UDC 539

LINKING THE PARAMETERS OF DIQUARK-QUARK MODEL TO THE CABIBBO ANGLE

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From two different modifications of the Gell-Mann—Okubo mass relation for $(\frac{1}{2})^+$ baryons: the first one given by a version of diquark-quark model and the second one being an optimal mass sum rule obtained using quantum groups $U_q(\mathrm{su}_n)$ in the role of hadronic flavor symmetries, we find a direct connection of the mass parameters of the diquark-quark model of Lichtenberg, Tassie and Keleman to the (proper value of) q-parameter and then to the Cabibbo angle.

Introduction

One of the earliest improved or generalized versions of the Gell-Mann—Okubo (GMO) mass relation for baryons forming the SU(3) octet has been obtained by Lichtenberg, Tassie and Keleman (LTK) in a particular version of 'diquark-quark' model [1]. The essence of the LTK result is that the correction to GMO combination is expressed in terms of basic parameters (of dimension of mass) characterizing both the diquark and the third quark. It is important to emphasize that viewing baryons as consisting of a diquark and a separate third quark provides a number of advantages [2] among which one also finds the important ability to account for some nonperturbative aspects of QCD.

On the other hand, quantum groups and quantum (q-deformed) algebras [3] provide a very useful tools not only for application to the spectroscopy of diatomic molecules and (super)deformed nuclei (see e.g. [4]) but, as it was demonstrated more recently, these are very useful when applied to phenomenological description of hadron properties [5]. In the framework of the approach initiated in [6] (where the case of vector mesons was first considered) and developed in more detail in subsequent papers, see [5] and references given therein, the q-algebras $U_q(\mathrm{su}_n)$ have been adopted, in place of the Lie algebras of the groups SU(n), as those describing hadronic flavor symmetries. Such replacement allowed to derive numerous results concerning hadron masses and mass sum rules, along with a number of interesting implications. Basic tool of this aproach is the representation theory of the q-algebras $U_q(\mathrm{su}_n)$ [3]. In the case of baryons, it was clearly demonstrated that the approach takes into account, in a uniform and natural way, the contributions in baryon masses which reflect essentially non-polynomial [7] (in fact, all-order) effects of SU(3) breaking. As another important consequence, the (phenomenologically) most adequate fixed value of the deformation parameter q is linked directly to the famous Cabibbo angle θ_c [8].

The goal of the present note is to find a direct connection between the basic parameters involved in the aforementioned two different extensions (modifications) of the octet baryon GMO mass formula derived within the *apparently differing* approaches: from the evaluation of hadron masses using q-deformed counterpart $U_q(\mathrm{su}_n)$ of flavor symmetries SU(n), and from the diquark-quark treatment of baryons within the LTK model.

1. Mass Relation from Diquark-Quark Model

Here we recapitulate some of the results from [1] which are relevant for our further discussion. With the notation

$$\zeta = (m_s - m_t + v_s - v_t)/2V_0,$$

$$\gamma_t = (\delta_t - \delta_q)/6V_0, \qquad \gamma_s = (\delta_s - \delta_q)/6V_0,$$

(here the subscript "s" or "t" refers to SU(3) sextet or SU(3) triplet diquark respectively, and "q" refers to the third quark), the octet baryon mass differences obtained in the LTK model are of the form

$$m_\Lambda - m_N = \tfrac{1}{6}(\delta_t + 3\delta_s + 2\delta_q) - v_8 + $$
$$+V_0\Big(2\zeta(3\gamma_s - \gamma_t) + 9\gamma_s^2 - 6\gamma_s\gamma_t + 5\gamma_t^2\Big), \qquad (1)$$

$$m_\Sigma - m_\Lambda = \tfrac{1}{3}(\delta_t - \delta_s) - 4V_0\Big(\zeta(\gamma_s + \gamma_t) + \gamma_s^2 - \gamma_t^2\Big), \quad (2)$$

$$m_\Xi - m_\Sigma = \tfrac{1}{3}(2\delta_s + \delta_q) - v_8 + 8V_0\left(\zeta\gamma_s + 3\gamma_s^2 - 3\gamma_s\gamma_t\right). \quad (3)$$

The parameters δ_t, δ_s and δ_q (of dimension of mass) in these expressions are a measure of the violation of SU(3) and, added properly to the respective SU(3) invariant masses m_t, m_s and m_q, provide necessary mass splittings in the diquark triplet, diquark sextet, and in the third quark SU(3) triplet. For further details concerning definition and physical meaning of all the involved parameters see [1].

