# EVALUATION OF PARTICLE/HOLE PROPAGATORS BY THE TSERKOVNIKOV'S METHOD

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By particle/hole propagator ($f_t/g_t$ for short), we imply the statistically averaged product of particle's creation/annihilation operator by its conjugate partner separated from it by the time interval $t$. As $f_t$ and $g_t$ are intimately linked to transport coefficients, their form at finite residual interactions is an issue of the utmost physical import. In contrast to the previous studies, having heuristic character, we find the desired expressions for $f_t$ and $g_t$ from the appropriate dynamic equations by means of a method devised by Yu.A. Tserkovnikov to solve the chains of equations for two-time Green's functions. Friction coefficients of a slowly evolving nuclear shape associated with the propagators obtained in this way are compared with heuristic expressions.

## Introduction

The main objects of this paper are the particle propagator and hole propagator defined correspondingly as follows

$$f_t = \langle a_{qt} a_q^+ \rangle, \quad g_t = \langle a_{qt}^+ a_q \rangle. \tag{1}$$

Here, $a_q^+$, $a_q$ are the creation and annihilation operators of nucleons in the state with the linear momentum $q$=$(q_1, q_2, q_3)$. The subscript $t$ near $A$= $a_q$, $a_q^+$ (or any other operator in the following) implies its Heisenberg representation

$$A_t = \mathrm{e}^{iHt} A \mathrm{e}^{-iHt}, \tag{2}$$

where $H$ is the Hamiltonian of the system. The angular brackets denote the averaging with the statistical operator: $\langle ... \rangle = \mathrm{Sp}(\rho ...)$, where

$$\rho = Z^{-1} \exp\left(-\beta(H - \mu N)\right),$$

$$Z = \mathrm{Sp} \exp\left(-\beta(H - \mu N)\right),$$

$N$ is the operator of the number of particles, $\mu$ is the chemical potential, $\beta = 1/T$ and $T$ is the temperature.

The usefulness of $f_t$ and $g_t$ is determined by the fact that they enter the microscopic formulas for dissipative functions of slow collective motion which should eventually replace the wall and window formulas [1] in the computer codes [2] used in the current analyses of nucleus-nucleus collisions at the beam energies below 10-20 MeV/u (see [3] for review).

According to Ref.[4], the propagators $f_t$ and $g_t$ may be presented as

$$f_t = \frac{1}{2\pi} \int d\omega \mathrm{e}^{-i\omega t} \bar{n}(\omega) A_q(\omega),$$

$$g_t = \frac{1}{2\pi} \int d\omega \mathrm{e}^{i\omega t} n(\omega) A_q(\omega), \tag{3}$$

where (we take $\hbar = 1$ throughout the paper)

$$n(\omega) = \frac{1}{1 + \exp[\beta(\omega - \mu)]}, \quad \bar{n}(\omega) = 1 - n(\omega), \tag{4}$$

while $A_q(\omega)$ is defined by the discontinuity of a chronological Green's function $G_q(\omega)$ across the real axis, $A_q(\omega) = i[G_q(\omega + i0) - G_q(\omega - i0)]$. Note that $A_q(\omega) = 2\pi\delta(\omega - \varepsilon_q)$ at negligable residual interactions, where $\varepsilon_q$ are the energy levels in the mean field potential.

A series of papers on dissipative functions summarized in [5] rests upon quantities, let us call them

$f_t^{\rm L}$ and $g_t^{\rm L}$, defined by Eqs.(3) with the Lorentzian shape for $A_q(\omega)$:

$$A_q^{\rm L}(\omega) = \frac{\Gamma}{(\omega - \varepsilon_q)^2 + \frac{1}{4}\Gamma^2}. \quad (5)$$

