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SUM RULES FOR PHOTOPRODUCTION PROCESSES

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Utilizing the analytic properties of a heavy photon forward Compton scattering amplitude, we derive a sum rule which connects the hadron electromagnetic formfactors with the differential cross section of the electroproduction process on a hadron. For the case of small transferred momenta, it can be expressed as a relation of the radius and eventually the electromagnetic moment with the integral over the total hadron photoproduction cross section.

Introduction

Let us consider, e.g., the peripheral very high energy electron-proton scattering with the production of some hadron state X moving closely to the direction of the initial proton (center of mass reference frame is implied). The matrix element of this process, $e(p_1)+P(p)\to e(p_1')+X$ (Fig. 1,a), is

$$M=i\frac{4\pi\alpha}{q^2}\bar{u}(p_1')\gamma^\mu u(p_1)J^\nu g_{\mu\nu}$$

with J^ν to be a hadron current. It can be rewritten into the following form:

$$M=is\frac{8\pi\alpha N_1|\boldsymbol{q}|}{q^2}\frac{\boldsymbol{e}.\boldsymbol{J}}{s_1},\quad N_1=\frac{1}{s}\bar{u}(p_1')\hat{p}u(p_1),$$

$$s=2p_1p\gg Q^2=-q^2. \tag{1}$$

The polarization vector of a heavy photon $\boldsymbol{e}$ is a 2-dimensional Euclidean vector $\boldsymbol{e}=\boldsymbol{q}/|\boldsymbol{q}|$.

Here we use the Sudakov expansion [1] of the transferred 4-vector in terms of the light-like 4 vectors $\tilde{p}=p-p_1\frac{p^2}{2p_1p}$ and $\tilde{p}_1=p_1-p\frac{p_1^2}{2p_1p}$, $q=\beta_q\tilde{p}_1+\alpha_q\tilde{p}+q_\perp$, with $q_\perp^2=-\boldsymbol{q}^2<0$ and the Euclidean 2-vector $\boldsymbol{q}$. Besides the Gribov representation of the photon Green function $\frac{g_{\mu\nu}}{q^2}=\frac{2}{sq^2}\tilde{p}_\mu\tilde{p}_{1\nu}$, the gauge invariance condition $qJ=0$ written in the form $p_1J=-s|\boldsymbol{q}|\boldsymbol{e}.\boldsymbol{J}/s_1$ with $s_1=s\beta_q$ is applied. The quantity s_1 is related to the invariant mass squared of the created hadronic state:

$$s_1=M_X^2+Q^2-m_p^2,\quad M_X^2=(p+q)^2. \tag{2}$$

One can transform the phase space volume of the final state to the form

$$d\Gamma=\frac{d^3p_1'}{(2\pi)^3 2\varepsilon_1'}d\Gamma_X=\frac{d^4p_1'}{(2\pi)^3}d^4q\,\delta((p_1-q)^2-m_e^2)d\Gamma_X=$$

$$=\frac{ds_1}{2s}d^2\boldsymbol{q}\frac{d\Gamma_X}{(2\pi)^3}, \tag{3}$$

where

$$d\Gamma_X=(2\pi)^4\delta^4(q+p-\Sigma_jp_j)\prod_j\frac{d^3p_j}{2\varepsilon_j(2\pi)^3}$$

is the phase space volume of the hadron system. Then it is transparent to see that the electroproduction cross section (see formula (D1) in [2])

$$\frac{d\sigma}{d\boldsymbol{q}^2}=\frac{\alpha\boldsymbol{q}^2}{4\pi^2}\int\frac{ds_1}{s_1^2(\boldsymbol{q}^2+(m_es_1/s)^2)^2}d^2\boldsymbol{q}\,\mathrm{Im}A(s_1,\boldsymbol{q}), \tag{4}$$

$$\mathrm{Im}\tilde{A}(s_1,\boldsymbol{q})=4\pi\alpha\int|\boldsymbol{e}.\boldsymbol{J}|^2d\Gamma_X$$

