{k})$ are simplified because only the electron-hole Coulomb interactions give the contribution different from zero. The electron-electron Coulomb interaction terms as well as the hole-hole interaction terms acting on the zero order exciton wave functions

$$\{|\psi^{0}_{i,\vec{k}}\rangle\} = \{|\psi^{0}_{n,m,\vec{k}}\rangle\}, \qquad (14)$$

will turn them into zero, because there are two annihilation operators in the mentioned Coulomb terms and only one creation operator of the same type in the wave functions (14).

2. The Energy Spectrum of Four Magnetoexciton Bands

Each magnetoexciton related with a well-defined pair of Landau levels (one for the electron and another for the hole) has a continuum spectrum of the levels of the $e - h$ relative motion. Instead of a hydrogen-line series of discrete levels of relative motion, we deal with an exciton band formed by a continuum of levels depending on the two-dimensional wave vector $\vec{k}(k_x, k_y)$. The calculations of the energy are based on the evaluation of the matrix elements of the full Hamiltonian $\hat{H}$ (4) and its components $\hat{H}_0$ (5) and $\hat{H}_{\text{Coul}}$ (6) using the zero order exciton wave functions (14). These matrix elements are

$$H_{i,j}(\vec{k}) = \langle\psi_i(\vec{k})|\hat{H}|\psi_j(\vec{k})\rangle =$$

$$= \delta_{i,j}\langle\psi_i(\vec{k})|\hat{H}_0|\psi_j(\vec{k})\rangle + V_{i,j}(\vec{k}),$$

$$V_{i,j}(\vec{k}) = \langle\psi_i,(\vec{k})|\hat{H}_{\text{Coul}}|\psi_j(\vec{k})\rangle, \quad i, j = 1, 2, 3, 4 \qquad (15)$$

Four exciton bands related with four combinations of two Landau levels for the electron and another two for the hole are

$$E^{(0,0)}_{\text{ex}}(k) = H_{1,1}(k) = V_{1,1}(k) = -I^{(0,0)}_{\text{ex}}(k),$$

$$E^{(1,1)}_{\text{ex}}(k) = H_{2,2}(k) = \hbar\omega_{c,e} + \hbar\omega_{c,h} + V_{2,2}(k) =$$

$$= \hbar\omega_{c,\mu} - I^{(1,1)}_{\text{ex}}(k),$$

$$\hbar\omega_{c,e} + \hbar\omega_{c,h} = \hbar\omega_{c,\mu},$$

$$E^{(1,0)}_{\text{ex}}(k) = H_{3,3}(k) = \hbar\omega_{c,e} + V_{3,3}(k) =$$

$$= \hbar\omega_{c,e} - I^{(1,0)}_{\text{ex}}(k),$$

$$E^{(0,1)}_{\text{ex}}(k) = H_{4,4}(k) = \hbar\omega_{c,h} + V_{4,4}(k) =$$

$$= \hbar\omega_{c,h} - I^{(0,1)}_{\text{ex}}(k). \qquad (16)$$

Here the exciton energies are accounted from the energies of the corresponding pairs of the Landau levels. Between the Landau levels of the electron-hole pair, there is a distance equal to the semiconductor energy gap E_g, which was dropped in (16). But it must be added if we intend to consider the optical quantum

transition in the exciton range of spectrum. The case of magnetoexcitons in the LLL approximation was discussed in [14 – 16]. The ionization potential was determined as

$$I_{\text{ex}}^{(0,0)}(k) = I_l\, e^{-k^2 l^2/2}\, {}_1F_1\left(\frac{1}{2}, 1; \frac{k^2 l^2}{2}\right) =$$

$$= I_l\, e^{-k^2 l^2/4}\, I_0\left(-\frac{k^2 l^2}{4}\right),$$

$$I_l = \frac{e^2}{c_0 l}\sqrt{\frac{\pi}{2}}. \qquad (17)$$

Here ${}_1F_1(a, b, x)$ is the degenerate hypergeometric function and $I_0(x)$ is the modified Bessel function. $I_{\text{ex}}^{(0,0)}(k)$ tends to zero as $\frac{1}{kl}$, when the dimensionless value kl tends to infinity. The ionization potentials $I_{\text{ex}}^{(n,m)}(k)$ are determined by the formula

