ON THE DEPENDENCE OF THE INDUCED VACUUM ENERGY-MOMENTUM TENSOR ON THE COUPLING TO THE CURVATURE SCALAR

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UDC 621.1

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Charged scalar field is quantized in the background of a static $(d-2)$-brane which is a core of magnetic flux lines in a flat $(d+1)$-dimensional space-time. We find that the vector potential of a magnetic core induces the energy-momentum tensor in the vacuum. Notwithstanding the flatness of the space-time, the tensor components depend on the coupling to the curvature scalar, and peculiarities of the behaviour at the conformal value of the coupling are revealed.

1. Introduction

Since Casimir's seminal paper [1] it has become clear that the effect of external boundary conditions in quantum field theory can be exposed as the emergence of a non-zero vacuum expectation value of the energy-momentum tensor (see, e.g., [2, 3]). This may have far reaching consequences; in particular, the vacuum energy-momentum tensor serves as a source of gravitation, and the so-called self-consistent cosmological models of the Universe are proposed, where matter is absent and its role is played by the vacuum quantum effects [4].

In this respect, it seems to be of interest to look for various situations where the vacuum energy-momentum tensor is calculable and finite. Let X be the base space manifold of dimension d and Y be a submanifold of dimension less than d. The matter field is quantized under a certain boundary condition imposed on Y. In most implications of the Casimir effect, Y is chosen to be noncompact disconnected (e.g., two parallel infinite plates, as generically in [1]) or closed compact (e.g., box or sphere), see [3]. In the present paper, we choose Y to be noncompact connected and possessing the dimension $(d-2)$, i.e. a $(d-2)$-brane in d-dimensional space. If such a brane is filled with magnetic flux lines, then it can be regarded as a generalization of the Bohm — Aharonov [5] singular magnetic vortex in 3-dimensional space. If the matter field vanishes at Y, then the region where the matter field is nonvanishing (out of Y) does not overlap with the region where the background magnetic field is nonvanishing (inside Y). Thus, there is no effect of the background field on the matter field in the framework of classical theory, and such an effect, if exists, is of purely quantum nature. The conventional Bohm — Aharonov effect pertains to the quantum-mechanical framework [5]. As is clear from the above, our interest will be in the quantum-field-theoretical framework (i.e. vacuum polarization in the background of the brane), which, therefore, may be generally denoted as the Casimir-Bohm-Aharonov effect, see also [6].

Throughout the present paper, we restrict ourselves to the case of scalar matter. A peculiarity of this case is that the energy-momentum tensor depends on the coupling (ξ) of the scalar field to the scalar curvature of space-time even when space-time is flat. If the scalar field is massless, then the conformal invariance of the theory is achieved at $\xi=\xi_c$, where [7–9]

$$\xi_c=\frac{d-1}{4d}\,; \tag{1.1}$$

note that ξ_c varies from 0 to 1/4 when d varies from 1 to ∞. Our analysis of vacuum polarization effects in the background of the brane will be carried out for arbitrary values of ξ; however, results will be most impressive in the case of conformal coupling, $\xi=\xi_c$. We shall find out components of the induced vacuum energy-momentum tensor as functions of the brane flux, distance from the brane, and space dimension.

2. Energy-Momentum Tensor and its Vacuum Expectation Value

The energy-momentum tensor for a quantized charged scalar field $\Psi(x)$ is given by the expression

$$T^{\mu\nu} = T^{\mu\nu}_{\rm can} + \xi\left(g^{\mu\nu}\Box - \nabla^\mu\nabla^\nu - R^{\mu\nu}\right)\left[\Psi^\dagger, \Psi\right]_+ , \quad (2.1)$$

where

$$T^{\mu\nu}_{\rm can} = \frac{1}{2}\left[\nabla^\mu\Psi^\dagger, \nabla^\nu\Psi\right]_+ + \frac{1}{2}\left[\nabla^\nu\Psi^\dagger, \nabla^\mu\Psi\right]_+ - g^{\mu\nu}L\,, \quad (2.2)$$

and

$$L = \frac{1}{2}\left[\nabla^\mu\Psi^\dagger, \nabla_\mu\Psi\right]_+ - \frac{1}{2}\left(m^2 + \xi R\right)\left[\Psi^\dagger, \Psi\right]_+ , \quad (2.3)$$