Here, $\Gamma$ is the spreading width obtained from the imaginary part $W$ of the optical potential: $\Gamma = -2W$ (see [6, 7] for details). In practice, $f_t^{\rm L}$ and $g_t^{\rm L}$ are obtained by contour integration over $\omega$. This gives (at $t \neq 0$):

$$f_t^{\rm L} = \bar{n}(\varepsilon_q - is\Gamma/2)\mathrm{e}^{-i\varepsilon_q t - \frac{\Gamma}{2}|t|} + \Sigma_t,$$

$$g_t^{\rm L} = n(\varepsilon_q + is\Gamma/2)\mathrm{e}^{i\varepsilon_q t - \frac{\Gamma}{2}|t|} - \Sigma_t^*, \quad (6)$$

where $s = t/|t|$ and $\Sigma_t$ is the sum of the terms $\sim \exp(i\mu t - 2\pi T(j+\frac{1}{2})|t|)$ over $j = 0,1,2,..\infty$, arising from the poles of $n(\omega)$ in the complex plane.

Critical assessment of the quantities $f_t^{\rm L}$ and $g_t^{\rm L}$ shows that they cannot be used, in fact, to model the true propagators, even in a crude approximation. Firstly, the equalities $f_0^{\rm L} = \frac{1}{2\pi}\int d\omega \bar{n} A_q^{\rm L}$ and $g_0^{\rm L} = \frac{1}{2\pi}\int d\omega n A_q^{\rm L}$ are in conflict with the initial conditions $f_0 = \langle a_q a_q^+\rangle$, $g_0 = \langle a_q^+ a_q\rangle$ which follow from Eq.(1). This is seen most easily in the case of a Fermi gas, when $\langle a_q a_q^+\rangle = \bar{n}(\varepsilon_q)$ and $\langle a_q^+ a_q\rangle = n(\varepsilon_q)$. Secondly, at $|t| \gg \tau_{\rm c}$, where $\tau_{\rm c}$ is the correlation time characteristic of a given problem, $|f_t|^2$ and $|g_t|^2$ must decay exponentially, namely, as $\exp(-\Gamma|t|)$. This follows from the general theory of thermodynamic two-time correlation functions [8], whose special case is represented by $f_t$ and $g_t$. Taking into account that $\tau_{\rm c}$ in our problem is given by the ratio of the two-particle interaction radius to the Fermi velocity, it is easy to show that $f_t^{\rm L}$ and $g_t^{\rm L}$ from Eqs.(6) violate this condition. Finally and mainly, as seen from [9] and will be shown later in this paper, the friction coefficients associated with $f_t^{\rm L}$, $g_t^{\rm L}$ tend to infinity as $1/\Gamma$ when $\Gamma \to 0$.

In this work we attempt a consistent, and physically plausible, description of particle/hole propagators. For this purpose we use the method proposed for decoupling the chains of equations for two-time Green's functions [10]. The general expressions for $f_t$ and $g_t$ obtained according to this method in Sec.1 are used in Sec.2 to derive explicit formulas for $f_t$ and $g_t$ valid at small residual interactions. In Sec.3, we study the short ($t \ll \tau_{\rm c}$) and long ($t \gg \tau_{\rm c}$) time limits for them. The implications of our results in terms of friction coefficients are discussed in the last section.

## 1. Tserkovnikov's Formalism

For definiteness, consider first the propagator of the particle $f_t = \left\langle a_{qt} a_q^+ \right\rangle$. There are different ways to derive the Tserkovnikov's representation for this quantity. According to the remark in [11], the most direct and clear one consists in taking, as a starting point, the following identity for $f_t$ :

$$f_t = f_0 \exp\left(\int_0^t dt' \frac{\dot{f}_{t'}}{f_{t'}}\right), \quad (7)$$

where the overdot denotes the time derivative.