$$I_{\text{ex}}^{(n,m)}(\vec{k}) = \sum_s F_{e-h}(p, n; k_x - p, m; ps, n;$$

$$k_x - p + s, m)\, e^{ik_y s l^2} =$$

$$= \frac{e^2}{\varepsilon_0 l}\int_0^\infty d x\, e^{-x^2/2}\left(1 - \frac{x^2}{2}\right)^{(n+m)} J_0(kl \cdot x). \qquad (18)$$

It leads to the expressions

$$I_{\text{ex}}^{(1,1)}(k) = I_l\, e^{-k^2 l^2/2}\left[{}_1F_1\left(\frac{1}{2}, 1; \frac{k^2 l^2}{2}\right) +\right.$$

$$\left. + \frac{3}{4}\,{}_1F_1\left(-\frac{3}{2}, 1; \frac{k^2 l^2}{2}\right) - {}_1F_1\left(-\frac{1}{2}, 1; \frac{k^2 l^2}{2}\right)\right],$$

$$I_{\text{ex}}^{(1,1)}(0) = \frac{3}{4} I_l,$$

$$I_{\text{ex}}^{(1,0)}(k) = I_{\text{ex}}^{(0,1)}(k) = I_l\, e^{-k^2 l^2/2}\left[{}_1F_1\left(\frac{1}{2}, 1; \frac{k^2 l^2}{2}\right) -\right.$$

$$\left. - \frac{1}{2}\,{}_1F_1\left(-\frac{1}{2}, 1; \frac{k^2 l^2}{2}\right)\right],$$

$$I_{\text{ex}}^{(1,0)}(0) = I_{\text{ex}}^{(0,1)}(0) = \frac{1}{2} I_l. \qquad (19)$$

One can observe that the ionization potentials for four types of excitons at the point $k = 0$ are different. In state 1 with the Landau level numbers (0,0), the ionization potential is equal to I_l, whereas, in another three states with their corresponding combinations, one has: state 2 or (1,1): $I_{\text{ex}}^{(1,1)}(0) = \frac{3}{4} I_l$; state 3 or (1,0): $I_{\text{ex}}^{(1,0)}(0) = \frac{1}{2} I_l$; state 4 or (0,1): $I_{\text{ex}}^{(0,1)}(0) = \frac{1}{2} I_l$.

Taking into account the off-diagonal matrix elements $V_{i,j}$ with $i \neq j$, we will find the more exact values of four starting exciton level $E_{\text{ex}}^{(n,m)}(k)$, as well as of their wave functions. Some words concerning the properties of the off-diagonal Coulomb matrix elements $V_{i,j}(k)$ are needed. While all of them are proportional to the ionization potential I_l and all of them tend to zero when kl tends to infinity, nevertheless one of them are different from zero at the point $k = 0$, whereas another ones tends to zero as (kl) or as $(kl)^2$. The first group includes two matrix elements $V_{12}(k)$ and $V_{21}(k)$:

$$V_{12}(k) = V_{21}(k) = -\, I_l \frac{1}{2}\, e^{-k^2 l^2/2}\, {}_1F_1\left(-\frac{1}{2}, 1; \frac{k^2 l^2}{2}\right). \qquad (20)$$

The second group includes another ten matrix elements:

$$V_{4,1}^*(\vec{k}) = V_{1,4}(\vec{k}) =$$

$$= -\frac{e^{-i\psi}}{2\sqrt{2}}(kl)\, I_l\, e^{-\frac{k^2 l^2}{2}}\, {}_1F_1\left(\frac{1}{2}, 2; \frac{k^2 l^2}{2}\right),$$

$$V_{3,1}^*(\vec{k}) = V_{1,3}(\vec{k}) =$$

$$= \frac{e^{i\psi}}{2\sqrt{2}}(kl)\, I_l\, e^{-\frac{k^2 l^2}{2}}\, {}_1F_1\left(\frac{1}{2}, 2; \frac{k^2 l^2}{2}\right),$$