does not decrease with increase of the energy. Here $\mathrm{Im}\tilde{A}$ is the imaginary part of the forward Compton scattering amplitude on the proton with the intermediate state X (Fig. 1,b). For the case of real photon $\boldsymbol{q}=0$, one obtains the optical theorem $\mathrm{Im}A=4\pi s_1\sigma_{\mathrm{tot}}^{\gamma p\to X}(s_1)$ and, as a result, the case of small $-t=Q^2=\boldsymbol{q}^2$, we have

$$\boldsymbol{q}^2\frac{d\sigma^{ep\to eX}}{d\boldsymbol{q}^2}\bigg|_{\boldsymbol{q}^2\to 0}=\frac{\alpha}{\pi}\int\limits_{s_{\mathrm{th}}}^{\infty}\frac{ds_1}{s_1}\sigma_{\mathrm{tot}}^{\gamma p\to X}(s_1), \tag{5}$$

with $s_{\rm th} = 2M_p m_\pi$, which is similar to the total cross section of the electroproduction process in the Weizsäcker–Williams approximation

$$\sigma_{\rm tot}^{ep\to eX} = \frac{2\alpha}{\pi}\ln\left(\frac{s}{m_e m_\pi}\right)\int\limits_{s_{\rm th}}^{\infty}\frac{ds_1}{s_1}\sigma_{\rm tot}^{\gamma p\to X}(s_1). \quad (6)$$

The important role in our consideration is play by the elastic cross section. The corresponding matrix element is

$$M_B = is\frac{8\pi\alpha}{q^2}N_1N_2, \quad N_2 = \frac{1}{s}\bar{u}(p')\Gamma_\mu p_1^\mu u(p), \quad (7)$$

$$\Gamma_\mu = \gamma_\mu F_1(q^2) + [\gamma_\mu,\gamma_\nu]q^\nu\frac{F_2(q^2)}{4M_p},$$

where F_1, F_2 are the Dirac and Pauli electromagnetic form factors of a proton. The differential cross section for a very high energy is

$$\frac{d\sigma_B}{d\boldsymbol{q}^2} = 4\frac{\alpha^2}{(\boldsymbol{q}^2)^2}\Big(F_1^2(q^2) + \frac{\boldsymbol{q}^2}{4M_p^2}F_2^2(q^2)\Big). \quad (8)$$

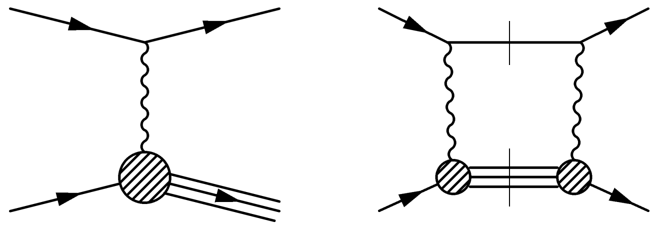
1. Sum Rule

Let us discuss now the analytical properties of the forward Compton scattering amplitude as a function of s_1. It has a pole at $s_1 = Q^2$ which corresponds to the one-proton intermediate state; a right-hand cut starting from $s_1 = Q^2 + 2M_pm_\pi$, corresponding to the pion-nucleon state; the right-hand cuts corresponding to several pions and the nucleon state and so on (see Fig. 2,*a,b,c*). Besides the later, it has a left-hand cut starting from the intermediate state in the u_1-channel ($u_1 = (q-p)^2$), corresponding to two nucleons and one antinucleon state $s_1 = Q^2 - 8M_N^2$ (see Fig. 2,*d*). The u_1-channel cut first appears for the Feynman amplitudes which contain the nucleon and two pions intermediate state in the s_1-channel.