Let the Hamiltonian of the system be given by

$$H = \sum_p T_p a_p^+ a_p + \frac{1}{2V}\sum_{pp'k} v_k a_p^+ a_{p'}^+ a_{p'+k} a_{p-k}, \quad (8)$$

where $T_p = \frac{p^2}{2m}$ is the kinetic energy of a particle, $v_k = \int dx \mathrm{e}^{i(kx)}\phi(x)$ is the Fourier transform of the two-particle interaction potential $\phi(x)$ having the symmetries

$$v_k^* = v_{-k} = v_k. \quad (9)$$

Then, using Eqs.(2),(8),(9), it is straightforward to show that

$$\dot{a}_{qt} = -iT_q a_{qt} - iJ_{qt}, \quad (10)$$

where $J_{qt}$ is the Heisenberg representation of the operator

$$J_q = \frac{1}{V}\sum_{pk} v_k a_p^+ a_{p-k} a_{q+k}. \quad (11)$$

Eq.(10) implies that $\dot{f}_t$ may be presented as

$$\dot{f}_t = -iT_q f_t - i\left\langle J_{qt} a_q^+ \right\rangle. \quad (12)$$

Inserting (12) into (7) yields

$$f_t = f_0 \exp\left\{-iT_q t - i\int_0^t dt' R_{t'}\right\}, \quad (13)$$

where

$$R_t = \frac{\left\langle J_{qt} a_q^+ \right\rangle}{\left\langle a_{qt} a_q^+ \right\rangle}. \quad (14)$$

In order to extract the Hartree-Fock potential from the time integral in Eq.(13), we use the identity

$$\int_0^t dt' R_{t'} = tR_0 - \int_0^t \tau d\tau \frac{d}{d\tau} R_{t-\tau}.$$

To find $dR_{t-\tau}/d\tau$, we use the formula $R_{t-\tau} = \left\langle J_{qt} a_{q\tau}^+ \right\rangle / \left\langle a_{qt} a_{q\tau}^+ \right\rangle$ following from Eq.(14) on accounting for the invariance of the trace under cyclic permutations of factors and the commutativity of $\rho$ with $H$. In combination with Eq.(10), this leads to the following expression for the particle propagator:

$$f_t = \langle a_{qt} a_q^+ \rangle = \langle a_q a_q^+ \rangle \exp\left(-iE_t t\right), \tag{15}$$

where

$$E_t = T_q + \frac{\left\langle J_q a_q^+ \right\rangle}{\left\langle a_q a_q^+ \right\rangle} -$$

$$-\frac{i}{t}\int_0^t \tau d\tau \left( \frac{\left\langle J_{qt} J_{q\tau}^+ \right\rangle}{\left\langle a_{qt} a_{q\tau}^+ \right\rangle} - \frac{\left\langle J_{qt} a_{q\tau}^+ \right\rangle \left\langle a_{qt} J_{q\tau}^+ \right\rangle}{\left\langle a_{qt} a_{q\tau}^+ \right\rangle^2} \right). \tag{16}$$

Proceeding in a completely similar manner, we can present the hole propagator as

$$g_t = \langle a_{qt}^+ a_q \rangle = \langle a_q^+ a_q \rangle \exp\left(itE_t^g\right), \tag{17}$$

where

$$E_t^g = T_q + \frac{\left\langle J_q^+ a_q \right\rangle}{\left\langle a_q^+ a_q \right\rangle} +$$

$$+\frac{i}{t}\int_0^t \tau d\tau \left( \frac{\left\langle J_{qt}^+ J_{q\tau} \right\rangle}{\left\langle a_{qt}^+ a_{q\tau} \right\rangle} - \frac{\left\langle J_{qt}^+ a_{q\tau} \right\rangle \left\langle a_{qt}^+ J_{q\tau} \right\rangle}{\left\langle a_{qt}^+ a_{q\tau} \right\rangle^2} \right). \tag{18}$$

## 2. Perturbation Theory

In this section, we evaluate $f_t$ and $g_t$ using the Hartree-Fock Hamiltonian $H_{\rm HF}$ instead of $H$ in calculating the Heisenberg representations and the statistical averages of the products of the creation and annihilation operators in Eqs.(15),(16) and (17),(18). This means that the residual interaction, $V_{\rm res} = H - H_{\rm HF}$, is treated as perturbation.

