
OFF-SHELL AND FACTORIZATION EFFECTS IN EXCLUSIVE VECTOR-MESON PRODUCTION

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It is shown that the universality of the initial and final state interactions responsible for the transition between the on- and off-mass shell states leads to the energy independence of the ratio of exclusive ρ electroproduction cross section to the total cross section. It is demonstrated that the above universality and explicit mass dependence of the exponent in the power-like energy behavior of the cross-section obtained in the approach based on unitarity is in a quantitative agreement with the high-energy HERA experimental data. We discuss also HERA results on angular distributions of vector-meson production.

independence was obtained in the approach based on the off-shell extension of the s -channel unitarity [8]. In this note, we provide an additional discussion pointing out to the origin of the above energy independence. We perform a quantitative comparison of the HERA data on vector-meson production with the results obtained in [8].

1. Vector-meson Electroproduction

Introduction

Exclusive vector meson production is an important process which can provide information on hadronic structure at large and small distances and nature of soft and hard interaction dynamics. As follows from the HERA data [1, 2], the integral cross section of the elastic vector meson production $\sigma_{\gamma^*p}^V(W^2, Q^2)$ increases with energy in a way similar to the $\sigma_{\gamma^*p}^{\text{tot}}(W^2, Q^2)$ dependence on W^2 [3]. It appeared also that the growth of the vector-meson electroproduction cross-section with energy is steeper for heavier vector mesons. Similar effect takes also place when the virtuality Q^2 increases.

The preliminary data of ZEUS Collaboration [4] provide an indication for an energy independence of the ratio of the cross section of exclusive ρ electroproduction to the total cross section. Such behavior of this ratio is at variance with perturbative QCD results [5], Regge and dipole approaches [4, 6]. A recent review of the related problems and successes of the various theoretical approaches can be found in [7].

Of course, the energy range of the available data is limited and the above-mentioned contradiction could probably be avoided due to a fine tuning of the appropriate models. Meanwhile, similar energy

There is no universal, generally accepted method to obey unitarity of the scattering matrix. However, long time ago, the arguments based on analytical properties of the scattering amplitude were put forward [9] in favor of the rational form of unitarization. Unitarity can be written for both the scattering amplitudes of real and virtual external particles. However, implications of unitarity are different for the scattering of real and virtual particles. The extension of the U -matrix unitarization scheme (rational form of unitarization) for the off-shell scattering was considered in [8]. It was supposed as usual that the virtual photon fluctuates into a quark-antiquark and this pair can be treated as an effective virtual vector meson state. There, limitations which the unitarity provides for the γ^*p -total cross-sections and geometrical effects in the energy dependence of $\sigma_{\gamma^*p}^{\text{tot}}$ were considered. It was shown that the solution of the extended unitarity is supplemented by an assumption of the Q^2 -dependent constituent quark interaction radius results in the following dependence at high energies:

$$\sigma_{\gamma^*p}^{\text{tot}} \sim (W^2)^\lambda \lambda(Q^2), \quad (1)$$

where $\lambda(Q^2)$ is saturated at large values of Q^2 and reaches unity. (We would like to note here that the concept of constituent quark has been used extensively since the very beginning of the quark era but has obtained just recently a possible direct experimental evidence at Jefferson Lab. [10].)

However, off-shell unitarity does not require a transformation of this power-like dependence into a logarithmic one at asymptotical energies. Thus, the power-like behavior of the cross-sections with the exponent dependent on virtuality could be of an asymptotical nature and have a physical ground. It should not be regarded merely as a transitory behavior or a convenient way to represent the data.

The extended unitarity for the off-mass-shell amplitudes F^{**} and F^* has a structure similar to the equation for the on-shell amplitude F but in the former case it relates to different amplitudes. We denoted in that way the amplitudes when both initial and final mesons are off mass shell, only initial meson is off mass shell and both mesons are on mass shell, respectively. Note that $\sigma_{\gamma^*p}^{\text{tot}}$ is determined by the imaginary part of the amplitude F^{**} , whereas $\sigma_{V^*p}^{\text{tot}}$ is determined by the square of another amplitude F^* . Disregard of this fact is the main reason leading to the difficulties mentioned in Introduction in explaining an energy independence of ratio of the cross section of exclusive ρ electroproduction to the total cross section.

