CONDITIONS FOR HADRONIZATION AND FREEZE-OUT

V.K. MAGAS[1], **H. SATZ**[1,2]

[1]**CFIF, Physics Department, Instituto Superior Tecnico**
(Av. Rovisco Pais, 1049-001 Lisbon, Portugal),

[2]**Fakultät fur Physik, Universität Bielefeld**
(Postfach 100 131, D-33501 Bielefeld, Germany)

UDC 621.1

The interacting hadronic matter implies the existence of a large-scale connected cluster of a uniform nature – the size of such clusters as a function of hadron density is specified by percolation theory. In this way, we can formulate the freeze-out and deconfinement conditions in terms of the percolation of hadronic clusters or vacuum, correspondingly. The resulting freeze-out condition as a function of temperature and baryochemical potential interpolates between the resonance gas behaviour at low baryon density and the repulsive nucleonic matter at low temperature. The results of our model are in a good agreement with data and lattice QCD calculations.

The hot quark-gluon plasma in thermodynamic equilibrium is specified in terms of temperature T and baryochemical potential μ. Reducing the temperature of this medium at constant μ eventually brings it to the confinement transition at $T_c(\mu)$. Below this temperature, the system consists of interacting hadrons. Further cooling finally leads to freeze-out, at $T_f(\mu)$; beyond this point, we have non-interacting hadrons. The resulting two dividing lines in the $T-\mu$ plane are schematically shown in Fig. 1.

While the deconfinement curve is, in principle, calculable in lattice QCD at finite temperature and density (although so far with considerable problems at large μ and low T), the freeze-out curve is less well-defined theoretically as well as experimentally. The aim of the present paper is to address how freeze-out can be specified conceptually.

Matter implies the existence of a large-scale interconnected medium of uniform nature. When such a system breaks up into fragments much smaller than the size of the volume in which it is contained, it has undergone a change of state. One possible way to define freeze-out is thus geometric: it occurs at the point at which the size of hadronic clusters falls below the size of the given overall spatial volume. This point is determined in percolation theory [1] and for three space dimensions becomes

$$n_f = \frac{0.34}{V_h}, \tag{1}$$

where $n = N/V$ specifies the hadron density, with N hadrons in the overall volume V. The volume of an individual hadron is denoted by $V_h = (4\pi/3)r_h^3$, and hadrons are allowed to overlap; thus V_h introduces the short range nature ($\sim r_h$) of hadronic forces [2].

When the hadron density has reached the percolation point $n = n_f$, the largest clusters reach the size of the overall volume. This, however, does not imply that they fill the volume. We can calculate that, at the percolation point, the following part of space remains empty: $V_{\rm empty}/V = \exp\{-0.34\} \simeq 0.71$. Thus, the vacuum, measured in terms of the hadronic scale r_h, also forms percolating clusters. We can, therefore, ask for what density vacuum percolation stops and vacuum effectively disappears. From percolation theory, we know that this occurs at [2]

$$n_c = \frac{1.24}{V_h}, \tag{2}$$

obtained in an analogous way as Eq. (1). Since the disappearance of the physical vacuum is a basic feature of deconfinement, it seems natural to relate this threshold to the confinement/deconfinement transition.