$$V_{3,4}(\vec{k}) = V_{4,3}^*(\vec{k}) =$$

$$= \frac{3\, e^{-2i\psi}}{16}(kl)^2\, I_l\, e^{-\frac{k^2 l^2}{2}}\, {}_1F_1\left(\frac{1}{2}, 3; \frac{k^2 l^2}{2}\right),$$

$$V_{3,2}(\vec{k}) = V_{2,3}^*(\vec{k}) =$$

$$= -\frac{e^{-i\psi}}{2\sqrt{2}}(kl)\, I_l\, e^{-\frac{k^2 l^2}{2}}\left[{}_1F_1\left(\frac{1}{2}, 1; \frac{k^2 l^2}{2}\right) -\right.$$

$$\left. - \frac{3}{2}(kl)^2\, {}_1F_1\left(-\frac{1}{2}, 1; \frac{k^2 l^2}{2}\right)\right],$$

$$V_{2,4}(\vec{k}) = V_{4,2}^*(\vec{k}) =$$

$$= \frac{e^{-i\psi}}{2\sqrt{2}}(kl)\, I_l\, e^{-\frac{k^2 l^2}{2}}\left[{}_1F_1\left(\frac{1}{2}, 2; \frac{k^2 l^2}{2}\right) - \right.$$

$$\left. - \frac{3}{2}(kl)^2\, {}_1F_1\left(-\frac{1}{2}, 2; \frac{k^2 l^2}{2}\right)\right]. \qquad (21)$$

One can see that these matrix elements depend on the in-plane orientation of the exciton wave vector $\vec{k} = (k_x = k\cos\psi, k_y = k\sin\psi)$. But the final results will not depend on its concrete orientation and the angle dependences in (21) can be dropped. These matrix elements are smaller than the previous two (20) in the range of small values of k.

But both groups of matrix elements (20) and (21) are small in comparison with the cyclotron frequencies $\hbar\omega_{c,i}$ and can be taken into account in the frame of perturbation theory. For example, taking into account only the matrix elements V_{12} and V_{21}, one can obtain the more exact expressions for the exciton bands $E_1(k)$ and $E_2(k)$ and for the corresponding wave functions:

$$\psi_i(k) = a_{1i}\,\psi^0_{1k} + a_{2i}\,\psi^0_{2k}; \qquad (22)$$

$$E_{12}(k) = \frac{H_{11} + H_{22}}{2} \mp \sqrt{(H_{11} - H_{22})^2 + 4H^2_{12}},$$

$$|a_{11}|^2 = |a_{22}|^2 = \frac{(H_{22} - H_{11})^2}{(H_{22} - H_{11})^2 + H^2_{22}},$$

$$|a_{21}|^2 = |a_{12}|^2 = \frac{H^2_{12}}{(H_{22} - H_{11})^2 + H^2_{12}}. \qquad (23)$$

After simplification in the limit $|V_{12}| < \hbar\omega_{c\mu}$, they become

$$E_1(k) \approx E^{(0,0)}_{\rm ex}(k) - \frac{V^2_{12}}{(\hbar\omega_{c,e} + \hbar\omega_{c,h})},$$

$$E_2(k) \approx E^{(1,1)}_{\rm ex}(k) + \frac{V^2_{12}}{(\hbar\omega_{c,e} + \hbar\omega_{c,h})},$$

$$|a_{11}|^2 = |a_{22}|^2 \approx 1 - \frac{V^2_{12}}{(\hbar\omega_{c,e} + \hbar\omega_{c,h})^2},$$

$$|a_{12}|^2 = |a_{21}|^2 \approx \frac{V^2_{12}}{(\hbar\omega_{c,e} + \hbar\omega_{c,h})^2}. \qquad (24)$$

In a similar way, one can take into account the matrix elements (21).