The important point in the solution of the extended unitarity is the factorization in the impact parameter representation at the level of the input dynamical quantity — U -matrix:

$$U^{**}(s, b, Q^2)U(s, b) - [U^*(s, b, Q^2)]^2 = 0. \quad (2)$$

Eq. (2) reflects universality of the initial and final state interactions when the transition between on- and off-mass shell states occurs. Despite that such factorization does not survive at the level of the amplitudes $F^{**}(s, t, Q^2)$, $F^*(s, t, Q^2)$ and $F(s, t)$ (i.e. after unitarity equations are solved and Fourier–Bessel transformation is performed), it is essential for the energy independence of the ratio of the exclusive ρ electroproduction cross section to the total cross section.

The above result (1) is valid when the interaction radius of the constituent quark Q from the virtual meson V^* increases with virtuality Q^2 . We use the same notation Q for the constituent quarks and virtuality, but it should not lead to misunderstanding. The dependence of the interaction radius

$$R_Q(Q^2) = \xi(Q^2)/m_Q. \quad (3)$$

on Q^2 comes through the dependence of the universal Q^2 -dependent factor $\xi(Q^2)$ (in the on-shell limit $\xi(Q^2) \rightarrow \xi$). The origin of the rising interaction radius of the constituent quark Q with virtuality Q^2 might be of a

dynamical nature and it would stem from the emission of the additional $q\bar{q}$ -pairs in the nonperturbative structure of a constituent quark. Available experimental data are consistent with the $\ln Q^2$ -dependence of the radius [8]:

$$R_Q(Q^2) = R_Q^0 + \frac{a}{m_Q} \ln(1 + Q^2/Q_0^2),$$

where $R_Q^0 = \xi/m_Q$ and parameters ξ , a and Q_0^2 are universal for all constituent quarks.

The introduction of the Q^2 dependent interaction radius of a constituent quark, which, in this approach, consists of a current quark surrounded by the cloud of quark-antiquark pairs of different flavors [11], is the main issue of the off-shell extension of the model, which provides at large values of W^2

$$\sigma_{\gamma^*p}^{\text{tot}}(W^2, Q^2) \propto G(Q^2) \left(\frac{W^2}{m_Q^2} \right)^{\lambda(Q^2)} \ln \frac{W^2}{m_Q^2}, \quad (4)$$

where

$$\lambda(Q^2) = 1 - \xi/\xi(Q^2), \quad \xi(Q^2) = \xi + a \ln(1 + Q^2/Q_0^2). \quad (5)$$

The value of parameter ξ in the model is determined by the slope of the differential cross-section of elastic scattering in the large t region [12], and it follows from the pp -experimental data that $\xi = 2$.

Inclusion of heavy vector meson production into this scheme is straightforward: the virtual photon fluctuates before the interaction with proton into the heavy quark-antiquark pair which constitutes the virtual heavy vector meson state. After the interaction with a proton, this state turns out into a real heavy vector meson.

Integral exclusive (elastic) cross-section of vector meson production in the process $\gamma^*p \rightarrow Vp$ when the vector meson in the final state contains not necessarily light quarks can be calculated directly:

$$\sigma_{\gamma^*p}^V(W^2, Q^2) \propto G_V(Q^2) \left(\frac{W^2}{m_Q^2} \right)^{\lambda_V(Q^2)} \ln \frac{W^2}{m_Q^2}, \quad (6)$$

where

$$\lambda_V(Q^2) = \lambda(Q^2)\tilde{m}_Q/\langle m_Q \rangle. \quad (7)$$

In Eq. (7), \tilde{m}_Q denotes the mass of the constituent quarks from the vector meson and $\langle m_Q \rangle = (2\tilde{m}_Q + 3m_Q)/5$ is the mean constituent quark mass of the vector meson and proton system. The function $G_V(Q^2)$ in Eq. (6) at fixed values of Q^2 was considered as an overall