∇_μ is the covariant derivative involving both affine and bundle connections, $\Box = \nabla_\mu\nabla^\mu$ is the covariant d'Alembertian, $R^{\mu\nu}$ is the Ricci tensor and $R = g_{\mu\nu}R^{\mu\nu}$ is the scalar curvature of space-time, signature of space-time metric $g_{\mu\nu}$ is chosen as $(+-\ldots-)$. Canonical tensor (2.2) is obtained by applying the Noether's theorem to Lagrangian (2.3), whereas tensor (2.1) is obtained by varying L (2.3) with respect to the metric tensor $g_{\mu\nu}$.

Eq.(2.1) can be rewritten in the form

$$T^{\mu\nu} = \tilde{T}^{\mu\nu} + (\xi - 1/4)\left(g^{\mu\nu}\Box - \nabla^\mu\nabla^\nu\right)\left[\Psi^\dagger, \Psi\right]_+ -$$

$$-\xi R^{\mu\nu}\left[\Psi^\dagger, \Psi\right]_+ , \quad (2.4)$$

where

$$\tilde{T}^{\mu\nu} = T^{\mu\nu}_{can} + \frac{1}{4}\left(g^{\mu\nu}\Box - \nabla^\mu\nabla^\nu\right)\left[\Psi^\dagger, \Psi\right]_+ \quad (2.5)$$

is the canonical tensor corresponding to the Lagrangian $\tilde{L}$ which differs from L(2.3) by a total divergence:

$$\tilde{L} = L - \frac{1}{4}\Box\left[\Psi^\dagger, \Psi\right]_+ = -\frac{1}{4}\left[\Psi^\dagger, \Box\Psi\right]_+ -$$

$$-\frac{1}{4}\left[\Box\Psi^\dagger, \Psi\right]_+ - \frac{1}{2}\left(m^2 + \xi R\right)\left[\Psi^\dagger, \Psi\right]_+ . \quad (2.6)$$

Both L(2.3) and $\tilde{L}$(2.6) yield the same equations of motion,

$$\left[\Box + \left(m^2 + \xi R\right)\right]\Psi = 0, \qquad \left[\Box + \left(m^2 + \xi R\right)\right]\Psi^\dagger = 0\,, \quad (2.7)$$

but $\tilde{L}$ is strictly vanishing on the solutions to the equations of motion. In fact, $\tilde{L}$ is used as a Lagrangian in the path integral approach to quantization, since namely $\tilde{L}$ is directly related to the inverse propagator of the quantized scalar field.

If Ψ is a solution to Eq.(2.7), then one gets

$$\tilde{T}^{\mu\nu} = \frac{1}{2}\left[\nabla^\mu\Psi^\dagger, \nabla^\nu\Psi\right]_+ + \frac{1}{2}\left[\nabla^\nu\Psi^\dagger, \nabla^\mu\Psi\right]_+ -$$

$$-\frac{1}{4}\nabla^\mu\nabla^\nu\left[\Psi^\dagger, \Psi\right]_+ , \quad (2.8)$$

and

$$g_{\mu\nu}T^{\mu\nu} = \left(\xi d - \frac{d-1}{4}\right)\Box\left[\Psi^\dagger, \Psi\right]_+ + m^2\left[\Psi^\dagger, \Psi\right]_+ , \quad (2.9)$$

where d is the dimension of space. Thus, there is a distinctive value of the parameter ξ ($\xi = \xi_c$, see Eq.(1.1)) under which the trace of the energy-momentum tensor becomes proportional to the mass squared,

$$g_{\mu\nu}T^{\mu\nu}|_{\xi=\xi_c} = m^2\left[\Psi^\dagger, \Psi\right]_+ , \quad (2.10)$$

and the tracelessness of $T^{\mu\nu}$ and conformal invariance are achieved in the massless limit ($m = 0$).