Consider now the path integral

$$I = \int_C \frac{ds_1}{(\boldsymbol{q}^2)^2}\frac{p_1^\mu p_1^\nu \tilde{A}_{\mu\nu}}{s^2} \quad (9)$$

with the integration contour C depicted in Fig. 3,*a*. The quantity $\tilde{A}_{\mu\nu}$ is a part of the total Compton scattering tensor, with a heavy photon first absorbed and then emitted counting along the fermion line. This quantity is not gauge invariant, whereas its light-cone projection $p_1^\mu p_1^\nu \tilde{A}_{\mu\nu}$ is gauge invariant as well as it is connected with differential cross section. Gauge invariance provides the convergence of integral (9) when integrating over s_1.



a **b**

Fig. 1. Feynman diagram for the process $e(p_1)+P(p)\to e(p_1')+X$ (*a*) and for the forward Compton scattering amplitude on the proton with intermediate state X (*b*)

The sum rule appears (see Fig. 3,*b*) when the contour C in (9) is closed to the upper half-plane and to the lower half-plane. In [2], the validity of this kind of sum rules in the framework of QED was shown.

Considering the formfactors and the inelastic cross sections as formal series in the strong coupling constant and keeping in mind the analytical properties of Feynman amplitudes, we can write down the general form of sum rules as:

$$\frac{d\sigma_0}{dQ^2} - \frac{d\sigma_{\rm el}}{dQ^2} = \frac{d\sigma_{\rm in}}{dQ^2} + {\rm LCC}_p, \quad (10)$$

where the abbreviation ${\rm LCC}_p$ means the left-cut contribution; $d\sigma_{\rm el(in)}/dQ^2$ means the elastic (inelastic) differential cross section. The quantity $d\sigma_0/dQ^2$ means the elastic cross section with strong interactions switched off. Then for the case of electron-proton scattering, we have

$$1 - \left(F_1^p(-Q^2)\right)^2 - \frac{Q^2}{4M_p^2}\left(F_2^p(-Q^2)\right)^2 =$$

$$= \frac{(Q^2)^2}{4\pi\alpha^2}\frac{d\sigma^{ep\to eX}}{dQ^2} + O\left(\frac{Q^2}{8M_p^2}\right). \quad (11)$$

The last term on the right-hand side of this equation represents the estimation of the contribution of the left-cut. Really, it follows from the realistic assumption about the decreasing high-energy behavior of the photoproduction cross section $\sigma^{\gamma^* p}(s_1, Q^2) \sim (1/s_1)$ as $s_1 \to \infty$. This last term determines the accuracy of the sum rule, which is of order 10% for the region of small $x \sim 10^{-2}$, $Q^2 < 1({\rm GeV/s^2})^2$. We note that the basis for our consideration is the analytical properties of the forward Compton scattering amplitude and the assumption about the photoproduction cross section. In

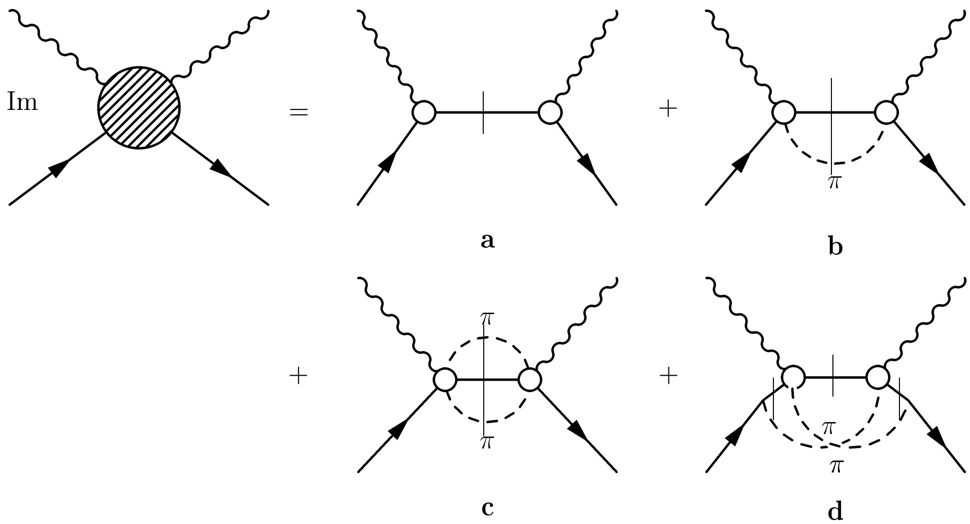


Fig. 2. Feynman diagram for the forward Compton scattering with one-proton intermediate state (a), pion-nucleon state (b), two pions and nucleon state (c), u_1-channel corresponding to the two nucleons and one antinucleon state (d)