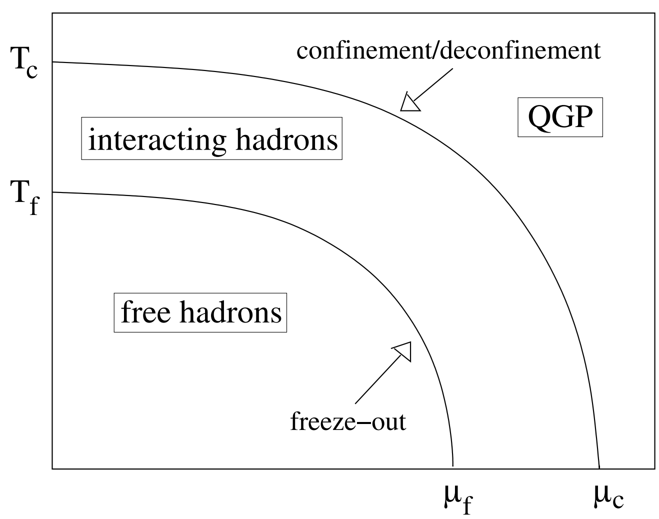

Fig. 1. States of matter in QCD — general view

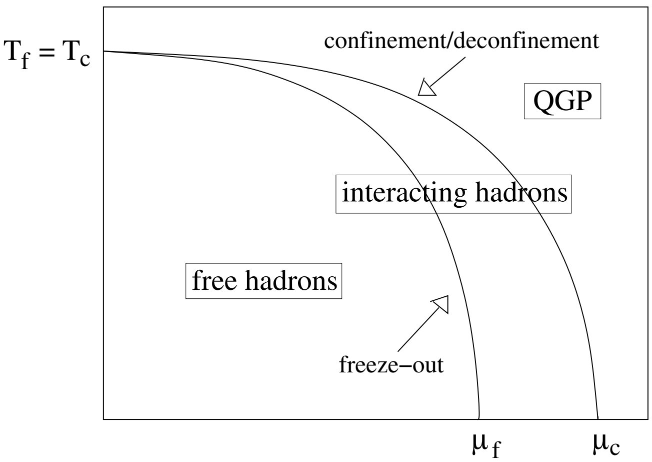

Fig. 2. States of matter in QCD, including resonance gas effects

Hence, on a purely geometric basis in terms of connected clusters, we have two thresholds: at $n = n_f$, there appear hadronic clusters of the size of the overall spatial volume and, at $n = n_c$, the vacuum percolation stops.

On a more dynamical level, there exists another approach to the problem. The confinement transition leads to an interacting hadronic medium. Now it is known that if the interactions between the constituents of such a medium are dominated by resonance formation and decay, the system can effectively be considered as an ideal gas of all possible resonances [3,4]. In our context, this means that if and when the interacting hadronic medium is resonance-dominated, freeze-out occurs at the point of confinement. In particular, the relative abundances of the different species of hadrons and hadronic resonances are in this case determined at the transition from QGP to hadronic matter.

For systems of vanishing or low baryon density, this appears quite well supported. On a theoretical level, it is in fact claimed that resonance formation dominates the interaction between hadrons, perhaps most clearly in the dual resonance model [5]. Experimentally, an analysis of the species abundances indicates that these are indeed determined by a single freeze-out temperature, $T_f \simeq 175$ MeV, obtained e^+e^- annihilation, $p - p$ and $p - \bar{p}$ interactions as well as in heavy ion collisions [6–8]. This temperature is moreover completely in agreement with that found for the confinement transition in finite temperature lattice QCD [9].

On the other extreme, for dense nuclear matter at low temperature, the situation is quite different. The interaction between two nucleons does not lead to resonance formation; instead, it is dominated by the Fermi statistics and baryon repulsion. Hence, here we expect freeze-out to occur only when nucleons no longer form interconnected matter. We can immediately check this. If we use the nucleon radius $r_n \simeq 0.8$ fm to determine V_h, Eq. (1) specifies $n_f \simeq 0.16$ fm^{-3} as the freeze-out density, and this is indeed just the density of normal nuclear matter. The correspondence between hadron percolation and freeze-out thus works well at $T = 0$. It defines nuclear matter as the most dilute system of nucleons which still forms connected matter.

Empirically, the determination of freeze-out parameters through the relative abundance of hadron species so far appears to be the only unambiguous approach to the problem. We therefore adopt this "chemical" freeze-out definition in the remainder of the paper. As an immediate consequence, we have to revise Fig. 1. At $\mu = 0$, $T_f = T_c$, while at $T = 0$, $\mu_f < \mu_c$, as shown in Fig. 2. Here the freeze-out point at $T = 0$ is specified by the nucleon percolation,

$$n(T = 0, \mu) = \frac{0.34}{(4\pi r_n^3/3)}, \tag{3}$$

which leads with $r_n \simeq 0.8$ fm for the nucleon radius to $\mu_f \simeq 0.97$ GeV. Can we also relate the other extreme, freeze-out and deconfinement at $\mu = 0$, to percolation?