In the case of a static background ($\nabla_0 = \partial_0, \quad g_{00} = 1$), the operator of the quantized charged scalar field is represented as

$$\Psi(x^0, \mathbf{x}) = \sum\!\!\!\!\!\!\int_\lambda \frac{1}{\sqrt{2E_\lambda}}\Big[e^{-iE_\lambda x^0}\psi_\lambda(\mathbf{x})\, a_\lambda +$$

$$+ e^{iE_\lambda x^0}\psi_{-\lambda}(\mathbf{x})\, b^\dagger_\lambda\Big]. \quad (2.11)$$

Here, $a^\dagger_\lambda$ and a_λ ($b^\dagger_\lambda$ and b_λ) are the scalar particle (antiparticle) creation and annihilation operators satisfying a commutation relation; λ is the set of parameters (quantum numbers) specifying the state; $E_\lambda = E_{-\lambda} > 0$ is the energy of the state; symbol $\sum\!\!\!\!\!\!\int_\lambda$ denotes the summation over discrete and integration (with a certain measure) over continuous values of λ; wave functions $\psi_\lambda(\mathbf{x})$ are the solutions to the stationary equation of motion,

$$\left\{-\triangle + \left[m^2 + \xi R(\mathbf{x})\right]\right\}\psi_\lambda(\mathbf{x}) = E^2_\lambda\psi(\mathbf{x}), \quad (2.12)$$

$\triangle = \boldsymbol{\nabla}^2$ is the covariant Laplacian. For components of the vacuum expectation value of the energy-momentum tensor,

$$t^{\mu\nu} = \langle \mathrm{vac}\,| T^{\mu\nu} |\,\mathrm{vac}\rangle\,, \quad (2.13)$$

one gets the expressions

$$t^{00} = \sum\!\!\!\!\!\!\int_\lambda E_\lambda \psi^*_\lambda(\mathbf{x})\,\psi_\lambda(\mathbf{x}) -$$

$$-(\xi - 1/4)\triangle \sum\!\!\!\!\!\!\int_\lambda E_\lambda^{-1}\psi^*_\lambda(\mathbf{x})\,\psi_\lambda(\mathbf{x}), \qquad (2.14)$$

$$t^{jj\,'} = \frac{1}{2}\sum\!\!\!\!\!\!\int_\lambda E_\lambda^{-1}\left\{\left[\nabla^j\psi_\lambda(\mathbf{x})\right]^*\left[\nabla^{j\,'}\psi_\lambda(\mathbf{x})\right] + \right.$$

$$\left. + \left[\nabla^{j\,'}\psi_\lambda(\mathbf{x})\right]^*\left[\nabla^{j}\psi_\lambda(\mathbf{x})\right]\right\} +$$

$$+\left\{\frac{1}{4}g^{jj\,'}(\mathbf{x})\triangle - \xi\left[g^{jj\,'}(\mathbf{x})\triangle + \nabla^j\nabla^{j\,'} + R^{jj\,'}(\mathbf{x})\right]\right\}\times$$

$$\times \sum\!\!\!\!\!\!\int_\lambda E_\lambda^{-1}\psi^*_\lambda(\mathbf{x})\,\psi_\lambda(\mathbf{x}), \quad j, j\,' = \overline{1, d}; \qquad (2.15)$$

note that the t^{0j} components are vanishing, and the relations $R^{00}(\mathbf{x}) = 0$ and $\partial_0\left[\Psi^+, \Psi\right]_+ = 0$ have been taken into account.

However, relations (2.14) and (2.15) can be regarded as purely formal and, strictly speaking, meaningless: they are ill-defined, suffering from ultraviolet divergencies. The well-defined quantities are obtained by inserting an inverse energy in a sufficiently high power:

$$t^{00}_{\rm reg}(s) = \sum\!\!\!\!\!\!\int_\lambda E_\lambda^{-2s}\psi^*_\lambda(\mathbf{x})\,\psi_\lambda(\mathbf{x}) -$$

$$-(\xi - 1/4)\triangle \sum\!\!\!\!\!\!\int_\lambda E_\lambda^{-2(s+1)}\psi^*_\lambda(\mathbf{x})\,\psi_\lambda(\mathbf{x}), \qquad (2.16)$$