3. Propagation Hanle Effect of Quadrupole Polaritons in Faraday Geometry

As was mentioned above, the upper and lower polariton branches (UPB, LPB) are related to the splitted components $m = \pm 1$ of the quadrupole-active ortho-exciton level in Cu_2O. It has the symmetry Γ^+_5 and three-fold degeneracy in the absence of the external magnetic field. The four quadrupole polariton branches of interest will be denoted as UPB (± 1) and LPB (± 1). Their dispersion curves were derived without taking into account of the antiresonant terms in the motion equations for the exciton and photon operators [19]. The dispersion curves are described by the common formula

$$(\omega - \omega_{{\rm or},\pm 1})(\omega - ck) = \frac{f(k)\,\omega^2_p}{4}, \qquad (25)$$

where the frequencies $\omega_{{\rm or}\pm 1} = \omega_{\rm or} \pm \omega_{\rm L}$ appear due to the Zeeman splitting of the three-fold degenerate ortho-exciton level Γ^+_5, ω_p is the plasmon frequency of the valence electrons, $f(k)$ is the oscillator strength of the quadrupole transition from the ground state of the crystal to the exciton state, and the Larmor frequency ω_L depends on the total g-factor $|g_c + g_v|$, Bohr magneton $\mu_{\rm B}$, and magnetic field strength H as follows:

$$2\hbar\omega_{\rm L} = |g_c + g_v|\,\mu_{\rm B} H. \qquad (26)$$

The exact solution of Eqs. (25) gives rise to two pairs of polariton branches

$$\omega^{(+1)}_{u,l} = \frac{\omega_{{\rm or},+1} + ck}{2} \pm \frac{1}{2}\sqrt{(\omega_{{\rm or},+1} - ck)^2 + f(k)\,\omega^2_p},$$

$$\omega^{(-1)}_{u,l} = \frac{\omega_{{\rm or},-1} + ck}{2} \pm \frac{1}{2}\sqrt{(\omega_{{\rm or},-1} - ck)^2 + f(k)\,\omega^2_p}. \qquad (27)$$

One pair of the branches is related to the ortho-exciton Zeeman component with the magnetic quantum number $m = 1$ and with the circular polarization σ^- of photons, whereas another pair of polariton branches is related to the circular polarization of photons σ^+ and magnetic quantum number $m = -1$. Here the $\sigma_\pm$ circular polarizations $(\vec{e_x} \pm i\vec{e_y})$ are oriented in the plane perpendicular to the longitudinal magnetic field directed along the light propagation direction. The group velocities of the quadrupole polariton branches will be determined neglecting the spatial dispersion

effects as well the changes of the quadrupole oscillator strength $f(k)$. It is supposed to be taken in the range of the wave vectors $\vec{k}$ in the vicinity of the value k_0 where the intersection of the photon and exciton branches takes place and where its value $f(k_0)$ in crystal Cu_2O equals $3.7 \cdot 10^{-9}$. These two neglected dependences are much slower than the dependence of the photon branch on the wave number k. The influences of the functions $\omega_{or,\pm 1}(k)$ and $f(k)$ on the polariton group velocities will be discussed elsewhere. In our approximations, the group velocities for the polariton branches are

$$\nu_{g;u,l}^{(+1)} = \frac{d\omega_{u,l}^{(+1)}(k)}{dk} =$$

$$= \frac{c}{2}\left(1 \pm \frac{ck - \omega_{or,+1}}{\sqrt{(ck - \omega_{or,+1})^2 + f(k_0)\,\omega_p^2}}\right),$$

$$\nu_{g;u,l}^{(-1)} = \frac{d\omega_{u,l}^{(-1)}(k)}{dk} =$$

$$= \frac{c}{2}\left(1 \pm \frac{ck - \omega_{or,-1}}{\sqrt{(ck - \omega_{or,-1})^2 + f(k_0)\,\omega_p^2}}\right). \qquad (28)$$

Here the signs '+" in the brackets concern the UPB and the signs '~" concern the LPB. Now we will consider the interference of two polariton wave packets with the same group velocity. When one of them belongs to UPB (+1) and another one LPB (~1), we suppose that

$$\nu_{g,u}^{(+1)}(k_u) = \nu_{g,l}^{(-1)}(k_l) = \nu_g = \frac{c}{2}(1 + x). \qquad (29)$$