principle, the latter can be violated in higher orders of perturbation theory due to possible Pomeron-type contributions. This situation, in our opinion, cannot be realized in the proton fragmentation region $s_1 \sim M_p^2$.

One can put the relation between the slope of the Dirac form factor of a proton at zero momentum transferred to be connected with the proton mean square Dirac radius $\langle r_{1p}^2 \rangle$, the proton anomalous magnetic moment $\mu_p = 1.793$, and the photoproduction cross section as

$$\frac{1}{3}\langle r_{1p}^2 \rangle - \frac{\mu_p^2}{M_p^2} = \frac{1}{2\pi^2\alpha}\int\limits_{\omega_p}^{\infty}\frac{d\omega}{\omega}\sigma_{\rm tot}^{\gamma p \to X}(\omega), \quad \omega_p = $$

$$= m_\pi + \frac{m_\pi^2}{2M_p}. \qquad (12)$$

The left-hand part of this equation is the positive quantity approximately equal to 1.8 mbarn.

For the case of electron-neutron scattering, we have

$$-\left(F_1^n(-Q^2)\right)^2 - \frac{Q^2}{M_n^2}\left(F_2^n(-Q^2)\right)^2 = $$

$$= \frac{(Q^2)^2}{4\pi\alpha^2}\frac{d\sigma^{en\to eX}}{dQ^2} + \mathrm{LCC}_n. \qquad (13)$$

Here we have some difficulty since the left-hand part of (13) is negative, whereas the right part is positive if one excludes LCC_n. So it's role here becomes important.

A reasonable assumption

$$\mathrm{LCC}_p = \mathrm{LCC}_n, \qquad (14)$$

can be used to avoid this difficulty. In such a way, we can obtain one combined sum rule for a proton and a neutron, which have a more extended region of applicability:

$$1 - (F_1^p(-Q^2))^2 + (F_1^n(-Q^2))^2 -$$

$$-\frac{Q^2}{4M_p^2}(F_2^p(-Q^2))^2 + \frac{Q^2}{4M_n^2}(F_2^n(-Q^2))^2 =$$

$$= \frac{((Q^2)^2}{4\pi\alpha^2}[\frac{d\sigma^{ep\to eX}}{dQ^2} - \frac{d\sigma^{en\to eX}}{dQ^2}];$$

$$\frac{1}{3}[\langle r_{1p}^2\rangle - \langle r_{1n}^2\rangle] - \frac{\mu_p^2}{M_p^2} + \frac{\mu_n^2}{m_n^2} =$$

$$= \frac{1}{4\pi^2\alpha}[\int\limits_{\omega_p}^{\infty}\frac{d\omega}{\omega}\sigma^{\gamma p\to X}(\omega) - \int\limits_{\omega_n}^{\infty}\frac{d\omega}{\omega}\sigma^{\gamma n\to X}(\omega)]. \qquad (15)$$

We expect the region of applicability of these sum rules to be larger than that for individual sum rules for the proton and the neutron.