$$t^{jj\,'}_{\rm reg}(s) = \frac{1}{2}\sum\!\!\!\!\!\!\int_\lambda E_\lambda^{-2(s+1)}\left\{\left[\nabla^j\psi_\lambda(\mathbf{x})\right]^*\left[\nabla^{j\,'}\psi_\lambda(\mathbf{x})\right] + \right.$$

$$\left. + \left[\nabla^{j\,'}\psi_\lambda(\mathbf{x})\right]^*\left[\nabla^{j}\psi_\lambda(\mathbf{x})\right]\right\} +$$

$$+\left\{\frac{1}{4}g^{jj\,'}(\mathbf{x})\triangle - \xi\left[g^{jj\,'}(\mathbf{x})\triangle + \nabla^j\nabla^{j\,'} + R^{jj\,'}(\mathbf{x})\right]\right\}\times$$

$$\times \sum\!\!\!\!\!\!\int_\lambda E_\lambda^{-2(s+1)}\psi^*_\lambda(\mathbf{x})\,\psi_\lambda(\mathbf{x}), \quad j, j\,' = \overline{1, d}. \qquad (2.17)$$

Sums (integrals) are convergent in the case of $\mathrm{Re}\, s > d/2$. Thus, the summation (integration) is performed in this case, and then the result is analytically continued to the case of $s = -1/2$. This way of dealing with ultraviolet divergencies is known as the zeta function regularization procedure [10–12].

It is amazing, as is already mentioned in Introduction, that the energy-momentum tensor and, consequently, its vacuum expectation value remain to be dependent on the parameter ξ even in the case of flat space-time ($R = 0$). If the scalar field is quantized in the background of a static magnetic field in flat space-time, then the covariant derivative is defined as

$$\begin{array}{l} \boldsymbol{\nabla}\Psi = (\boldsymbol{\partial} - i\mathbf{V})\,\Psi\,, \quad \boldsymbol{\nabla}\Psi^\dagger = (\boldsymbol{\partial} + i\mathbf{V})\,\Psi^\dagger\,, \\ \boldsymbol{\nabla}\left[\Psi^\dagger, \Psi\right]_+ = \boldsymbol{\partial}\left[\Psi^\dagger, \Psi\right]_+\,, \end{array} \qquad (2.18)$$

and the magnetic field strength reads

$$B^{j_1\cdots j_{d-2}}(\mathbf{x}) = -\varepsilon^{j_1\cdots j_d}\partial_{j_{d-1}}V_{j_d}(\mathbf{x})\,, \qquad (2.19)$$

where $\mathbf{V}(\mathbf{x})$ is the bundle connection (vector potential of the magnetic field), and $\varepsilon^{j_1\cdots j_d}$ is the totally antisymmetric tensor, $\varepsilon^{12\cdots d} = 1$.

In the present paper, we consider the bundle curvature (magnetic field strength) to be nonvanishing in the $(d-2)$-brane (i.e. point in the $(d = 2)$ case, line in the $d = 3$ case, plane in the $d = 4$ case, and $(d-2)$-hypersurface in the $d > 4$ case). Denoting the location of the $(d-2)$-brane by $x^1 = x^2 = 0$, one gets

$$B^{3\cdots d}(\mathbf{x}) = 2\pi\Phi\delta(x^1)\delta(x^2)\,, \qquad (2.20)$$

where Φ is the total flux (in the units of 2π) of the bundle curvature; then the bundle connection can be chosen as:

$$V^1(\mathbf{x}) = -\Phi\frac{x^2}{(x^1)^2 + (x^2)^2}\,, \quad V^2(\mathbf{x}) = \Phi\frac{x^1}{(x^1)^2 + (x^2)^2}\,,$$
$$V^j(\mathbf{x}) = 0\,, \quad j = \overline{3, d}\,. \qquad (2.21)$$

The complete set of regular solutions to Eq.(2.12) in background (2.20), (2.21) is given by the functions (see, e.g., [6])

$$\psi_{kn\mathbf{p}}(\mathbf{x}) = (2\pi)^{\frac{1-d}{2}}J_{|n-\Phi|}(kr)e^{in\varphi}e^{i\mathbf{p}\mathbf{x}_{d-2}}\,, \qquad (2.22)$$

where

$$0 < k < \infty\,, \quad n \in \mathbb{Z}\,, \quad -\infty < p^j < \infty\,, \quad j = \overline{3,d}\,, \tag{2.23}$$

$J_\mu(u)$ is the Bessel function of order μ, $r = \sqrt{(x^1)^2 + (x^2)^2}$, $\varphi = \arctan(\frac{x^2}{x^1})$, $\mathbf{x}_{d-2} = (0, 0, x^3, \dots x^d)$, and $\mathbb{Z}$ is the set of integers.