The explicit form of the Hartree-Fock Hamiltonian in our problem is

$$H_{\rm HF} = \sum_q \varepsilon_q a_q^+ a_q, \tag{19}$$

where

$$\varepsilon_q = T_q + \frac{1}{V}\sum_p (v_0 - v_{p-q}) n_p, \tag{20}$$

$$n_p = \frac{1}{1+{\rm e}^{x_p}}, \quad x_p = \frac{\varepsilon_p - \mu}{T}. \tag{21}$$

The Heisenberg representations of $a_q$ and $a_q^+$ at $H = H_{\rm HF}$ become

$$a_{qt} = {\rm e}^{-i\varepsilon_q t} a_q, \quad a_{qt}^+ = {\rm e}^{i\varepsilon_q t} a_q^+ \tag{22}$$

and the basic statistical brackets are given by

$$\left\langle a_q^+ a_{q'} \right\rangle = n_q \delta_{qq'}, \quad \left\langle a_q a_{q'}^+ \right\rangle = \bar{n}_q \delta_{qq'},$$

$$\left\langle a_q^+ a_{q'}^+ \right\rangle = \langle a_q a_{q'} \rangle = 0, \tag{23}$$

where

$$\bar{n}_q = 1 - n_q = \frac{1}{{\rm e}^{-x_q}+1}. \tag{24}$$

The calculation of the statistical brackets for a product of even number, $s$, of the creation and annihilation operators is performed using the Bloch—De Dominicis decomposition [12]:

$$\langle \alpha_1 \alpha_2 \alpha_3 \alpha_4 ... \alpha_s \rangle = \langle \alpha_1 \alpha_2 \rangle \langle \alpha_3 \alpha_4 ... \alpha_s \rangle -$$

$$- \langle \alpha_1 \alpha_3 \rangle \langle \alpha_2 \alpha_4 ... \alpha_s \rangle + ... + \langle \alpha_1 \alpha_s \rangle \langle \alpha_2 \alpha_3 \alpha_4 ... \rangle, \tag{25}$$

where $\alpha_i$ with $i = 1, 2, ..., s$, are the creation or annihilation operators.

The application of Eqs.(22),(23),(25) to Eqs.(15),(16) and (17),(18) leads to the following results. For the particle propagator, we obtain

$$f_t = \left\langle a_{qt} a_q^+ \right\rangle = \bar{n}_q \exp(-iE_t t), \tag{26}$$

$$E_t = \varepsilon_q + \frac{1}{V^2}\sum_{pk} v_k (v_k - v_{p-q-k}) \times$$

$$\times \frac{n_p \bar{n}_{p-k} \bar{n}_{q+k}}{\bar{n}_q} I_t(\varepsilon_{p,q;k}), \tag{27}$$

where $\varepsilon_{p,q;k} = \varepsilon_p + \varepsilon_q - \varepsilon_{p-k} - \varepsilon_{q+k}$ and

$$I_t(\varepsilon) = -\frac{i}{t}\int_0^t t' dt' {\rm e}^{i\varepsilon(t-t')}. \tag{28}$$

For the hole propagator, we find

$$g_t = \left\langle a_{qt}^+ a_q \right\rangle = n_q \exp(iE_t^g t), \tag{29}$$

$$E_t^g = \varepsilon_q + \frac{1}{V^2}\sum_{pk} v_k (v_k - v_{p-q-k}) \times$$

$$\times \frac{\bar{n}_p n_{p-k} n_{q+k}}{n_q} I_t^*(\varepsilon_{p,q;k}). \tag{30}$$

Using the explicit expressions (21),(24) for occupation numbers, it is easy to prove the identities

$$\frac{n_p \bar{n}_{p-k} \bar{n}_{q+k}}{\bar{n}_q} = n_p \bar{n}_{p-k} \bar{n}_{q+k} +$$

$$+ \exp\left(-\frac{\varepsilon_{p,q;k}}{T}\right) \bar{n}_p n_{p-k} n_{q+k}, \tag{31}$$

$$\frac{\bar{n}_p n_{p-k} n_{q+k}}{n_q} = \bar{n}_p n_{p-k} n_{q+k} +$$

$$+ \exp\left(\frac{\varepsilon_{p,q;k}}{T}\right) n_p \bar{n}_{p-k} \bar{n}_{q+k}, \tag{32}$$

which may be inserted in Eqs.(27) and (30), respectively.