It means that the relation

$$\frac{ck_u - \omega_{or,+1}}{\sqrt{(ck_u - \omega_{or,+1})^2 + f(k_0)\,\omega_p^2}} =$$

$$= \frac{\omega_{or,-1} - ck_l}{\sqrt{(ck_l - \omega_{or,-1})^2 + f(k_0)\,\omega_p^2}} = x \qquad (30)$$

is valid. It reflects the rules that x is positive on the (UPB)(+1) in the region $ck_u > \omega_{or,+1}$ and on the LPB(~1) in the range of wave vectors $ck_l < \omega_{or,-1}$. Equation (30) allows us to find the expressions

$$ck_u - \omega_{or,+1} = \frac{x}{\sqrt{1 - x^2}}\sqrt{f(k_0)\,\omega_p^2},$$

$$\omega_{or,-1} - ck_l = \frac{x}{\sqrt{1 - x^2}}\sqrt{f(k_0)\,\omega_p^2} \qquad (31)$$

in accordance with the above-mentioned rules concerning the behavior of the group velocity along the polariton branches.

Expressions (31) permit us to determine the needed difference of two wave vectors

$$k_u - k_l = \frac{2\omega_L}{c} + \frac{2x\sqrt{f(k_0)\,\omega_p^2}}{c\sqrt{1 - x^2}}, \qquad (32)$$

which plays the key role in the propagation Hanle effect.

Side-by-side with the difference of two wave vectors (32), it is necessary to determine the difference of the frequencies of carrier waves $\omega_u^{(+1)}(k_u) - \omega_l^{(-1)}(k_l)$. Substituting the values k_u and k_l from (31) into (27), we obtain

$$\omega_u^{(+1)}(k_u) = \omega_{or,+1} + \frac{1}{2}\sqrt{\frac{1 + x}{1 - x}}\,\sqrt{f(k_0)\,\omega_p^2},$$

$$\omega_u^{(-1)}(k_l) = \omega_{or,-1} - \frac{1}{2}\sqrt{\frac{1 + x}{1 - x}}\,\sqrt{f(k_0)\,\omega_p^2}, \qquad (33)$$

which determines the needed difference

$$\omega_u^{(+1)}(k_u) - \omega_l^{(-1)}(k_l) = 2\omega_L\sqrt{\frac{1 + x}{1 - x}}\,\sqrt{f(k_0)\,\omega_p^2}. \qquad (34)$$

The quantum interference phenomena will lead to the Hanle effect if the polariton branches belong to the different magnetic numbers $m = \pm 1$, and the differences of their frequencies depend on the magnetic field. For example, one can study the interference between the waves emitted by UPB(+1) and LPB(-1), or by another pair LPB(+1) and UPB(~1). Another combinations of two, three, or four waves are also possible. But below we will restrict ourselves by the interference only of one UPB(+1) polariton wave with circular polarization $(\vec{e_x} - i\vec{e_y})$ named as σ^- and of another LPB(~1) polariton wave with the circular polarization $(\vec{e_x} + i\vec{e_y})$ named as σ^+. The propagation Hanle effect arises due to the interference of two wave packets with the same group velocity and differs from the usual Hanle effect, where only the interference of two plane waves is considered and their propagation effects are neglected.

Following formula (34) [19], we willl start with the summary electric field strength

$$\vec{E}(z, t) = (\vec{e_x} - i\vec{e_y})\,E_u(k_u)\,\frac{\sin x_u}{x_u} \times$$

$$\times \exp[-i\omega_u^{(+1)}(k_u) + ik_u z + i\varphi_u - \gamma_u t] +$$

$$+ (\vec{e_x} + i\vec{e_y})\,E_l(k_l)\,\frac{\sin x_l}{x_l} \times$$

$$\times \exp [- i\omega_l^{(-1)}(k_l) + ik_l z + i\varphi_l - \gamma_l t], \quad (35)$$

which represents the interference of two wave packets with opposite circular polarizations and has two components E_x and E_y:

$$\vec{E} = \vec{e_x} E_x + \vec{e_y} E_y, \quad (36)$$

where

$$E_x = E_u(k_u) \frac{\sin x_u}{x_u} \exp(- i\omega_u^{(+1)}(k_u) + ik_u z +$$

$$+ i\varphi_u - \gamma_u t) + E_l(k_l) \frac{\sin x_l}{x_l} \times$$

$$\times \exp(- i\omega_l^{(-1)}(k_l) + ik_l z + i\varphi_l - \gamma_l t),$$

$$E_y = i\left[E_l(k_l) \frac{\sin x_l}{x_l} \exp(- i\omega_l^{(-1)}(k_l) + ik_l z + \right.$$

$$+ i\varphi_l - \gamma_l t) - E_u(k_u) \frac{\sin x_u}{x_u} \times$$

$$\left. \times \exp(- i\omega_u^{(+1)}(k_u) + ik_u z + i\varphi_u - \gamma_u t) \right]. \quad (37)$$

Here the values $\omega_i^{(\pm 1)}(k_i)$, k_i, γ_i, $x_i = \nu_{g,i}^{(\pm 1)}(k_i) \Delta k_i t$ with $i = u$ correspond to UPB(+1), whereas the values with $i = l$ correspond to LPB(˜1). The wave vectors k_u and k_l correspond to the selected states on two polariton branches discussed above, where two group velocities $\nu_{g,u}^{(+1)}(k_u)$ and $\nu_{g,l}^{(-1)}(k_l)$ coincide, as was written in (29). $2\Delta k_i$ are the wave packet widths, which were supposed to be equal, $\Delta k_u = \Delta k_l = \Delta k$. The values $\omega_i^{(\pm 1)}(k_i)$, k_i and γ_i (the frequencies, wave vectors and damping rates of two carrier waves taking part in the interference effect) depend essentially on the selected polariton branches.

Starting with the components E_x and E_y, one can determine the light beam intensity in two perpendicular polarizations:

$$I_x = |E_x|^2 = e^{-2\gamma t}\left(I_{0u} \frac{\sin^2 x_u}{x_u^2} + I_{0l} \frac{\sin^2 x_l}{x_l^2} + \right.$$

$$\left. + 2\sqrt{I_{0u} I_{0l}} \frac{\sin x_u}{x_u} \frac{\sin x_l}{x_l} \cos(\Omega(x) t - \Theta(x)) \right),$$

$$I_y = |E_y|^2 = e^{-2\gamma t}\left(I_{0u} \frac{\sin^2 x_u}{x_u^2} + I_{0l} \frac{\sin^2 x_l}{x_l^2} - \right.$$

$$\left. - 2\sqrt{I_{0u} I_{0l}} \frac{\sin x_u}{x_u} \frac{\sin x_l}{x_l} \cos(\Omega(x) t - \Theta(x)) \right). \quad (38)$$

The frequency splitting $\Omega(x)$ and the phase angle $\Theta(x)$ depend on the group velocity parameter x as follows:

$$\Omega(x) = \omega_u^{(+1)}(k_u) - \omega_l^{(-1)}(k_l) =$$

$$= 2\omega_L + \sqrt{\frac{1+x}{1-x}} \sqrt{f(k_0)\omega_p^2},$$

$$\Theta(x) = (k_u - k_l) d + \varphi = \frac{2\omega_L d}{c} +$$

$$+ \frac{2dx}{c\sqrt{1-x^2}} \sqrt{f(k_0)\omega_p^2} + \varphi. \quad (39)$$

The parameter x following its definition (30) can approach unity but cannot be equal to 1. Contrary to the case of the Voigt geometry, where the results published in [19] are true only in the case of Zeeman splitting smaller than the quadrupole polariton effect, our results in the frame of the Faraday geometry are true at the arbitrary relation between the Zeeman effect and quadrupole polariton effect. The applied magnetic field strength must be less than the critical value H_{cr}, which for crystal Cu_2O and the 1S yellow series ortho-exciton Γ_5^+ level is unattainable under laboratory conditions, as was discussed above.