Now, to compute renormalized components of the vacuum expectation value of the energy-momentum tensor, we have to substitute (2.22) into Eqs.(2.16), (2.17), subtract the quantities that correspond to the case of noninteracting quantized field, and take limit $s \to -\frac{1}{2}$, i.e. to obtain $t^{\mu\nu}_{\rm ren} = \lim_{s\to -\frac{1}{2}}[t^{\mu\nu}_{\rm reg}(s) - t^{\mu\nu}_{\rm reg}(s)|_{F=0}]$. Its nonvanishing components are given by the expressions:

$$t^{00}_{\rm ren} = -t^{jj}_{\rm ren} = \frac{16\sin(F\pi)}{(4\pi)^{\frac{d+3}{2}}}\left(\frac{m}{r}\right)^{\frac{d+1}{2}} \times$$

$$\times \int\limits_1^\infty \frac{dv}{\sqrt{v^2-1}} \cosh[(2F-1)\,\mathrm{arccosh}\, v] \times$$

$$\times v^{-\frac{d+3}{2}} \Big\{ [1 + 2(1-4\xi)v^2] K_{\frac{d+1}{2}}(2mrv) -$$

$$-2(1-4\xi) mrv^3 K_{\frac{d+3}{2}}(2mrv) \Big\}\,, \tag{2.24}$$

$$t^{rr}_{\rm ren} = -\frac{16\sin(F\pi)}{(4\pi)^{\frac{d+3}{2}}}\left(\frac{m}{r}\right)^{\frac{d+1}{2}} \times$$

$$\times \int\limits_1^\infty \frac{dv}{\sqrt{v^2-1}} \cosh[(2F-1)\,\mathrm{arccosh}\, v] \times$$

$$\times v^{-\frac{d+3}{2}} (1 - 4\xi v^2) K_{\frac{d+1}{2}}(2mrv)\,, \tag{2.25}$$

$$t^{\varphi\varphi}_{\rm ren} = -\frac{16\sin(F\pi)}{(4\pi)^{\frac{d+3}{2}}}\frac{1}{r^2}\left(\frac{m}{r}\right)^{\frac{d+1}{2}} \times$$

$$\times \int\limits_1^\infty \frac{dv}{\sqrt{v^2-1}} \cosh[(2F-1)\,\mathrm{arccosh}\, v] \times$$

$$\times v^{-\frac{d+3}{2}} (1 - 4\xi v^2) \Big\{ K_{\frac{d+1}{2}}(2mrv) - 2mrv K_{\frac{d+3}{2}}(2mrv) \Big\}\,, \tag{2.26}$$

where $F = \Phi - [\![\Phi]\!]$ $(0 \le F < 1)$ is the fractional part of the flux, and $K_\mu(u)$ is the Macdonald function of order μ.

The temporal component of the vacuum tensor (i.e. energy density) is positive at $\xi \ge \frac{1}{4}$ and negative at $\xi \le 0$.[1] Transverse components of the vacuum tensor are also of opposite signs at $\xi \ge \frac{1}{4}$ and at $\xi \le 0$: the radial one is of the same and the angular one is of the opposite to the sign of the temporal component. The region $0 < \xi < \frac{1}{4}$ or, more precisely, the vicinity of $\xi = \xi_c$ is distinguished as the region where all components change their signs. Transverse components change their signs at a certain value of ξ simultaneously for all distances from the brane. In contrast to this, the temporal component is positive at small distances and negative at large distances for a certain, dependent on the value of the brane flux, vicinity of the point $\xi = \xi_c$.