## 3. Limiting Cases

Depending on the position of $\varepsilon_p$ and $\varepsilon_q$ relative to the Fermi energy, the deviation of the quantity $\varepsilon_{p,q;k}$ from zero characterizes the magnitude of p-p, h-h or p-h correlations owing to residual interactions. The characteristic value $\varepsilon_c$ of $\varepsilon_{p,q;k}$ can be estimated from the relation $\varepsilon_c = 1/\tau_c$, where $\tau_c$ is the dynamic correlation time given in our problem by the ratio of the nucleon-nucleon interaction radius to the Fermi velocity.

Let us consider $I_t(\varepsilon)$ in the short and long time regimes, that is, at $|t| \ll \tau_c$ and $|t| \gg \tau_c$. Making use of the explicit form of this function,

$$I_t(\varepsilon) = \frac{1}{\varepsilon}\left(1 - \frac{\sin \varepsilon t}{\varepsilon t}\right) - i\frac{1-\cos\varepsilon t}{\varepsilon^2 t},$$

we find that $I_t(\varepsilon) \to -it/2$ for small times. To find the large time limit of $I_t(\varepsilon)$, it is better to return to its integral representation (28). Taking into account that, at large $t$, this integral is determined by contributions at $t' \approx t$, because otherwise the exponent oscillates strongly, we conclude that, in the $|t| \to \infty$ limit,

$$I_t(\varepsilon) \to -i\int_0^t dt' \mathrm{e}^{i\varepsilon(t-t')} =$$

$$= -i\int_0^t d\tau \mathrm{e}^{i\varepsilon\tau} \to \frac{P}{\varepsilon} - i\frac{t}{|t|}\pi\delta(\varepsilon),$$

where $P$ stands for the principal value of the integral.

These properties of $I_t(\varepsilon)$ lead to the following expressions for the particle propagator. If $|t| \ll \tau_c$, then

$$\langle a_{qt} a_q^+\rangle \cong \bar{n}_q \exp\left(-i\varepsilon_q t - \sigma_q t^2\right), \tag{33}$$

where

$$\sigma_q = \frac{1}{2V^2}\sum_{pk} v_k(v_k - v_{p-q-k})\frac{n_p\bar{n}_{p-k}\bar{n}_{q+k}}{\bar{n}_q}. \tag{34}$$

If $|t| \gg \tau_c$, then

$$\langle a_{qt} a_q^+\rangle \cong \bar{n}_q \exp\left(-i\tilde{\varepsilon}_q t - \frac{\Gamma_q}{2}|t|\right), \tag{35}$$

where

$$\tilde{\varepsilon}_q = \varepsilon_q + P\frac{1}{V^2}\sum_{pk} v_k(v_k - v_{p-q-k})\times$$

$$\times\frac{1}{\varepsilon_{p,q;k}}\frac{n_p\bar{n}_{p-k}\bar{n}_{q+k}}{\bar{n}_q}, \tag{36}$$

$$\Gamma_q = \frac{2\pi}{V^2}\sum_{pk} v_k(v_k - v_{p-q-k})\delta(\varepsilon_{p,q;k})\times$$

$$\times(n_p\bar{n}_{p-k}\bar{n}_{q+k} + \bar{n}_p n_{p-k} n_{q+k}). \tag{37}$$

Note that Eq.(37) coincides with the expressions in [4, 13] and with the damping of the advanced Green's function obtained in [10].