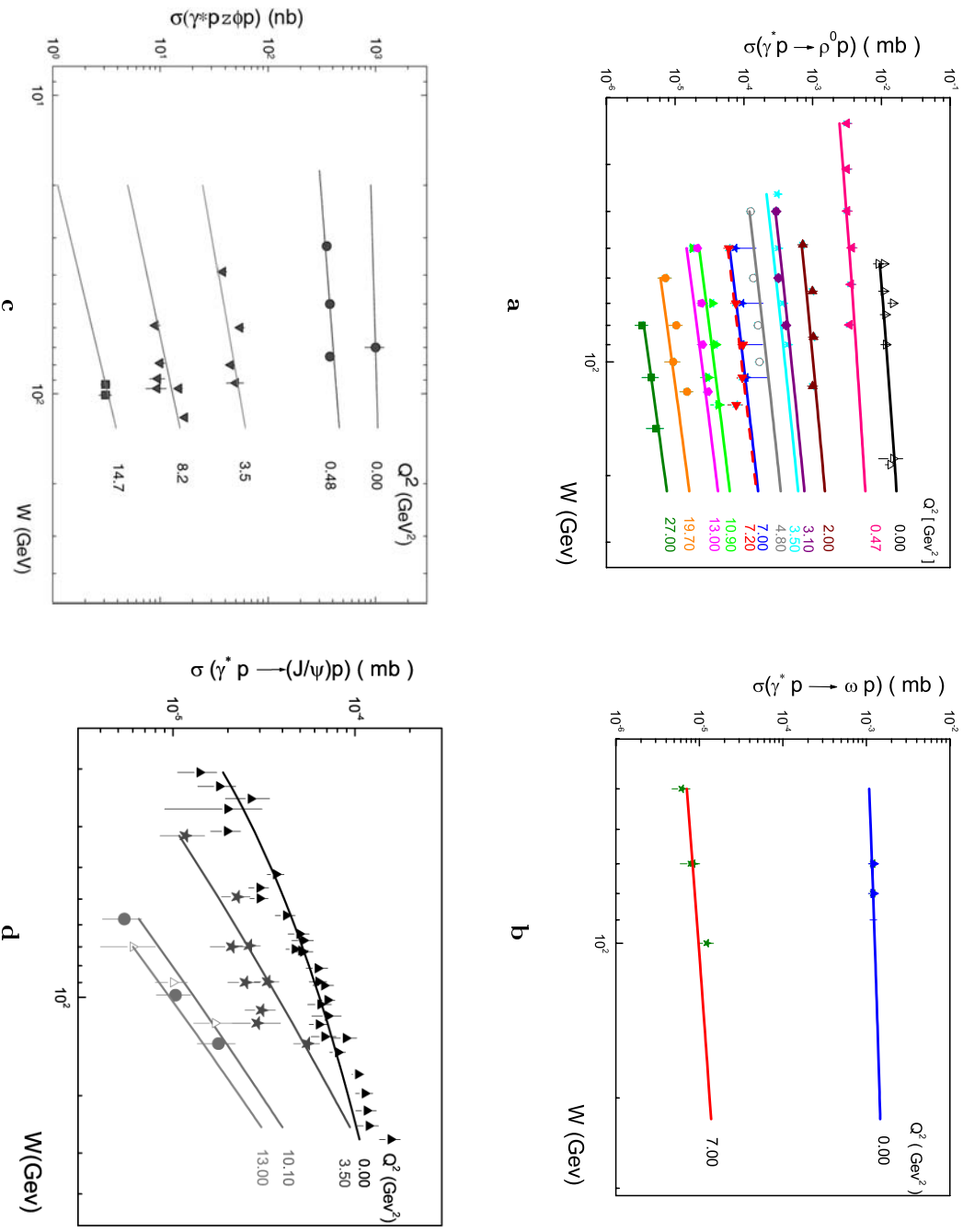


Fig.1. Energy dependences of the elastic cross-section of exclusive ρ -meson (a), ω -meson (b), φ -meson (c), and J/ψ -meson (d) production. Experimental data are taken from [1, 2] (a); [13] (b); [14] (c), and [15] (d)

constant to be fixed by the experimental data. Its dependence on Q^2 can be described then by a simple formula

$$G_V(Q^2) = G_0(1 + Q^2/Q_0^2)^{-\alpha}.$$

Of course, for the on-shell scattering ($Q^2 = 0$), we have a standard Froissart-like asymptotic energy dependence.

