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FREE-CARRIER MAGNETOABSORPTION IN QUANTUM WELL STRUCTURES

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The theory of free-carrier absorption is developed for a quasi-two-dimensional quantum well in a quantizing magnetic field for the case where the carriers are scattered by polar optical phonons, piezoelectric phonons, and nonpolar optical phonons and the radiation field is polarized perpendicularly to the plane of the layer. The free-carrier absorption coefficients found for the case of a nondegenerate electron gas. The dependence of the evaluated absorption on the magnetic field, thickness of the well, and temperature is shown explicitly. For polar and nonpolar optical phonons, the free-carrier absorption coefficient oscillates as a function of the magnetic field and photon frequency with resonances occurring when $P\omega_c = \Omega \pm \omega_0$, where ω_c, Ω, and ω_0 are the cyclotron, photon, and phonon frequencies, respectively, and where P is an integer. For elastic scattering by piezoelectrical phonons, resonances are expected when $P\omega_c = \Omega$. The obtained results are compared with those of the theory of free-carrier absorption in the absence of a magnetic field.

Introduction

In the past years, quasi-two-dimensional systems in the presence of a magnetic field have been the subject of numerous experimental and theoretical investigations [1 – 10]. In the two-dimensional case, a magnetic field, which is perpendicular to the quantum well, transforms the continuum electron and hole energy spectrum to a discrete spectrum. Due to the complete quantization of the electronic states, a magnetic field should essentially influence the optical properties of quantum well structures. The quantum theory of free-carrier absorption (ECA) in size-confined systems in the absence of magnetic fields is well developed [11 – 20]. These papers have dealt with FCA of electromagnetic radiation which is polarized parallel to the layer plane. The theory of FCA *n*-type GaAs films have investigated [16] in two special cases: the radiation field polarized parallel to the layer plane and the radiation field polarized perpendicularly to the layer plane. The theory of FCA in semiconductors in the presence of a quantizing magnetic field when acoustic phonon scattering is important has been studied in [21]. It was found that the FCA coefficient depends upon the polarization of the radiation field relative to the direction of the magnetic field. Other theoretical work has been done by Mycielski et al. [22, 23] for the case where impurity scattering is important and by Bass and Levinson [24], Enck et al. [25], and Zawadski et al. [26] for the case where polar optical phonon scattering is important. When the radiation field is polarized parallel to the applied magnetic field, the dependence of the FCA on the magnetic field only appears when the quantization of electronic energy levels in the magnetic field becomes important [21]. In [7], we have extended the theory of FCA in quasi-two-dimensional systems to take into account the presence of quantizing magnetic fields. The theory was then applied to calculate the FCA of radiation polarized parallel to the magnetic field when acoustic phonon scattering is important. When the electromagnetic radiation polarized transversely to the direction of the applied magnetic field, the cyclotron-resonance absorption of the radiation can occur.

The purpose of this paper is to evaluate the FCA coefficient in a quasi-two-dimensional quantum well in the presence of a magnetic field perpendicular to the quantum-well layer. We consider only the scattering of electrons by phonons (polar optical, nonpolar optical, piezoelectrical) in the deformation potential model. We will present a calculation of the FCA coefficient for electromagnetic radiation polarized parallel to the applied magnetic field.

1. General Relations

For the size-confined systems, we assume the quantum well-like structure with an infinite quantum well in

the z-direction. The magnetic field H is applied in the direction perpendicular to the quantum-well layer (i.e., perpendicularly to the interfaces). Adopting a single-band spherical effective mass model for electrons, the one-electron eigenfunctions ψ_{nlk_y} and energy eigenvalues E_{nl} are given by

$$E_{nl} = \left(n + \frac{1}{2}\right)\hbar\omega_c + l^2 E_0, \qquad E_0 = \frac{\pi^2\hbar^2}{2m^* d^2},$$