From Eq. (2), we can specify the density of hadrons for which the vacuum disappears. Using the arguments given above, we know that, at $\mu = 0$, we can consider the interacting hadron system as an ideal resonance gas, containing all observed resonances [10] up to masses of 2.5 GeV. From the condition

$$n(T, \mu = 0) = \frac{1.24}{(4\pi r_h^3/3)}, \tag{4}$$

for the density of such a gas, we then obtain $T_c = T_f \simeq$ 170 MeV as the threshold temperature for confinement and (species abundance) freeze-out at $\mu = 0$. We have used here in Eq. (4) the nucleon radius $r_n = 0.8$ fm for baryons, and, for mesons, a value smaller by a factor of $(2/3)^{3/4} \simeq 1.1$, as suggested by bag model arguments [11].

At this point, we note that, in Fig. 2, we have so far only specified a way to determine $T_f = T_c$ at $\mu = 0$ and μ_f at $T = 0$. To determine the entire freeze-out curve in the $T - \mu$ plane, we have to know how contributions from non-resonant baryon interactions modify the ideal resonance gas picture. For the vanishing baryon number, at $\mu = 0$, we have an ideal resonance gas, and Eq. (4) determines the freeze-out as well as the deconfinement temperature. For a non-vanishing baryon density, the gas is no longer ideal, since now baryon interactions are

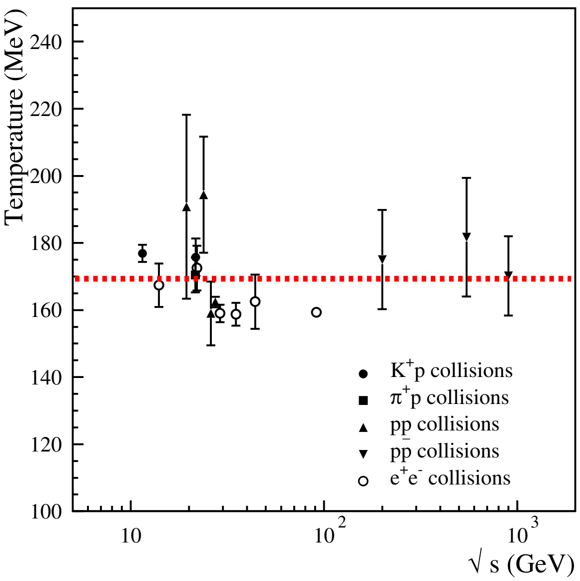

Fig. 3. Freeze-out temperature from species abundance in elementary hadron collisions, from [8]

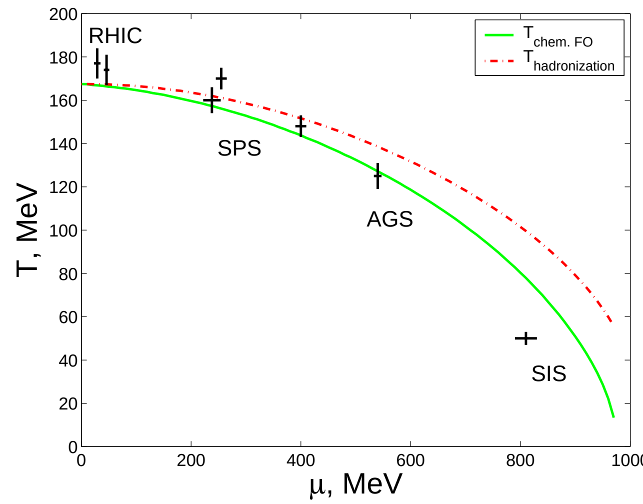

Fig. 4. Percolation freeze-out and data [12] . The calculations are performed under assumption of the completely equilibrated strangeness, i.e. $\gamma_s = 1$

present, which is not accountable in terms of resonances. For freeze-out, the system has to expand and cool off enough to stop these non-resonant baryon contributions. It is thus clear that now freeze-out will occur for $T_f < T_c$, below deconfinement. In particular, as is seen from Fig. 2, when we reach $\mu = \mu_c$, the interacting hadronic medium formed at the confinement point, still contains mesons; but these have disappeared when the medium has cooled off enough to freeze out all dynamical baryon repulsion, leaving cold nuclear matter of standard density, as determined by Eq. (1).