The limiting expressions for the hole propagator are as follows. If $|t| \ll \tau_c$, then

$$\langle a_{qt}^+ a_q\rangle \cong n_q \exp\left(i\varepsilon_q t - \sigma_q^g t^2\right), \tag{38}$$

where

$$\sigma_q^g = \frac{1}{2V^2}\sum_{pk} v_k(v_k - v_{p-q-k})\frac{\bar{n}_p n_{p-k} n_{q+k}}{n_q}. \tag{39}$$

If $|t| \gg \tau_c$, then

$$\langle a_{qt}^+ a_q\rangle \cong n_q \exp\left(i\tilde{\varepsilon}_q^g t - \frac{\Gamma_q}{2}|t|\right), \tag{40}$$

where

$$\tilde{\varepsilon}_q^g = \varepsilon_q + P\frac{1}{V^2}\sum_{pk} v_k(v_k - v_{p-q-k})\times$$

$$\times\frac{1}{\varepsilon_{p,q;k}}\frac{\bar{n}_p n_{p-k} n_{q+k}}{n_q}. \tag{41}$$

## 4. Discussion and Conclusion

To illustrate the implications of our results, consider the rate $\dot{Q}$ at which collective energy converts into thermal energy in a hot nucleus, whose shape (specified by the parameter $\alpha$) evolves adiabatically with time. The adiabaticity implies that the system passes through the sequence of states of partial thermal equilibrium (the equilibrium at fixed $\alpha$). This makes it meaningful to introduce, for each value of $\alpha$, the eigenstates $|\mu\rangle$ and eigenvalues $E_\mu$ of the single-particle Hamiltonian $h = -\nabla^2/2m + V$, the spreading $\Gamma_\mu$ of $E_\mu$ being due to (presumably small) residual interactions, and to characterize the nuclear states by Fermi-gas occupation numbers $n_\mu$. Using the methods of non-equilibrium thermodynamics, one can show that $\dot{Q} = \gamma\dot{\alpha}^2$, where $\dot{\alpha}$ is the collective velocity and

$$\gamma = 2\pi\sum_{nn'} \rho_n |\langle n|\mathcal{F}|n'\rangle|^2\delta'(\Omega_n - \Omega_{n'}) \tag{42}$$

is the friction coefficient (e.g., see [14]). Here, $|n\rangle$ and $\Omega_n$ are the eigenstates and eigenvalues of the many-body Hamiltonian $H$ at a frozen shape, $\rho_n = \langle n|\rho|n\rangle$ is the statistical operator in the energy representation,

$\delta'(x) = d\delta(x)/dx$, $\mathcal{F} = \sum_{\mu\nu} F_{\mu\nu} a^+_\mu a_\nu$, $F_{\mu\nu}$ are the single-particle matrix elements of $F = \partial V/\partial\alpha$.

The above expression for $\gamma$ can be rewritten as

$$\gamma = \lim_{\omega\to+0} \frac{d}{d\omega} \sum_{\mu\nu} |F_{\mu\nu}|^2 \times$$

$$\times \int_{-\infty}^{\infty} dt \exp(i\omega t) \left\langle a^+_{\mu t} a_{\nu t} a^+_\nu a_\mu \right\rangle . \quad (43)$$

The time interval, contributing to (43), is determined by the life time, $\tau_{\rm ph}$, of a particle-hole excitation which should be close to $1/\Gamma$. At reasonable temperatures ( $T < 3 - 4$ MeV) this quantity essentially exceeds the p-h correlation time $\tau_{\rm c}$. In this case, the particle-hole propagator $\langle a^+_{\mu t} a_{\nu t} a^+_\nu a_\mu \rangle$ may be replaced by the product $\langle a_{\nu t} a^+_\nu \rangle \langle a^+_{\mu t} a_\mu \rangle$ of the particle propagator by the hole propagator; the exchange term $\langle a^+_{\mu t} a_{\nu t} \rangle \langle a^+_\nu a_\mu \rangle = \langle a^+_\mu a_\nu \rangle \langle a^+_\nu a_\mu \rangle$ does not depend on time and therefore makes no contribution to $\gamma$ at finite $\omega$. Taking into account that the particle/hole propagators at $|t| \gg \tau_c$ are described by Eqs.(35), (40), we obtain

$$\gamma = -\sum_{\mu\nu} (n_\mu - n_\nu) |F_{\mu\nu}|^2 \frac{2(E_\mu - E_\nu)\Gamma}{[(E_\mu - E_\nu)^2 + \Gamma^2]^2}, \quad (44)$$

where $2\Gamma = \Gamma_\mu + \Gamma_\nu$.