$$\psi_{nlk_y} = \left(\frac{2}{L_y d}\right)^{1/2} \Phi_n (x - x_0)\, e^{ik_y y} \sin\left(\frac{l\pi z}{d}\right), \qquad (1)$$

where $n = 0, 1, 2, \ldots$, $l = 1, 2, 3, \ldots$, d is the thickness of the layer and m^* is the effective mass of an electron, Φ_n represents the harmonic oscillator wave function centered at $x_0 = R^2 k_y$ with $R = (\hbar c/eH)^{1/2}$ being the cyclotron radius, $\omega_c = eH/m^* c$ is the cyclotron frequency, n denotes the Landau level index, and l denotes the level quantization in the z direction.

The FCA coefficient α, which is related to the quantum-mechanical transition probabilities in which the carriers absorb or emit a photon with the simultaneous scattering of the carriers by phonons, is given in [11, 21, 27] as

$$\alpha = \frac{\varepsilon^{1/2}}{n_0 c} \sum_i W_i f_i . \qquad (2)$$

Here ε is the dielectric constant of material, n_0 is the number of photons in the radiation field, and f_i is the free-carrier distribution function. The sum is over all the possible initial states i of the system. The transition probabilities W_i can be calculated using the standard second-order Born golden rule approximation:

$$W_i = \frac{2\pi}{\hbar} \sum_{fq} [\,|\langle f | M_+ | i\rangle|^2 \delta(E_f - E_i - \hbar\Omega - \hbar\omega_q) +$$

$$+ |\langle f | M_- | i\rangle|^2 \delta(E_f - E_i - \hbar\Omega + \hbar\omega_q)]. \qquad (3)$$

Here E_i and E_f are the initial and final state energies of electrons, respectively, $\hbar\Omega$ is the photon energy, $\hbar\omega_q$ is the phonon energy, and $\langle f | M_\pm | i\rangle$ are the transition matrix elements from the initial state to the final state for the interaction between electrons, photons, and phonons. These transition matrix elements can be represented by

$$\langle f | M_\pm | i\rangle = \sum_\alpha \left(\frac{\langle f | H_R | \alpha\rangle\langle\alpha | V_s | i\rangle}{E_i - E_\alpha \mp \hbar\omega_q} + \right.$$

$$\left. + \frac{\langle f | V_s | \alpha\rangle\langle\alpha | H_R | i\rangle}{E_i - E_\alpha - \hbar\Omega} \right), \qquad (4)$$

where H_R is the interaction Hamiltonian between electrons and the radiation field, V_s is the scattering potential due to the electron-phonon interaction.

The matrix elements of the electron-photon interaction Hamiltonian using wave functions are

$$\langle k'_y n' l' | H_R | k_y nl\rangle = -\frac{ie\hbar}{m^*}\left(\frac{2\pi\hbar n_0}{\varepsilon\Omega\Omega_0}\right)^{1/2} \frac{l}{d} \times$$

$$\times \left[\frac{1 - \cos[\pi(l' - l)]}{l' - l} + \frac{1 - \cos[\pi(l' + l)]}{l' + l} \right] \delta_{k_y k'_y} \delta_{nn'}, \qquad (5)$$

where Ω_0 is the volume of the crystal. Here the radiation field is polarized perpendicularly to the plane of the layer.

We shall use three different scattering processes: polar-optical scattering, piezoelectric scattering, and nonpolar-optical scattering. The matrix elements $\langle k'_y n' l' | V_s | k_y nl\rangle$ of the electron-phonon interaction corresponding to the above three processes are equal to

$$\langle k'_y n' l | V_s | k_y nl\rangle = C'_j \delta_{k'_y, k_y \pm q_y} J_{nn'}(q_x q_y) \Lambda_{ll'}(q_z) \qquad (6)$$

where $J_{n',n}(q_x, q_y)$ is the overlap integral of the harmonic wave functions,