It is evident from Eqs. (4) and (6) that $\lambda_V(Q^2) = \lambda(Q^2)$ for the light vector mesons, i.e. the ratio

$$r_V(W^2, Q^2) = \sigma_{\gamma^*p}^V(W^2, Q^2)/\sigma_{\gamma^*p}^{\text{tot}}(W^2, Q^2) \quad (8)$$

¹We are indebted to I. Ivanov for pointing out to this fact.

does not depend on energy for $V = \rho, \omega$. Eq. (6) and consequently (8) are in a good agreement (Fig. 1, a, b) with the experimental data of HI and ZEUS Collaborations [1, 2]. It should be noted, however, that the experimental definition of the ratio r_V adopted by the ZEUS Collaboration [4] considers cross-sections at the different virtualities¹ and does not directly correspond to (8).

For the case of the heavy vector meson production J/ψ and Υ , the respective cross-section rises about two

times faster than the total cross-section; Eq. (7) results in

$$\lambda_{J/\psi}(Q^2) \simeq 2\lambda(Q^2), \quad \lambda_{\Upsilon}(Q^2) \simeq 2.2\lambda(Q^2),$$

i.e.

$$r_{J/\psi}(W^2, Q^2) \propto (W^2)\lambda(Q^2), \quad r_{\Upsilon}(W^2, Q^2) \propto (W^2)^{1.2}\lambda(Q^2).$$

Corresponding relations for the φ -meson production are the following:

$$\lambda_{\varphi}(Q^2) \simeq 1.3\lambda(Q^2), \quad r_{\varphi}(W^2, Q^2) \propto (W^2)^{0.3}\lambda(Q^2).$$

In the limiting case where the vector meson is very heavy, i.e. $\tilde{m}_Q \gg m_Q$, the relation between exponents is $\lambda_V(Q^2) = 2.5\lambda(Q^2)$.

The quantitative agreement of Eq. (6) with experimental data for the case of φ -meson production can be seen in Fig. 1,c. Corresponding cross-section of J/ψ -meson production is presented in Fig. 1,d.

This agreement is in favor of relation (7) which provides explicit mass dependence of the exponent in the power-like energy dependence of cross-sections. Thus, the power-like parameterization of the ratio r_V ,

$$r_V(W^2, Q^2) \sim (W^2)\lambda(Q^2)(\tilde{m}_Q - (m_Q))^{1/(m_Q)},$$

with m_Q and Q^2 -dependent exponent could also have a physical ground. It would be interested therefore to check experimentally the predicted energy dependence of the ratio r_V .

The dependence of the constituent quark interaction radius (3) on its mass and virtuality gets an experimental support and the non-universal energy asymptotical dependence (6) and (7) predicted in [8] does not contradict to the high-energy experimental data on elastic vector-meson electroproduction. Of course, as was already mentioned in Introduction, the limited energy range of the available experimental data allows other parameterizations, e.g. universal asymptotical Regge-type behavior with Q^2 -independent trajectories (cf. [21–23]), to treat the experimental regularities as transitory ones.

It seems, however, that the scattering of virtual particles reaches the asymptotics much faster than the scattering of the real particles and the Q^2 -dependent exponent $\lambda(Q^2)$ reflects the asymptotical dependence and not the “effective” preasymptotical one. Despite that the relation between $\xi(Q^2)$ and $\lambda(Q^2)$ implies a saturation of the Q^2 -dependence of $\lambda(Q^2)$ at large values of Q^2 , the power-like energy dependence itself will survive at asymptotical energy values. The early

asymptotics of virtual particle scattering is correlated with the peripheral impact parameter behavior of the scattering amplitude for virtual particles. The respective profiles of the amplitudes F^{**} and F^* are peripheral when $\xi(Q^2)$ increases with Q^2 [8].

The energy independence of the ratio $r_{\rho}(W^2, Q^2)$ reflects universality of the initial and final state interactions responsible for the transition between the on- and off-mass shell states. This universality is a natural assumption leading to factorization (2) at the level of the U -matrix [8]. Under this, the off-shell unitarity is the principal origin of the energy independence of the ratio $r_{\rho}(W^2, Q^2)$.