Eq.(44) looks similar to the expression in [15], in which, however, one finds the parameter $\eta \approx 1/\tau_{\rm r}$ in place of $\Gamma$, where $\tau_{\rm r}$ is the duration time of the reaction. Moreover, Eq.(44) coincides with one of two heuristic formulas for $\gamma$ studied numerically in [16] and with the formula implied in [17] as a starting point in the derivation of the classical approximation for $\dot{Q}$. The heuristic formula in [18, 19] is also equivalent to Eq.(44) but accounts for pairing.

For the friction coefficient $\gamma^{\rm L}$ associated with $f^{\rm L}_t$, $g^{\rm L}_t$, the above procedure gives

$$\gamma^{\rm L} = \frac{1}{2\pi} \sum_{\mu\nu} |F_{\mu\nu}|^2 \int dx \int dy \delta'(y-x) \times$$

$$\times \bar{n}(x) n(y) A^{\rm L}_\nu(x) A^{\rm L}_\mu(y).$$

Since $|F_{\mu\nu}|^2$ is symmetric with respect to the permutation of $\mu$, $\nu$, the contribution to the sum is made by the symmetric component of $A^{\rm L}_\nu(x) A^{\rm L}_\mu(y)$:

$$\frac{1}{2}(A^{\rm L}_\nu(x) A^{\rm L}_\mu(y) + A^{\rm L}_\mu(x) A^{\rm L}_\nu(y)) \approx$$

$$\approx A^{\rm L}_\nu(E) A^{\rm L}_\mu(E),$$

where $E = (x + y)/2$. Noting that this quantity does not change under exchange of $x$ with $y$, while $\delta'(y - x)$ changes its sign, we replace $\bar{n}(x)n(y)$ with its antisymmetric component:

$$\frac{1}{2}[\bar{n}(x)n(y) - \bar{n}(y)n(x)] =$$

$$= \frac{1}{2}[n(y) - n(x)] \approx \frac{1}{2} n'(E)\omega,$$

where $\omega = y - x$. The approximate values above follow from the closeness between $y$ and $x$, which is granted by the factor $\delta'(y - x)$. Putting these estimates into the equation for $\gamma^{\rm L}$, making the change of variables from $x$, $y$ to $E = (x + y)/2$, $\omega = y - x$, integrating over $\omega$, and using (5) give

$$\gamma^{\rm L} = -\pi \int dE n'(E) \sum_{\mu\nu} |F_{\mu\nu}|^2 \times$$

$$\times \delta_{\Gamma_\mu}(E - E_\mu) \delta_{\Gamma_\nu}(E - E_\nu), \quad (45)$$

where $\delta_\Gamma(x) = \Gamma/[2\pi(x^2 + \frac{1}{4}\Gamma^2)]$ is the soft $\delta$-function.

Eq.(45) was first explicitly presented in [9]. In the limiting case $\Gamma \to 0$, $\gamma^{\rm L}$ exhibits a $1/\Gamma$ singularity. Indeed, since $\delta^2_\Gamma(E_\mu - E) \to (\pi\Gamma)^{-1} \delta(E_\mu - E)$ at $\Gamma \to 0$, the energy integral of the sum of terms with $E_\nu = E_\mu$ behaves in this limit like $1/\Gamma$. In practical applications [5, 9, 20, 21], the $E_\nu = E_\mu$ terms in $\gamma^{\rm L}$ have been left out. The regularized expression, let us call it $\gamma^{\rm L}_{\rm r}$, coincides at $\Gamma \to 0$ with the corresponding limit of (44). At finite $\Gamma$, however, $\gamma^{\rm L}_{\rm r}$ significantly differs from Eq.(44) (see [17] for details).