$$J_{n',n}(q_x, q_y) = \int_{-\infty}^{\infty} dx \exp(iq_x x)\, \Phi_{n'}(x - R^2 k_y -$$

$$- R^2 q_y)\, \Phi_n (x - R^2 k_y), \qquad (7)$$

$$\Lambda_{ll'}(q_z) = \frac{2}{d}\int_0^d dz \exp(iq_z z) \sin\left(\frac{l'\pi z}{d}\right) \sin\left(\frac{l\pi z}{d}\right), \qquad (8)$$

$$C'^2_j = C^2_j F_j(q).$$

The function $\Lambda_{ll'}(q_z)$ given by Eq. (8) is crucial for our calculation whose suitable approximation is discussed in [28].

For the electron-polar-optical phonon interaction, we have

$$C^2_{\mathrm{POL}} = 2\pi e^2 \hbar\omega_0 \left\{\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right\}, \quad F_{\mathrm{POL}} = \frac{N^\pm_0}{q^2}.$$

Here, ε_∞ and ε_0 are the high-frequency and static dielectric constants of the semiconductor, respectively. As usual, we take the phonon energy $\hbar\omega_q = \hbar\omega_0 \approx \text{const}$.

$$N_0 = \left[\exp\left(\frac{\hbar\omega_0}{K_B T}\right) - 1\right]^{-1}, \quad N_0^- = N_0, \quad N_0^+ = N_0 + 1,$$

where N_0^- (N_0^+) describes the annihilation (creation) of a phonon.

For the electron-nonpolar optical phonon interaction, we have

$$C_{np}^2 = \frac{\hbar D^2}{2\rho\omega_0\Omega_0}, \quad F_{np}(q) = N_0^{\pm},$$

where D is the nonpolar optical deformation potential constant.

When the piezoelectric coupling is dominant, one may obtain

$$C_{\text{PE}}^2 = \frac{e^2 K_B T \beta_p}{2\rho\upsilon_s^2\Omega_0\varepsilon^2}, \quad F_{\text{PE}}(q) = \frac{1}{q^2},$$

where β_p is the appropriate piezoelectric constant. In the case of bulk materials and at extremely strong magnetic fields, the electronic wave functions have small absolute values of momentum components parallel to the applied magnetic field. Therefore, we can neglect the q_z dependence in the interaction potential given by C_j'.

The electron distribution function for a quasi-two-dimensional nondegenerate electron gas in the presence of a magnetic field can be shown to be

$$f_{nl} = 2\pi n_e dR^2 \text{sh}(\hbar\omega_c/2k_B T)\,\gamma^{-1} \times \exp\left\{-\frac{1}{k_B T}\left[\left(n + \frac{1}{2}\right)\hbar\omega_c + E_0 l^2\right]\right\}. \quad (9)$$

Here, $\gamma = \sum_l \exp(-E_0 l^2/k_B T)$ and n_e is the concentration of electrons.

Below, we will use the following identities:

$$\int_0^\infty |J_{nn'}(q_x, q_y)|^2 q_\perp dq_\perp = \frac{1}{R^2},$$

$$\int_0^\infty |\Lambda_{ll'}(q_z)|^2 dq_z = \frac{2\pi}{d}\left(1 + \frac{1}{2}\delta_{ll'}\right),$$

$$\int_0^\infty \frac{|J_{nn'}(q_x q_y)|^2 dq_\perp}{q_\perp} = \frac{1}{n' - n}. \quad (10)$$