The modified/improved version of GMO relation obtained in the LTK diquark-quark model is

$$\tfrac{3}{2}m_\Lambda+\tfrac{1}{2}m_\Sigma-m_N-m_\Xi=\mathcal{C}_{\rm LTK}\equiv\mu_s\big(3\xi_{ts}^2+18\xi_{ts}-13\big) \quad (4)$$

where for convenience we set

$$\mu_s=V_0\gamma_s^2, \qquad \xi_{ts}\equiv\gamma_t/\gamma_s. \quad (5)$$

The quantity μ_s must be positive since it can also be inferred by using the decuplet mass combination [1]: $8\mu_s=2m_{\Xi^*}-m_\Omega-m_{\Sigma^*}>0$. From the viewpoint of agreement with data, it is clear that there exists a continuum of values for the pair (μ_s,ξ_{ts}), determined in the $\mu_s-\xi_{ts}$ plane by the curve $\mu_s\big(3\xi_{ts}^2+18\xi_{ts}-$ $-13\big)=$ const, which provide the agreement of eq.(4) with data. To reduce maximally such sort of non-uniqueness, one needs some additional criteria. To this end, LTK exploited the mass relation for decuplet baryons. Namely, taking the ratio of decuplet mass combination to the octet one, then maximizing its r.h.s. as a function of ξ_{ts}, they inferred the value $\underline{\xi_{ts}=-3}$ (remark, it is not clear why one has to just maximize).

Negative sign of the solution $\xi_{ts}=-3$ implies that the mass difference δ_t must be greater (less) than δ_q when the mass difference δ_s is less (greater) than δ_q. However, this value $\xi_{ts}=-3$ is in conflict with empirics: it supplies negative value to the r.h.s. of (4) thus providing the correction to GMO *in wrong direction.*

2. Quantum-Group Based Baryon Mass Relation

Now let us turn over to another modification of the GMO mass sum rule, namely the q-deformed mass relation obtained, using the quantum algebras $U_q(\mathrm{su}_n)$ taken for flavor symmetries, in the form [9, 10, 5]

$$[2]M_N+\frac{[2]M_\Xi}{[2]-1}=[3]M_\Lambda+\Big(\frac{[2]^2}{[2]-1}-[3]\Big)M_\Sigma+$$
$$+\frac{A_q}{B_q}\left(M_\Xi+[2]M_N-[2]M_\Sigma-M_\Lambda\right) \quad (6)$$

with $[3]\equiv[3]_q=([2]_q)^2-1$, and A_q, B_q being certain polynomials in $[2]\equiv[2]_q=q+q^{-1}$ whose sets of zeros are completely different. This q-analog yields, as particular cases, the familiar Gell-Mann—Okubo relation [11] $M_N+M_\Xi=\frac{3}{2}M_\Lambda+\frac{1}{2}M_\Sigma$ (known to hold with the 0.58% accuracy) at the 'classical' value $q=1$, and the whole infinite series of new mass sum rules [5, 7, 9]

$$m_N+\frac{1}{[2]_{q_n}-1}m_\Xi=\frac{[3]_{q_n}}{[2]_{q_n}}m_\Lambda+\Big(\frac{[2]_{q_n}}{[2]_{q_n}-1}-\frac{[3]_{q_n}}{[2]_{q_n}}\Big)m_\Sigma,$$
$$6\le n<\infty, \quad (7)$$

where $q_n=\exp(i\pi/n)$. It should be mentioned that for each such value q_n the respective sum rule shows better agreement with data than GMO one.