In summary, we found the shell-model expressions for particle/hole propagators and friction coefficients. This was done using Tserkovnikov's formalism and treating occupation numbers, particle/hole energies, and their spreading to the lowest non-vanishing order in the residual interaction. Incorporation of the residual interactions by means of replacing the $2\pi\delta(\omega - \varepsilon_q)$ expression for $A_q(\omega)$ with the Lorentzian function (5) is shown to lead to friction coefficients, which tend to infinity at $\Gamma \to 0$. From Eqs.(3),(5), (35), (40), it follows however that such a replacing results in the correct *long-time* expressions for $f_t$ and $g_t$ if in addition $\bar{n}(\omega)$ and $n(\omega)$ are replaced by their values at $\omega = \varepsilon_q$. It is worthwhile emphasizing that it is this 'combined' substitution that was used in [16, 18, 19].

According to the general analysis of two-time correlation functions [8], the time derivatives of the quantities $E_t$, $E_t^g$ from Eqs.(16), (18) vanish at $|t| \gg \tau_c$. This means, as seen from Eqs.(15), (17), that $f_t$, $g_t$ at $|t| \gg \tau_c$ will always have the form given by Eqs.(35), (40), although the formulas for average occupation numbers, single-particle excitation energies, and their spreading will generally differ from Eqs.(21), (36), (37), (41). For instance, by using the technique devised in [10] to deal with certain potentials not amenable to the perturbative treatment, one can express $\Gamma$ in terms of the $T$-matrix of particle-particle scattering in the medium. Thus, Eq.(44) for the friction coefficient is valid not only at small residual interactions but in all those cases when the condition $\Gamma^{-1} \gg \tau_c$ is fulfilled.

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## ОЦІНКА ПРОПАГАТОРІВ ЧАСТИНКИ ТА ДІРКИ ЗА МЕТОДОМ ЦЕРКОВНИКОВА

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Р е з ю м е

У даній роботі під пропагатором частинки/дірки $f_t/g_t$ маємо на увазі статистичне середнє добутку оператора народження/знищення частинки і спряженого оператора, відділеного від нього часовим інтервалом $t$. Оскільки $f_t$ і $g_t$ тісно зв'язані з кінетичними коефіцієнтами, їх форма при скінченній залишковій взаємодії має виняткове фізичне значення. На відміну від попередніх досліджень, які носять евристичний характер, ми знаходимо бажані вирази для $f_t$ і $g_t$ із відповідних динамічних рівнянь за допомогою запропонованого Ю.А. Церковниковим методу розв'язання ланцюжків рівнянь для двочасових температурних функцій Гріна. Коефіцієнти тертя при повільній зміні форми ядра, одержані із знайденими таким чином пропагаторами, ми порівнюємо з евристичними виразами.

## ОЦЕНКА ПРОПАГАТОРОВ ЧАСТИЦЫ И ДЫРКИ МЕТОДОМ ЦЕРКОВНИКОВА

*В.П. Алешин*

Р е з ю м е

В настоящей работе под пропагатором частицы/дырки $f_t/g_t$ подразумевается статистическое среднее произведения оператора рождения/уничтожения частицы на сопряженный ему оператор, отделенный от него временным интервалом $t$. Поскольку $f_t$ и $g_t$ тесно связаны с кинетическими коэффициентами, их форма при конечном остаточном взаимодействии имеет крайне важное физическое значение. В отличие от предыдущих исследований, имеющих эвристический характер, мы находим требуемые выражения для $f_t$ и $g_t$ из соответствующих динамических уравнений посредством предложенного Ю.А. Церковниковым метода решения цепочек уравнений для двухвременных температурных функций Грина. Коэффициенты трения при медленном изменении формы ядра, полученные с найденными таким образом пропагаторами, мы сравниваем с эвристическими выражениями.