2. The Absorption Coefficient for Different Scattering Mechanisms

Using Eqs. (3) – (6) and (9) in [2] and also identities (10), we obtain the following expression for the FCA for polar, nonpolar, and piezoelectric phonon scattering:

$$\alpha_{\text{POL}}(H) = \frac{2\pi^2 e^4 \hbar\omega_0 n_e \text{sh}(\hbar\omega_c/2K_B T)}{\varepsilon' m^{*2}\varepsilon^{1/2}\Omega^3 V d^3 c\gamma} \times \sum_{n_f n_i}\frac{1}{n_f - n_i}\sum_{l_f l_i}\sum_{l''}(l'')^2 g_{l_f l_i l''}\exp\left\{-\frac{1}{K_B T}\left[\left(n_i + \frac{1}{2}\right)\times\right.\right.$$

$$\left.\left.\times\,\hbar\omega_c + E_o l_i^2\right]\right\}\{N_0\,\delta((n_f - n_i)\hbar\omega_c + (l_f^2 - l_i^2)E_0 -$$

$$-\,\hbar\Omega + \hbar\omega_0) + (N_0 + 1)\,\delta((n_f - n_i)\hbar\omega_c +$$

$$+\,(l_f^2 - l_i^2)E_0 - \hbar\Omega - \hbar\omega_0)\}, \quad (11)$$

where

$$g_{l_f l_i l''} = \left[\frac{1 - \cos\pi(l_f - l'')}{l_f - l''} + \frac{1 - \cos\pi(l_f + l'')}{l_f + l''}\right]^2 \times \frac{1}{(l_f^2 - (l'')^2)E'_0 - 1} \times \left(1 + \frac{\delta_{l_i l''}}{2}\right) +$$

$$+\left[\frac{1 - \cos\pi(l'' - l_i)}{l_i - l''} + \frac{1 - \cos\pi(l'' + l_i)}{l_i + l''}\right]^2 \times \frac{1}{1 + (l_i^2 - (l'')^2)E'_0} \times \left(1 + \frac{\delta_{l_f l''}}{2}\right), \quad E'_0 = E_0/\hbar\Omega.$$

Here, we assumed $q_\perp >> q_z$ for the transport in the (x, y) plane [9].

$$\alpha_{np} = \frac{2\pi e^2 D^2 n_e \omega_c \text{sh}(\hbar\omega_c/2K_B T)}{\varepsilon^{1/2} m^* c\rho\omega_0 d^3\Omega^3\gamma} \times \sum_{n_f n_i}\sum_{l_f l_i}\sum_{l''}(l'')^2 g_{l_f l_i l''}\exp\left\{-\frac{1}{K_B T}\left[\left(n_i + \frac{1}{2}\right)\times\right.\right.$$

$$\left.\left.\times\,\hbar\omega_c + E_o l_i^2\right]\right\}\{N_0\,\delta((n_f - n_i)\hbar\omega_c + (l_f^2 - l_i^2)E_0 -$$

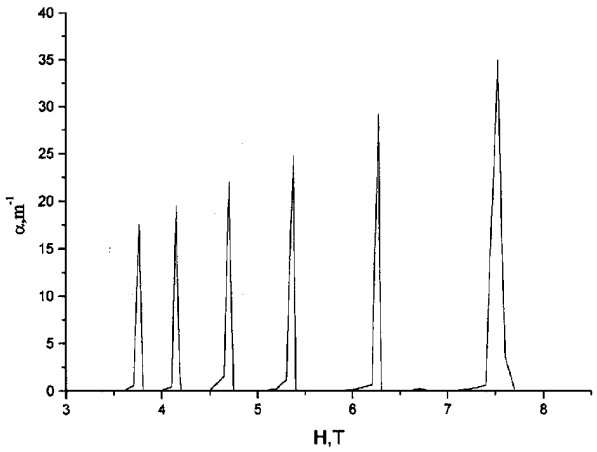

Fig. 1. FCA coefficient in the presence of the magnetic field vs the magnetic field at $T = 100$ K due to the scattering of electrons by nonpolar optical phonons