This situation is illustrated by Figs. 1 — 3. Here, the variable mr is along the x-axis, and dimensionless products of tensor components at half-integer values of the brane flux and appropriate powers of r are along the y-axis: $r^{d+1} t^{00}_{\rm ren}\big|_{F=\frac{1}{2}}$ is presented by a solid line, $r^{d+1} t^{rr}_{\rm ren}\big|_{F=\frac{1}{2}}$ — by a dotted line, and $r^{d+3} t^{\varphi\varphi}_{\rm ren}\big|_{F=\frac{1}{2}}$ — by a dashed line. We consider cases of $\xi = 0$, $\xi = \xi_c$, $\xi = \frac{1}{4}$, each one at $d = 2$ and $d = 3$. We see from Fig.2 that the vacuum energy density at $\xi = \xi_c$ has minimum at $mr \approx 1.0$ $(d = 2)$ or $mr \approx 1.2$ $(d = 3)$; the minimal value is $r^{d+1} t^{00}_{\rm ren}\big|_{F=\frac{1}{2}} \approx -0.0018$ $(d = 2)$ or $r^{d+1} t^{00}_{\rm ren}\big|_{F=\frac{1}{2}} \approx -0.0039$ $(d = 3)$.

3. Summary

We have shown that the vacuum of the quantized charged scalar field is polarized in the background of a static magnetic $(d-2)$-brane in flat $(d+1)$-dimensional space-time. Vector potential of the brane induces a finite energy-momentum tensor in the vacuum; therefore, this effect may be denoted as the Casimir—Bohm—Aharonov effect. Tensor components depend periodically on the brane flux (Φ), vanishing at its integer values $(\Phi = n)$, and attaining maximal absolute values at its half-integer values $(\Phi = n + \frac{1}{2})$.

Qualitatively, the temporal component (energy density) is divergent with positive power at small

[1] The vacuum energy density at $\xi = \frac{1}{4}$ was considered in [6].