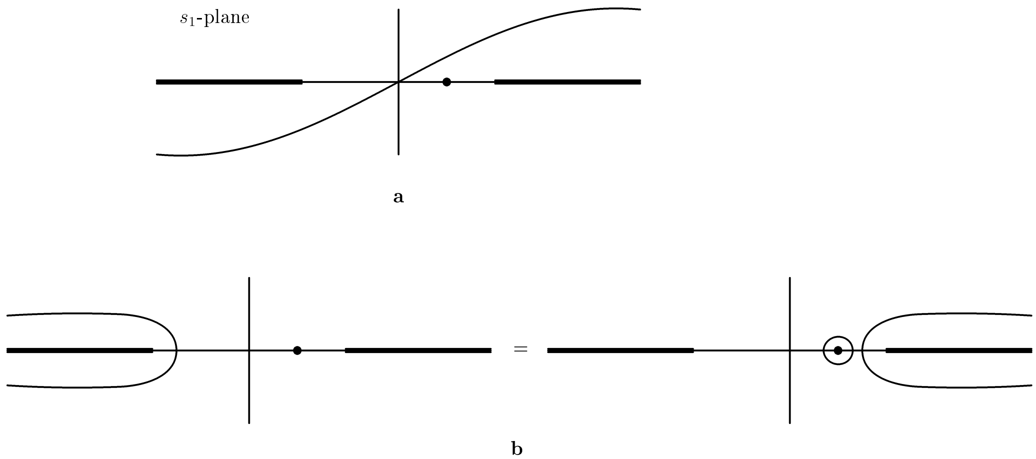


Fig. 3. Sum rule interpretation in s_1-plane

In conclusion, we consider the electroproduction process on a deuteron. For this case, the continuous spectrum starts from $s_1 = 2\omega_0 M_d = (M_p + M_n)^2 - M_d^2$, which corresponds to the desintegration threshold. The left-hand cut in the s_1-plane starts from $s_1 = -8M_d^2 \approx -32\,\mathrm{GeV}^2$. Neglecting its contribution, we obtain

$$1 - F_{CH}^2(-Q^2) - \frac{1}{18}\frac{(Q^2)^2}{M_d^4}F_Q^2(-Q^2) +$$

$$+\frac{1}{6}\frac{Q^2}{M_d^2}\left(1 - \frac{Q^2}{4M_d^2}\right)F_M^2(-Q^2) =$$

$$= \frac{(Q^2)^2}{4\pi\alpha^2}\frac{d\sigma^{ed\to eX}}{dQ^2}(1 + O\left(\frac{Q^2}{32M_p^2}\right)), \qquad (16)$$

with the charge F_{CH}, quadrupole F_Q and magnetic F_M form factors of a deuteron [3]. Then for the photoproduction process, one finds

$$2F_{CH}'(0) + \frac{1}{6M_d^2}F_M^2(0) = \frac{1}{2\pi^2\alpha}\int\limits_{\omega_0}^{\infty}\frac{d\omega}{\omega}\sigma_{\mathrm{tot}}^{\gamma d\to X}(\omega). \qquad (17)$$

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1. *Sudakov V.V.*// Sov. Phys. JETP **3** (1956) 65.
2. *Baier V.N., Fadin V.S., Khoze V.A., Kuraev E.A.*// Phys. Repts. **78** (1981) 323.
3. *Gourdin M.*// Nuovo cim. XXVIII (1963) 533.

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ПРАВИЛА СУМ ДЛЯ ПРОЦЕСІВ ФОТОНАРОДЖЕННЯ

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Р е з ю м е

Грунтуючись на аналітичних властивостях амплітуди комптонівського розсіяння вперед важких фотонів, було отримано правило сум, яке зв'язує електромагнітні форм-фактори адронів з диференціальним перерізом процесу народження електрона на адроні. Для випадку малих переданих імпульсів його можна записати у вигляді співвідношення між радіусом і, врешті-решт, електромагнітним моментом з інтегралом по повному перерізу фотонародження адронів.

ПРАВИЛА СУММ ДЛЯ ПРОЦЕССОВ ФОТОРОЖДЕНИЯ

Е. Бартош, Е. А. Кураев, С. Дубничка

Р е з ю м е

На основании аналитических свойств амплитуды комптоновского рассеяния вперед тяжелых фотонов получено правило сумм, которое связывает электромагнитные форм-факторы адронов с дифференциальным сечением процесса рождения электрона на адроне. Для случая малых переданных импульсов его можно записать в виде соотношения между радиусом и конечным электромагнитным моментом с интегралом по полному сечению фоторождения адронов.