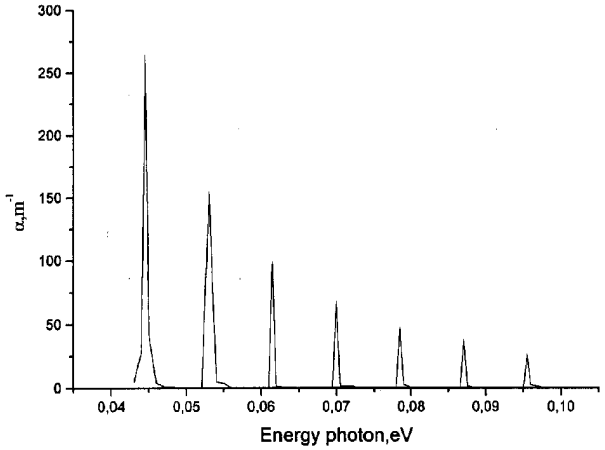

Fig. 2. FCA coefficient in the presence of the magnetic field vs the photon energy at $T = 100$ K due to the scattering of electrons by nonpolar optical phonons

$$-\hbar\Omega+\hbar\omega_0)+(N_0+1)\delta((n_f-n_i)\hbar\omega_c-\hbar\Omega-\hbar\omega_0)\},\tag{12}$$

$$\alpha_{\mathrm{PE}}=\frac{2\pi K_{\mathrm{B}}T e^4\beta_P^2\hbar^2 n_e \operatorname{sh}(\hbar\omega_c/2K_{\mathrm{B}}T)}{\rho\upsilon_s^2\varepsilon^{5/2}m^{*2}\Omega^3 d^3 c\gamma}\times$$

$$\times\sum_{n_f n_i}\frac{1}{n_f-n_i}\sum_{l_f l_i}\sum_{l''}(l'')^2 g_{l_f l_i l''}\times$$

$$\times\exp\left\{-\frac{1}{K_{\mathrm{B}}T}\left[\left(n_i+\frac{1}{2}\right)\hbar\omega_c+E_0 l_i^2\right]\right\}\times$$

$$\times\delta((n_f-n_i)\hbar\omega_c+(l_f^2-l_i^2)E_0-\hbar\Omega).\tag{13}$$

In the case of a two-dimensional system, the FCA coefficient given by Eqs. (11) – (13) diverges under resonance condition in a quantizing magnetic field through the delta function which reflects the singularites of the density of states [29]. It may be noted that the replacement of the δ functions by Lorentzian functions in Eqs. (11) – (13) are rigorously justifed in [30].

3. Discussion

Thus, we have found from the above three equations that the FCA coefficient is inversely proportional to the thickness of the well and photon frequency. Our results show that the FCA varies as d^{-3}. This dependence of the FCA coefficients on the well thickness has been obtained when the radiation field is polarized perpendicularly to the layer plane in the absence of the magnetic field as well [16]. It is well known [11 – 20] that when the radiation field is polarized in the layer plane, the FCA coefficient in $Q2D$ structures varies as d^{-1}. The difference of the dependence of the FCA coefficient on d in the two cases is connected, in the first case, with the dependence of the matrix elements of the electron-photon interaction on d.

Using the zero-field FCA coefficient for longitudinally polarized radiation in n-type GaAs films in the case where a carrier is scattered by piezoelectric phonons ([16] and (13)), we can express our results in terms of the dimensionless ratio of the FCA coefficient in the presence of a magnetic field to that in the absence of the field

$$\frac{\alpha_{\mathrm{PE}}(H)}{\alpha_{\mathrm{PE}}(0)}=F(T,\omega_c,\Omega).\tag{14}$$

In this form, the ratio depends only upon the magnetic field, absolute temperature, and photon frequency and does not depend upon such material parameters as the values of the deformation potential, sound velocity, or density of the material, although, of course, the absolute value of the absorption coefficient does depend upon the numerical values of these parameters.