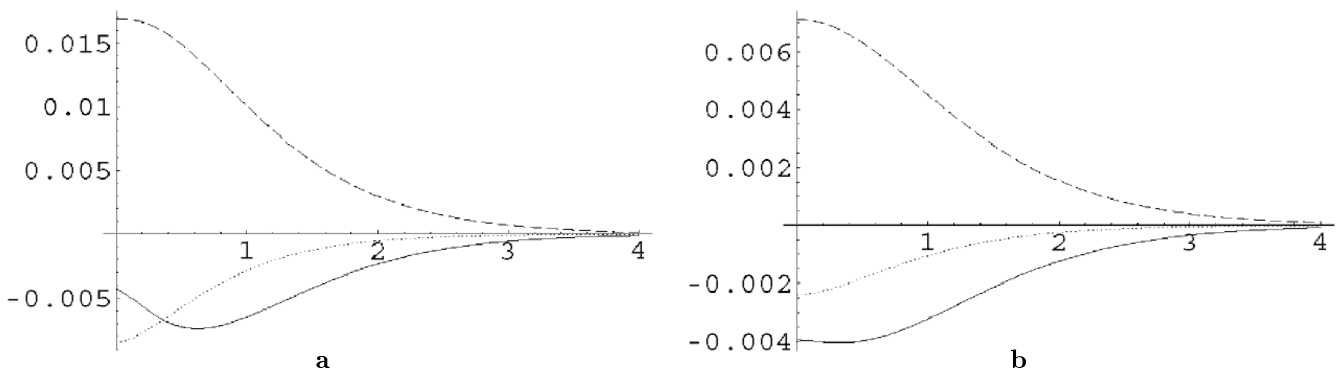

Fig. 1. $\xi = 0$: $a - d = 2$, $b - d = 3$

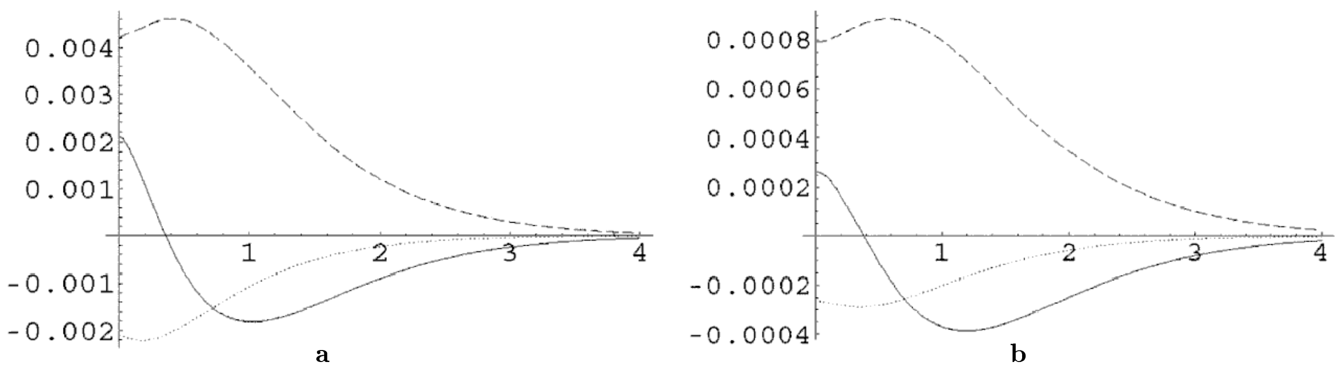

Fig. 2. $\xi = \xi_c$: $a - d = 2$, $b - d = 3$

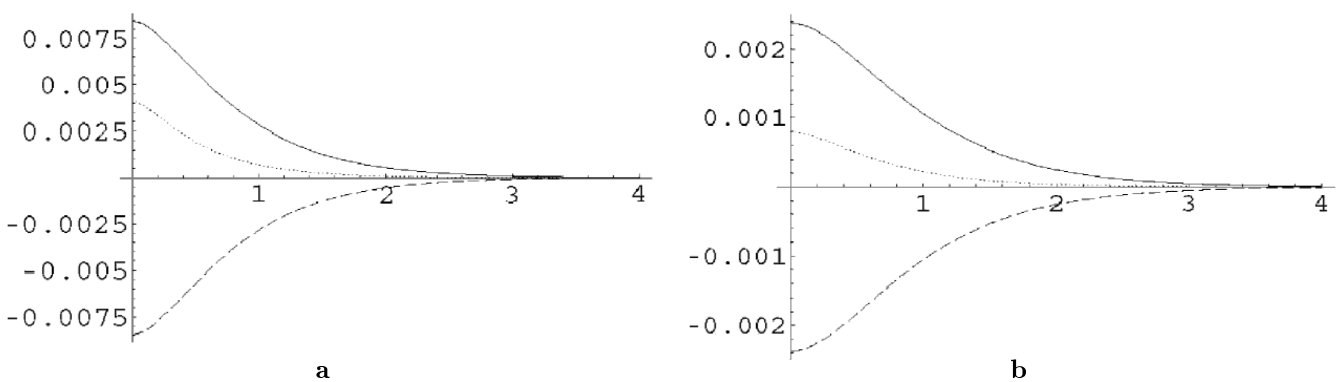

Fig. 3. $\xi = 1/4$: $a - d = 2$, $b - d = 3$

distances from the brane, decreases with the increase of the distance, passes zero, becomes negative and reaches a minimum at $r \sim m^{-1}$, then increases and reaches zero asymptotically from below with exponential behaviour. This is distinct from the conventional Casimir effect, which corresponds to the vacuum energy density being space independent constant, either negative or positive [3].

An intriguing question is about the underlying physics of the negativeness of the vacuum energy density. Recent examples of low-dimensional (without magnetic background) models [13] indicate that the negativeness of the vacuum energy density at large distances is related to the effectively perfect reflection of quantized matter at small distances; the negativeness might persist self-consistently for the whole, classical plus quantum, energy density. In this respect, work [14] is worth mentioning, where the vacuum energy density for the quantized spinor matter in the same background, as in the present paper, but exclusively in the $d = 2$ case, was considered. As is known, a spinor field cannot be made zero at the location of a singular magnetic brane (which is a point in the $d = 2$ case). The whole set of permissible boundary conditions is parametrized by a real quantity Θ, and, at $\cos\Theta < 0$, a bound state appears in the gap between positive and negative frequency continua [15]. Thus, for sure, the perfect reflection at the point of singularity is excluded at $\cos\Theta < 0$. As is shown in [14], namely at these values of Θ, the vacuum energy density is strictly positive at all distances, whereas, otherwise, its behaviour is similar to that of the vacuum energy density for the quantized scalar matter with $\xi = \xi_c$, see, qualitatively, solid curves in Fig.2.