In the relativistic, multicomponent gas, where the conservation of the number of distinct particles is not the main interest, we associate the chemical potentials to the conserved quantum numbers B, Q and S. The situation with strangeness is more complicated than with others due to the large mass of strange quarks. Even at most energetic heavy ion collisions nowadays, we cannot expect to create such a hot and long-lived system that strange quarks will also be in thermal equilibrium at hadronization time. In order to take into account the degree of equilibration of the strange quarks, the parameter γ_s ($\gamma_s \le 1$) was introduced [6]. For the equilibrated strangeness, $\gamma_s = 1$; the smaller it is the less strange quarks we have in the system in comparison with equlibrated situation. The γ_s is connected to every strange quark, and thus, for the particles containing f_s strange quarks, we have an additional multiplier in fugacity — $\gamma_s^{f_s}$ [1].

We assume that the sector of vanishing baryon density freezes out according to the resonance gas approximation and by vacuum percolation, the sector of finite baryon number freezes out according to baryon percolation in such a way to give

$$n(T,\mu) = \frac{1.24}{V_h}\left[1 - \frac{n_B(T,\mu)}{n(T,\mu)}\right] + \frac{0.34}{V_h}\left[\frac{n_B(T,\mu)}{n(T,\mu)}\right] \qquad (5)$$

as the defining equation for the freeze-out curve in the whole $T-\mu$ plane. It is clear that when $\mu = 0$, we recover condition (4), while in the limit of a cold nucleon gas, with $n/n_B = 1$, we get back Eq. (3). Moreover, it determines freeze-out fully in terms of geometric clustering based on the intrinsic hadronic scale and it contains no adjustable parameters of any kind.

Finally, we compare our results to the relevant data. Starting with the data for freeze-out temperatures at $\mu = 0$, we show a recent compilation of results from elementary hadron collisions [7] in Fig. 3; it is seen that the value $T \simeq 170$ MeV obtained from the vacuum percolation condition (4) is in complete agrreement with the data. Turning to heavy ion collisions, we show results

[1]Notice that the production of neutral hadrons with a fraction of f of $s\bar{s}$ content will also be suppressed by the γ_s^{2f} factor, i.e. $f_s = 2f$ for such hadrons.

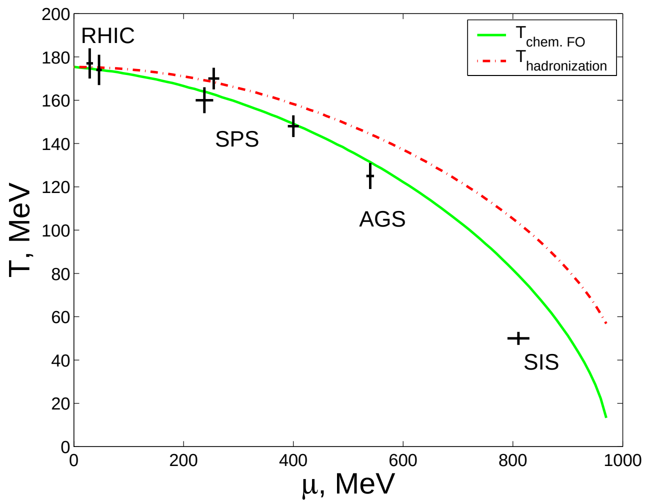

Fig. 5. The same as in Fig. 4, but for $\gamma_s = 0.5$

from the four different facilities [12][2] in Figs. 4, 5. The experimental results agree very well with our prediction, except the lowest energy point (SIS data), which is clearly overestimated[3]. Unless there are kinematic effects at SIS energy, the discrepancies at high baryon density seem to indicate that our assumption of an ideal nucleon gas is an oversimplification. Besides the repulsive effect due to the Fermi statistics of nucleons, there might be dynamical contributions which cannot be neglected.