We will consider that d is so small that no transitions between levels l can take place due to thermal excitations or phonons. For GaAs, E_0 is about 0.05 eV for $d = 100$ Å, $\hbar\omega_c \approx 1.7H$ (meV), with H measured in Tesla units. That is, we consider that all the carriers are in the lowest subband $l = l' = 1$. We have evaluated numerically the FCA coefficients, in the extreme quantum limit, for GaAs. The parameters used in our calculation are $m^* = 0.07m_0$, $\hbar\omega_0 = 0.036$ eV.

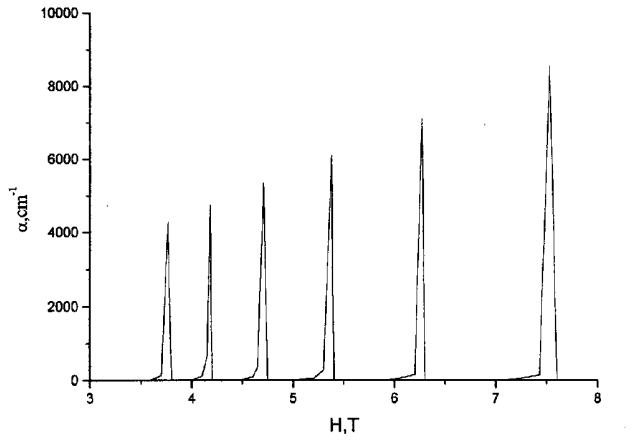

Fig. 3. FCA coefficient in the presence of the magnetic field vs the magnetic field at $T = 100$ K due to the scattering of electrons by polar optical phonons

The width parameters Γ_s were assumed to be the same for all intermediate levels [10].

From Eqs. (11) – (13), it can be seen that, in the extreme quantum limit ($\hbar\omega_c >> k_B T$, $n_i = 0$, $l_i = 1$) for nonpolar and polar optical phonons, the FCA coefficient oscillates as a function of the magnetic field and photon frequency with resonances occuring when $P\omega_c = \Omega \pm \omega_0$. For the elastic scattering by piezoelectrical phonons, resonances are expected when $P\,\omega_c = \Omega$.

In Fig. 1, the FCA coefficient due to nonpolar optical phonons is plotted as a function of the magnetic field in the range 3 – 8 T at $\hbar\Omega = 0.1$ eV. The resonances in α vs H are noted. It is shown that the amplitude of the oscillation increases with magnetic field. Since we introduced a broadening to the delta function, the divergence at the resonance is removed. As the magnetic field and therefore ω_c, increases, there are fewer and fewer subbands to which the transition can place until finally. Every time that the ratio $(\Omega \pm \omega_0)/\omega_c$ equals an integer value, the transition can take place with an additional subband ending as a final state.

In Fig. 2, the FCA coefficient due to nonpolar optical phonons is plotted as a function of the photon energy at $H = 5$ T. Similar to the magnetic field dependence, the resonances in α vs $\hbar\Omega$ are noted. It is shown that the amplitude of the oscillation decreases with increasing photon energy. The most dominant resonance occurs for the lowest Landau level. Higher subbands lead only to small contributions to the resonance structure.

In Fig. 3, the FCA coefficient due to polar optical phonons is plotted as a function of the magnetic field in the range 3 – 8 T at $\hbar\Omega = 0.1$ eV. The resonances in α vs H are noted. Similar to the nonpolar phonon scattering, the FCA coefficient increases with the magnetic field. As seen from Figs. 1 – 3, the resonance amplitude decreases with increasing value of the Landau subband index n.

For piezoelectric phonon scattering, the oscillations in the FCA coefficient as a fuction of the magnetic field and photon energy should be similar to that for other optical phonon scattering.

Therefore, the FCA coefficient in quantizing magnetic fields due to nonpolar, polar optical, and piezoelectric phonons as that for acoustic phonons [7] under resonance condition is directly proportional to the magnetic field. The magnetic field dependence of the FCA is explained in terms of phonon-assisted transitions between the various Landau levels of carriers.

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