There are also other interesting manifestations of the off-shell unitarity effects, e.g. the behavior of the differential cross-sections at large t is to a large extent determined by the off-shell unitarity effects. Indeed, a smooth power-like dependence on t has been predicted in [8]:

$$\frac{d\sigma_V}{dt} \simeq \tilde{G}(Q^2) [1 - \tilde{\xi}^2(Q^2)t/\tilde{m}_Q^2]^{-3}, \quad (9)$$

where

$$\tilde{\xi}(Q^2) = \xi\xi(Q^2)/[\xi - \xi(Q^2)]. \quad (10)$$

Smooth power-like t -dependence was obtained also in the approaches based on the perturbative QCD [24–26]. We expect that Eq. (9) should be applicable for the J/ψ vector meson elastic electroproduction not only at the large but at moderate values of t also. This is due to a large mass of a charmed quarks which enhances in the model contribution of the region of small impact parameters and suppresses the contribution from the region $b \sim R(s)$. As is seen from Fig. 2,a, Eq. (9) indeed corresponds to experimental data for J/ψ vector meson even at the very moderate t -values.

It appears that the differential cross section does not depend on energy and depends on t in a simple power-like way $(-t)^{-3}$. (A similar power-like behaviour is expected for differential cross section of process with proton dissociation [27], which however will numerically be dominating at large t .) This dependence differs from the corresponding dependence in the case of on-shell exclusive scattering [11] which approximates the quark counting rule [28]. The ratio of differential cross-sections for the exclusive production of the different vector mesons

$$\frac{d\sigma_{V_1}}{dt} / \frac{d\sigma_{V_2}}{dt}$$

does not depend on the variables W^2 and t at large enough values of $-t$.