The phenomenologically most plausible mass relation among those contained in the series (7), namely

$$M_N+M_\Xi/([2]_{q_7}-1)=M_\Lambda/([2]_{q_7}-1)+M_\Sigma, \quad (8)$$

shows the remarkable 0.07% accuracy. This most accurate mass sum rule (8) corresponds to the value $q_7=e^{\mathrm{i}\pi/7}$, for which a clear physical meaning has been given in ref. [9, 12] where the value q_7 was directly linked to the Cabibbo angle, i.e. $\frac{1}{i}\ln q_7=\frac{\pi}{7}{=}2\theta_C$.

Now we present the q-deformed mass relation (8) in the form of GMO combination with a correction to it:

$$\tfrac{3}{2}m_\Lambda+\tfrac{1}{2}m_\Sigma-m_N-m_\Xi=\mathcal{C}_{q_7}, \quad (9)$$

$$\mathcal{C}_{q_7}\equiv\left(([2]_7-1)^{-1}-1\right)(m_\Xi-m_\Lambda)-\tfrac{1}{2}(m_\Sigma-m_\Lambda).$$

Formulae (6)-(8) encode *highly nonlinear dependence* of mass on SU(3)-breaking. This makes them radically different from the classical GMO result accounting only first order effects in SU(3)-breaking.

Such *nonpolynomiality* in SU(3)-breaking effectively accounted by the quantum-group based model in the case of octet baryon masses, was demonstrated in [7]. For this goal, the explicit dependence on hypercharge Y and isospin I of matrix elements for isoplet masses is to be analyzed. Typical contribution to octet baryon mass contains such terms as, e.g., $([Y/2]_q[Y/2{+}1]_q-[I]_q[I{+}1]_q)$ or $([Y/2-1]_q[Y/2-2]_q-[I]_q[I+1]_q)$, with multipliers depending on the labels m_{15},m_{55} of a chosen dynamical representation. This shows explicit dependence on hypercharge and the factor $[I]_q[I+1]_q$ (q-deformed SU(2) Casimir). Since the q-bracket $[n]_q=\frac{\sin(nh)}{\sin(h)}$ if $q=\exp(\mathrm{i}h)$, baryon masses depend on Y and I (that is, on SU(3)-breaking effects) in highly nonlinear — *nonpolynomial* — fashion. The ability to account highly nonlinear SU(3)-breaking effects, due to the use of the quantum counterpart $U_q(\mathrm{su}_n)$ of usual flavor symmetries, is analogous to the result [13] that by exploiting *appropriate free* q-deformed structure one is able to efficiently describe the properties of (undeformed) quantum-mechanical system with complicated interaction.

3. Diquark-Quark Model Parameters and q-parameter

Now let us connect the results (4) and (9) of two different approaches. To this end, we form from the mass differences $m_\Xi - m_\Lambda$ (multipled with some w) and $m_\Sigma - m_\Lambda$ from (1)-(3), the particular combination involved in the r.h.s. $\mathcal{C}_{q_7}$ of (9) and equate it to the r.h.s. of (4). Then, imposing

$$4w\ \mu_s\ (\xi_{ts}^2 - 6\xi_{ts} + 5) = \mu_s\ (5\xi_{ts}^2 + 18\xi_{ts} - 15), \qquad (10)$$

$$w\big[\tfrac{1}{3}(\delta_t + \delta_s + \delta_q) - v_8 +$$

$$+2(\gamma_s - \gamma_t)(m_s - m_t + v_s - v_t)\big] + \tfrac{1}{6}(\delta_s - \delta_t) +$$

$$+(\gamma_s + \gamma_t)(m_s - m_t + v_s - v_t) = 0, \qquad (11)$$

with some w, guarantees validity of eq.(4) and its correspondence with eq.(9). The solution of (10), namely

$$w = -1 + 4(\xi_{ts}^2 - 6\xi_{ts} + 5)/(9\xi_{ts}^2 - 6\xi_{ts} + 5) \qquad (12)$$

put in (11) gives a particular constraint on the parameters of the LTK quark-diquark model.