We thank Krzysztof Redlich (Bielefeld and Wroclaw) for helpful discussions and Jaakko Manninen (Oulu) for his valuable assistence with programing.

1. See e.g., *Isichenko M.S.* // Rev. Mod. Phys. – 1992. – **64**. – P. 961.
2. *Baym G.* // Physica A. – 1979. – **96**. – P. 131; *Çelik T., Karsch F., Satz H.* // Phys. Lett. B. – 1980. – **97**. – P. 128; *Satz H.* // Nucl. Phys. A. – 1998. – **642**. – P. 130.
3. *Beth E., Uhlenbeck G.E.* // Physica. – 1937. – **4**. – P. 915.
4. *Hagedorn R.* // Nuovo cim. Suppl. – 1965. – **3**. – P. 147; Nuovo cim. A. – 1968. – **56**. – P. 1027.
5. See e.g., *Del Giudice E. et al.* // Ann. Phys. – 1972. – **70**. – P. 378.
6. *Becattini F.* // Z. Phys. C. – 1996. – **69**. – P. 485.
7. *Becattini F., Heinz U.* // Ibid. – 1997. – **76**. – P. 269.
8. *Becattini F.* // Nucl. Phys. A. – 2002. – **702**. – P. 336.
9. See e.g., *Karsch F.* // Lect. Notes Phys. – 2002. – **583**. – P. 209.
10. *Hagiwara K. et al.* // Phys. Rev. D. – 2002. – **66**. – P. 010001.
11. *Chodos A. et al.* // Ibid. – 1974. – **9**. – P. 3471.
12. *Becattini F. et al.* // Phys. Rev. C. – 2001. – **64**. – P. 02490.

УМОВИ ДЛЯ АДРОНІЗАЦІЇ І ЗАМОРОЖУВАННЯ

В. Магас, Х. Затц

Р е з ю м е

Адронна матерія зі взаємодією припускає існування крупномасштабного зв'язаного кластера, однорідного за своїми властивостями; розмір таких кластерів як функція адронної густини визначається за допомогою теорії перколяції. В рамках цієї теорії можна сформулювати умови заморожування і деконфайнменту в термінах перколяції відповідно адронних кластерів чи вакууму. Одержана умова заморожування як функція температури і баріонного хімічного потенціалу зв'язує між собою поведінку резонансного газу при низькій баріонній густині та нуклонну матерію з відштовхуванням при низькій температурі. Результати запропонованої нами моделі добре узгоджуються з експериментальними даними і обчисленнями в КХД на гратці.

УСЛОВИЯ ДЛЯ АДРОНИЗАЦИИ И ЗАМОРАЖИВАНИЯ

В. Магас, Х. Затц

Р е з ю м е

Взаимодействующая адронная материя предполагает существование крупномасштабного связанного кластера, однородного по своим свойствам; размер таких кластеров как функция адронной плотности определяется с помощью теории перколяции. В рамках этой теории можно сформулировать условия замораживания и деконфайнмента в терминах перколяции соответственно адронных кластеров или вакуума. Полученное условие замораживания как функция температуры и барионного химического потенциала связывает между собой поведение резонансного газа при низкой барионной плотности и нуклонную материю с отталкиванием при низкой температуре. Результаты предложенной нами модели хорошо согласуются с экспериментальными данными и вычислениями в КХД на решетке.

[2] Figs. 4, 5 are calculated for different γ_s. While γ_s varies from one point to another in the data, as a function of experimental energy, here we assume it to be constant for all the μ. This approximation is, in some sense, justified "a posteriori" by the fact that the resulting curves are found to be close for $\gamma_s = 1$ and 0.5.

[3] One has to take the SIS point into account very carefully, since there are only 4 particle ratios measured in this experiment and the fit with T and μ (γ_s was fixed to 1, although one could expect a lower value) through these 4 points was not so good - $\chi^2/d.o.f. \approx 14-15$.