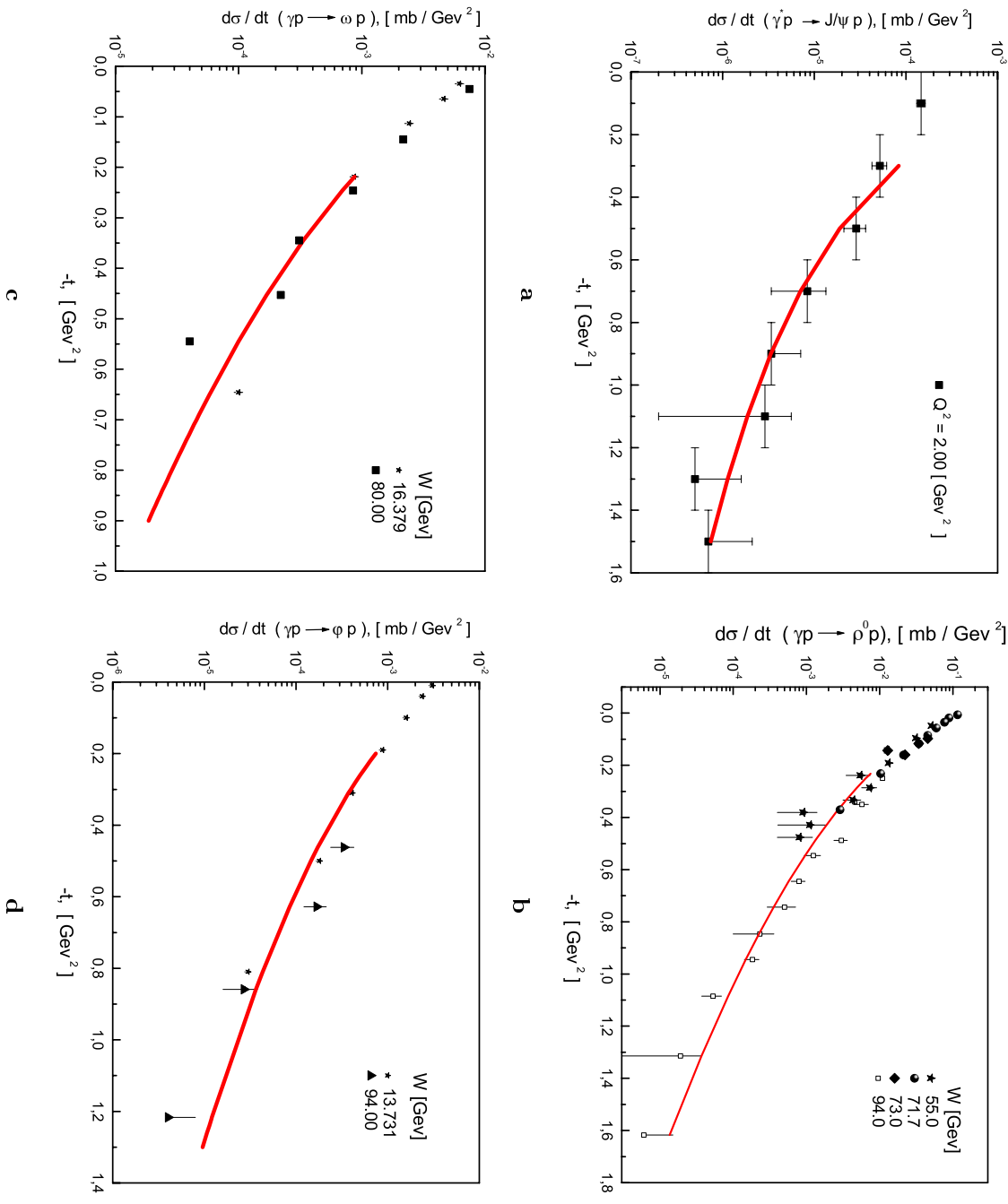


Fig.2. Angular dependences of the elastic cross-sections of exclusive J/ψ -meson (a), ρ -meson (b), ω -meson (c), and φ -meson (d) electroproduction. Experimental data are taken from [16](a), [17] (b), [18,19] (c), and [19,20] (d)

Meanwhile the Orear-type behavior of the differential cross-section of the vector meson photoproduction obtained in [8],

$$\frac{d\sigma_V}{dt} \propto \exp \left[-\frac{2\pi\xi}{M} \sqrt{-t} \right], \quad (11)$$

is in a good agreement with the HERA experimental data at moderate values of t (cf. Fig.2,b,c, and d). Note that the parameters ξ and $M = 2\tilde{m}_Q + 3m_Q$ are fixed.

Thus, we have shown that the model [8] which leads to the energy independence of the ratio of exclusive ρ -

meson electroproduction cross-section to the total cross-section as the result of the adopted universality of the initial and final state interactions responsible for the transition between the on- and off-mass shell states, is in a quantitative agreement with experimental data for the vector-meson production.

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ЕФЕКТИ ПОЗАМАСОВОЇ ПОВЕРХНІ ТА ФАКТОРИЗАЦІЇ В ЕКСКЛЮЗИВНОМУ НАРОДЖЕННІ ВЕКТОРНИХ МЕЗОНІВ

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Резюме

Показано, що універсальність взаємодій в початковому та кінцевому станах, відповідальних за перехід між станами на- і позамасовий поверхні, приводить до незалежності від енергії відношення ексکلюзивного поперечного перерізу електроннародження ρ до повного поперечного перерізу. Показано, що ця універсальність і явна залежність від маси показника в степеневій поведінці поперечного перерізу від енергії, одержаного з унітарного підходу, знаходяться в кількісному узгодженні з високоенергетичними експериментальними даними, одержаними на прискорювачі HERA. Ми обговорюємо також результати вимірювань на HERA кутового розподілу народжуваних векторних мезонів.

ЭФФЕКТЫ ВНЕМАССОВОЙ ПОВЕРХНОСТИ И ФАКТОРИЗАЦИИ В ЭКСКЛЮЗИВНОМ РОЖДЕНИИ ВЕКТОРНЫХ МЕЗОНОВ

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Резюме

Показано, что универсальность взаимодействий в начальном и конечном состояниях, отвечающих за переход между состояниями на- и внемассовой поверхности, приводит к независимости от энергии отношения эксключаивного поперечного сечения электронорождения ρ к полному поперечному сечению. Показано, что эта универсальность и явная зависимость от массы показателя в степеневом поведении поперечного сечения от энергии, полученного из основанного на унитарности подходе, находятся в количественном согласии с высокоэнергетическими экспериментальными данными, полученными на ускорителе HERA. Мы обсуждаем также результаты измерений на HERA углового распределения рождаемых векторных мезонов.