Finally, we gain the explicit relation between the (value $q_7 = \exp(i\pi/7)$ of) q-parameter in our q-GMO and the ratio $\xi_{ts} = \gamma_t/\gamma_s$ of LTK quark-diquark model:

$$([2]_7 - 1) = \frac{4(\xi_{ts}^2 - 6\xi_{ts} + 5)}{9\xi_{ts}^2 - 6\xi_{ts} + 5}. \qquad (13)$$

Since $[2]_7 = 2\cos\frac{\pi}{7} = 1.80194$, the obtained relation yields as its solutions the two values $\xi_{ts}^{(+)} \approx 0.741$ and $\xi_{ts}^{(-)} \approx -6.705$. By the very derivation, both these values guarantee the validity of sum rule (4) to within 0.07%. The both values differ substantially from that adopted by LTK and *provide positive correction to* GMO, see the comment in sec.1 on negative value of ξ_{ts}. Moreover, our positive value ξ_{ts}^{+} reflects (even qualitatively) different (from the case of ξ_{ts}^{-}) physical situation. This value implies that the mass difference δ_t is greater (less) than δ_q at the same time as the mass difference δ_s is greater (less) than δ_q, since

$$\delta_t - \delta_q = 0.741(\delta_s - \delta_q),$$

i.e.,

$$\delta_t - \delta_s = 0.259(\delta_q - \delta_s).$$

Thus, it should necessarily be either

$$\delta_q > \delta_t > \delta_s \qquad \text{or} \qquad \delta_s > \delta_t > \delta_q. \qquad (14)$$

The established inequalities give another perspective on the interrelation between the parameters involved in the LTK diquark-quark model.

4. Linking LTK Parameters to Cabibbo Angle

Now let us discuss the already mentioned connection between the q-parameter and the Cabibbo angle. As it was shown in [14], the *weak mixing is adequately modelled by the q-deformation.* On the other hand, there is the important relation $\theta_W = 2(\theta_{12} + \theta_{23} + \theta_{13})$, found in [15], which connects θ_W with the Cabibbo angle $\theta_{12} \equiv \theta_C$ (and the θ_{13}, θ_{23} of mixing with 3rd family). This relates the mixing in *bosonic* (interaction) with that in *fermionic* (matter) sectors of the electroweak model. Combined with (8) this implies: the Cabibbo angle can be linked to q-parameter of a quantum-group (or q-algebra) based structure *applied in the fermion sector.* We thus infer (see [5, 12])

$$\theta_{\mathbf{8}} = \frac{\pi}{7} = 2\ \theta_C. \qquad (15)$$

The latter formula suggests for Cabibbo angle the exact value $\frac{\pi}{14}$. (Remark that for the q-deformed analog of decuplet mass formula [16, 5], we have $\theta_{\mathbf{10}} = \theta_C$.)

As a final result of this note we infer the following corollary concerning *direct link of the Cabibbo angle to the (optimized) parameters of the diquark-quark model.*

Indeed, using $\theta_{\mathbf{8}} = 2\theta_{\mathrm{C}}$ as given in (15), from the relation (13) we obtain the formula under question, i.e.,

$$\cos 2\theta_{\mathrm{C}} = \frac{13\tilde{\xi}_{ts}^2 - 30\tilde{\xi}_{ts} + 25}{2(9\tilde{\xi}_{ts}^2 - 6\tilde{\xi}_{ts} + 5)}, \qquad \tilde{\xi}_{ts} \equiv \frac{\tilde{\delta}_t - \tilde{\delta}_q}{\tilde{\delta}_s - \tilde{\delta}_q}. \qquad (16)$$

This remarkable formula connects the Cabibbo angle with the (mass) parameters of the LTK diquark-quark model. It should be stressed that the tilda over δ_t, δ_s and δ_q in the relation (16) means that now *these values are the optimized ones corresponding to all-order account of* SU(3) symmetry breaking in octet baryon masses.

5. Diquark-Quark Model Parameters and q-parameter: General Case

In this section we demonstrate that, in principle, the connection with the result (4) of LTK model *can be established in the general case* of an arbitrary member from the infinite discrete set presented in (7).