As was determined earlier, the polarization rate in the Faraday geometry is determined by the formula

$$R = \frac{\int_0^\infty I_x(t)\, dt - \int_0^\infty I_y(t)\, dt}{\int_0^\infty I_x(t)\, dt + \int_0^\infty I_y(t)\, dt}, \quad (40)$$

which equals

$$R = \frac{\cos\Theta J_1(p, b) + \sin\Theta J_2(p, b)}{J_1(p, 0)}, \quad (41)$$

$$J_1(p, b) = \int_0^\infty e^{-px} \frac{\sin^2 x}{x^2} \cos bx\, dx =$$

$$= \frac{b+2}{4} \operatorname{arctg}\left(\frac{b+2}{p}\right) + \frac{b-2}{4} \operatorname{arctg}\left(\frac{b-2}{p}\right) -$$

$$- \frac{b}{2}\operatorname{arctg}\frac{b}{p} + \frac{p}{8}\ln\left(\frac{p^2 + b^2}{(b+2)^2 + p^2}\right) +$$

$$+ \frac{p}{8}\ln\left(\frac{b^2 + p^2}{(b-2)^2 + p^2}\right),$$

$$J_2(p, b) = \int_0^\infty e^{-px}\frac{\sin^2 x}{x^2}\sin bx\, dx =$$

$$= \frac{p}{4}\operatorname{arctg}\left(\frac{b+2}{p}\right) + \frac{p}{4}\operatorname{arctg}\left(\frac{b-2}{p}\right) -$$

$$- \frac{b}{2}\operatorname{arctg}\frac{b}{p} + \frac{b+2}{8}\ln((b+2)^2 + p^2) +$$

$$+ \frac{b-2}{8}\ln((b-2)^2 + p^2) - \frac{b}{4}\ln(b^2 + p^2). \qquad (42)$$

In the particular case $b = 0$, we have

$$J_1(p, 0) = \int_0^\infty e^{-px}\frac{\sin^2 x}{x^2}dx = \operatorname{arctg}\frac{2}{p} + \frac{p}{4}\ln\left(\frac{p^2}{p^2 + 4}\right),$$

$$J_2(p, 0) = 0. \qquad (43)$$

Here the constants p and b are determined as

$$p = \frac{2\gamma}{v_g \Delta k}, \quad b = \frac{\Omega}{v_g \Delta k}. \qquad (44)$$

From expressions (39) and (41), one can see that the trigonometric function changes periodically as a function of magnetic field strength H. The period ΔH approximately equals

$$\Delta H = \frac{2\pi\hbar c}{d\,|g_c + g_v|\,\mu_B}. \qquad (45)$$

The periodicity on the magnetic field is inversely proportional to the sample thickness d.

Conclusions

We considered two causes of excitons interacting with an external magnetic field in two limits: $H < H_{cr}$ and $H > H_{cr}$. In the first case, the propagation Hanle effect was studied. The developed theory is applicable if the main condition $v_g > 2\gamma d$ holds. This means the existence of a polariton group velocity cutoff from below, so that the propagation time is less than the coherence time $\tau_{pr} < \tau_{coh}$. Contrary to the case of the Voigt geometry considered in [19] where the Zeeman effect was supposed to be smaller than the quadrupole-polariton effect, in the case of Faraday geometry discussed above, this restriction does not exist. We can consider arbitrary magnetic fields and Zeeman splitting greater or smaller than the quadrupole-polariton effect. In both the Voigt and Faraday geometries, the propagation Hanle effect is characterized by the new periodic dependence on magnetic field strength, which can be even more important than the quasiresonant dependence well known for the usual Hanle effect. The new periodic dependence can be observed only in sufficiently thick samples. In another limiting case $H > H_{cr}$, the cyclotron frequencies of electrons and holes in 2*D* structures are supposed to be greater than the exciton Rydberg constant in the absence of the external perpendicular magnetic field. Under these conditions, magnetoexcitons are formed.

The 2*D* magnetoexcitons in the two lowest Landau levels approximation and in the second order approximation of perturbation theory on the electron-hole Coulomb interaction represent the superposition of four states related to four combinations of two Landau levels for electrons and of two Landau levels for holes. The ionization potentials for four types of magnetoexcitons in the zero order approximation as well as the corrections to these values are determined.

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