The research was supported by INTAS (grant INTAS OPEN 00-00055) and Swiss National Science Foundation (grant SCOPES 2000-2003 7 IP 62607).

1. *Casimir H.B.G.*//Proc. Kon. Ned. Akad. Wetenschap B **51** (1948) 793; Physica **19** (1953) 846 .
2. *Birrel N.D., Davies P.C.W.* Quantum Fields in Curved Space. — Cambridge: Cambridge Univ. Press, 1982.
3. *Mostepanenko V.M., Trunov N.N.* The Casimir Effect and Its Applications.— Oxford: Clarendon Press, 1997.
4. *Starobinsky A.A.*// Phys. Lett. B **91** (1980) 99; *Zeldovich Ya.B., Starobinsky A.A.*// Astron. Lett. **10** (1984) 135.
5. *Aharonov Y., Bohm D.*// Phys. Rev. **115** (1959) 485.
6. *Sitenko Yu.A., Babansky A.Yu.*// Mod. Phys. Lett. A **13** 379 (1998); Phys. Atom. Nucl. **61** (1998) 1594.
7. *Penrose R.*// Relativity, Groups and Topology/ Ed. by B.S. DeWitt, C. DeWitt. — New York: Gordon and Breach, 1964.
8. *Chernikov N.A., Tagirov E.A.*// Ann. Inst. Henri Poincare **9A** (1968) 109.
9. *Callan C.G., Coleman S., Jackiw R.*// Ann. Phys. (N.Y.) **59** (1970) 42.
10. *Salam A., Strathdee J.*// Nucl. Phys. B **90** (1975) 203.
11. *Dowker J.S., Critchley R.*// Phys. Rev. D. **13** (1976) 3224.
12. *Hawking S.W.*// Commun. Math. Phys. **55** (1977) 133.
13. *Olum K.D., Graham N.*// Phys. Lett. B **554** (2003) 175; *Graham N., Olum K.D.*// Phys. Rev. D. **67** (2003) 085014.
14. *Sitenko Yu.A.*// Phys. Atom. Nucl. **62** (1999) 1056.
15. *De Sousa G.P.*// Phys. Rev. D. **40** (1989) 1346.

ПРО ЗАЛЕЖНІСТЬ ІНДУКОВАНОГО ВАКУУМНОГО ТЕНЗОРА ЕНЕРГІЇ-ІМПУЛЬСУ ВІД ЗВ'ЯЗКУ ЗІ СКАЛЯРОМ КРИВИЗНИ

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Р е з ю м е

Заряджене скалярне поле квантується в присутності статичної $(d-2)$-брани, що містить лінії магнітного потоку, в $(d+1)$-вимірному плоскому просторі-часі. Векторний потенціал магнітної брани індукує тензор енергії-імпульсу у вакуумі. Незважаючи на те, що простір-час є плоским, компоненти тензора залежать від константи зв'язку зі скаляром кривизни. Показано, що значення константи зв'язку, яке відповідає конформній симетрії, є виділеним, і досліджено особливості поведінки компонент при цьому значенні.

О ЗАВИСИМОСТИ ИНДУЦИРОВАННОГО ВАКУУМНОГО ТЕНЗОРА ЭНЕРГИИ-ИМПУЛЬСА ОТ СВЯЗИ СО СКАЛЯРОМ КРИВИЗНЫ

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Р е з ю м е

Заряженное скалярное поле квантуется в присутствии $(d-2)$-браны, содержащей линии магнитного потока, в плоском $(d+1)$-мерном пространстве-времени. Векторный потенциал магнитной браны индуцирует тензор энергии-импульса в вакууме. Несмотря на плоский характер пространства-времени, компоненты тензора зависят от константы связи со скаляром кривизны. Показано, что значение константы связи, отвечающее конформной симметрии, является выделенным, и исследованы особенности поведения компонент при этом значении.