Indeed, the eq.(7) can be rewritten as

$$\tfrac{3}{2}m_\Lambda + \tfrac{1}{2}m_\Sigma - m_N - m_\Xi = \Big(\frac{[3]_{q_n}}{[2]_{q_n}} - \frac{1}{2}\Big)(m_\Sigma - m_\Lambda) + $$

$$+\frac{1}{[2]_{q_n}-1}(m_\Xi - m_\Sigma) - m_\Xi + m_\Lambda. \qquad (7')$$

Take mass differences from (1)—(3). Multiply $m_\Xi - m_\Sigma$ with some x^{-1}, and $m_\Sigma - m_\Lambda$ with definite[1]

$$y = \frac{x(x+2)}{x+1} - \frac{1}{2},$$

then form the specific combination that corresponds to the r.h.s. of (7′) and equate it to the r.h.s. of (4). The outlined procedure gives that the relation

$$4\Big(\frac{x(x+2)}{x+1} - \frac{1}{2}\Big)\left(\xi_{ts}^2 - 1\right) - 24x^{-1}(\xi_{ts} - 1) =$$

$$= 7\xi_{ts}^2 - 6\xi_{ts} + 7 \qquad (17)$$

and the additional constraint

$$\Big(\frac{x(x+2)}{x+1} - \frac{1}{2}\Big)\Big\{\frac{1}{3}(\delta_t - \delta_s) - 4\mu_s\zeta(1+\xi_{ts})\Big\} +$$

$$+x^{-1}\Big\{\frac{1}{3}(2\delta_s - \delta_q) - v_8 + 8\mu_s\zeta\Big\} -$$

$$-\frac{1}{3}(\delta_t + \delta_s + \delta_q) + v_8 + 4\mu_s\zeta(\xi_{ts} - 1) = 0 \qquad (18)$$

with ξ_{ts} given in (5), when hold simultaneously, provide validity of eq.(4). Moreover, the latter is directly correspondent with the eq.(7′) if the identification

$$x \longleftrightarrow [2]_{q_n} - 1$$

is made. Not going into further details, let us only mention that only at $q = \exp(i\pi/7)$ (i.e., in the case most adequate phenomenologically) we have the equality

$$[3]_{q_7}/[2]_{q_7} = ([2]_{q_7} - 1)^{-1}$$

or, put in another way, $\frac{x(x+2)}{x+1} = x^{-1}$. As a result, we recover the relations (12)—(13) of the distinguished particular case considered above.

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Received 27.12.02

ПРО ЗВ'ЯЗОК ПАРАМЕТРІВ ДИКВАРК-КВАРКОВОЇ МОДЕЛІ З КУТОМ КАБІББО

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Р е з ю м е

Виходячи з двох різних модифікацій відомої мас-формули Гелл-Манна і Окубо для баріонів $(\frac{1}{2})^+$ (однієї, виведеної в рамках деякої версії дикварк-кваркової моделі, і другої, яка є оптимальним правилом сум для мас, знайденим на основі квантових (q-деформованих) аналогів алгебр унітарних груп у ролі адронних ароматових симетрій) отримано нове співвідношення, яке пов'язує мас-параметри дикварк-кваркової моделі із значенням параметра деформації q і, тим самим, із кутом Кабіббо.

[1]Just this explicit dependence of y on x is motivated by the fact that, in the sequel, x is to be juxtaposed with $[2]_q - 1$, while the q-number $[3]_q$ is expressible in terms of $[2]_q$ as $[3]_q = ([2]_q + 1)([2]_q - 1)$.

О СВЯЗИ ПАРАМЕТРОВ ДИКВАРК-КВАРКОВОЙ МОДЕЛИ С УГЛОМ КАБИББО

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Р е з ю м е

Исходя из двух различных модификаций известной масс-формулы Гелл-Манна и Окубо для барионов $(\frac{1}{2})^+$ (одна из которых выведена в рамках некоторой версии дикварк-кварковой модели, а другая является оптимальным правилом сумм для масс, найденным на основе квантовых или q-деформированных аналогов алгебр унитарных групп в роли адронных ароматовых симметрий), получено новое соотношение, которое связывает масс-параметры дикварк-кварковой модели со значением параметра деформации q и, тем самым, с углом Кабиббо.