THE QCD GLUON LADDERS AND HERA STRUCTURE FUNCTION

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UDC 539.12

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We report on the extension of the data fitting considering a QCD inspired model based on the summation of gluon ladders applied to the ep scattering. In lines of a two Pomeron approach, the structure function F_2 has a hard piece given by the model and the remaining soft contribution: a soft Pomeron and non-singlet content. In this paper, we carefully estimate the relative role of the hard and the soft pieces from a global fit in a large span of x and Q^2.

Introduction

The HERA small-x data [1] have introduced a challenge to the phenomenologists in order to describe the strong growth of the inclusive structure function F_2 as the Bjorken scale x decreases, supplemented by the scaling violations on the hard scale given by the photon virtuality Q^2. Concerning the Regge approach, the ep scattering process at high energy is dominated by the exchange of the Pomeron trajectory in the t-channel. From the hadronic phenomenology, this implies that the structure function would present a mild increasing on energy ($s \simeq Q^2/x$) since the soft Pomeron intercept ranges around $\alpha_{I\!P}(0) \approx 1.08$. Such behavior is in contrast with high energy ep data, where the effective intercept takes values $\lambda_{\rm eff} \simeq 1.3 \div 1.4$. In the Regge language, this situation can be solved by introducing the idea of new poles in the complex angular momentum plane, for instance rendered in the multipoles models [2–4], producing a quite successful description of data. Other proposition is the two Pomeron model [5], introducing an additional hard intercept and corresponding residue. However, a shortcoming from these approaches is the poor knowledge about the behavior on virtuality, in general modeled in an empirical way through the vertex functions.

On the other hand, the high photon virtuality allows the applicability of QCD perturbative methods. The DGLAP formalism (see [6]) is quite successful in describing most of the measurements on structure functions at HERA and hard processes in the hadronic colliders. This feature is even intriguing, since its theoretical limitations at high energy are well known. Other perturbative approach is the BFKL formalism (see, for example, [7]), well established at LO level but not yet completely understood at NLO accuracy. The main issue in the NLO BFKL effects is the correct account of the subleading corrections in all the orders of resummation. The LO BFKL approach can describe the HERA structure function in a limited kinematical range, i.e. at not so large Q^2 and small-x. A consistent treatment considering higher order resummations is currently being available and applications should be allowed in a near future.

In this contribution, we report on the extension of the data fitting to the HERA structure function F_2 using the finite sum of gluon ladders as a model for the hard Pomeron [8]. The model is based on the truncation of the BFKL series considering only the first few orders in the strong coupling perturbative expansion, where subleading contributions can be absorbed in the adjustable parameters. From the phenomenology on hadronic collisions [9], just three orders, $\sim [\alpha_s \ln(1/x)]^2$, are enough to describe current accelerator data. The hard Pomeron model should be supplemented by a soft piece accounting for the non-perturbative contributions to the process. The description therefore turns out similar to the two Pomeron model [5], with the advantage of a complete knowledge of the behavior on x and Q^2. The original model contains a reduced number of adjustable parameters: the normalization $\mathcal{N}_p$ and the non-perturbative scale μ^2 from the proton impact factor and the parameter x_0 scaling the logarithms on energy.

In spirit of a global analysis, in [10], two distinct choices for the soft Pomeron were analyzed. The resulting two Pomeron model was successful in describing data on the structure function F_2 and its derivatives (slopes on Q^2 and x) for $x \leq 2.5 \cdot 10^{-2}$

and $0.045 \leq Q^2 \leq 1500$ GeV^2. Here, we extend the description to the whole x range and analyze the relative role of the hard and soft pieces in the results. This work is organized as follows. In the next section, we shortly review the main expressions for the hard piece given by the summation up to the two-rung ladder contribution. In Sec. 2, an overall fit to the recent deep inelastic data is performed based on the above-mentioned hard contribution supplemented by the remaining soft Pomeron and non-singlet contributions. In the last section, we draw up our conclusions.

1. The Hard Contribution: Summing Gluon Ladders

Here we review the elements needed to calculate the structure function using the finite sum of gluon ladders in the ep collision, with center of mass energy W. The proton inclusive structure function written in terms of the cross sections for the scattering of transverse or longitudinal polarized photons, reads as [11]

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{\rm em}}\left[\sigma_T(x,Q^2) + \sigma_L(x,Q^2)\right], \qquad (1)$$

$$\sigma_{T,\,L}(x,Q^2) =$$

$$= \frac{\mathcal{G}}{(2\pi)^4}\int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2}\,\frac{d^2\boldsymbol{k'}}{\boldsymbol{k'}^2}\,\Phi^{\gamma^*}_{T,\,L}(\boldsymbol{k})\,F(x,\boldsymbol{k},\boldsymbol{k'})\,\Phi_p(\boldsymbol{k'}), \qquad (2)$$

where $\mathcal{G}$ is the color factor for the color singlet exchange and $\boldsymbol{k}$ and $\boldsymbol{k'}$ are the transverse momenta of the exchanged reggeized gluons in the t-channel. The $\Phi^{\gamma^*}_{T,\,L}(\boldsymbol{k})$ is the virtual photon impact factor and $\Phi_p(\boldsymbol{k'})$ is the proton impact factor. The first one is well known in perturbation theory at leading order, while the latter is modeled since, in the proton vertex, there is no hard scale to allow pQCD calculations. The kernel $F(x,\boldsymbol{k},\boldsymbol{k'})$ contains the dynamics of the process, for instance, the BFKL kernel. The amplitudes can be calculated order by order: for instance the Born contributions coming from the two gluon exchange and the one-rung ladder contribution read as

$$\mathcal{A}^{(0)} = \frac{2\,\alpha_s\,W^2}{\pi^2}\sum_f e_f^2\int\frac{d^2\boldsymbol{k}}{\boldsymbol{k}^4}\,\Phi^{\gamma^*}_{T,\,L}(\boldsymbol{k})\,\Phi_p(\boldsymbol{k}),$$

$$\mathcal{A}^{(1)} = \frac{6\alpha_s^2W^2}{8\pi^4}\sum_f e_f^2\,\ln\left(\frac{W^2}{W_0^2}\right)\times$$

$$\times\int\frac{d^2\boldsymbol{k}}{\boldsymbol{k}^4}\,\frac{d^2\boldsymbol{k'}}{\boldsymbol{k'}^4}\,\Phi^{\gamma^*}_{T,\,L}(\boldsymbol{k}),\mathcal{K}(\boldsymbol{k},\boldsymbol{k'})\,\Phi_p(\boldsymbol{k'}),$$

where α_s is considered to be fixed since we are in the framework of the LO BFKL approach. The perturbative kernel $\mathcal{K}(\boldsymbol{k},\boldsymbol{k'})$ can be calculated order by order in the perturbative expansion [11]. The Pomeron is attached to the off-shell incoming photon through the quark loop diagrams, where the Reggeized gluons are attached to the same and to different quarks in the loop. The virtual photon impact factor averaged over the transverse polarizations reads as [12]

$$\Phi^{\gamma^*}_{T,\,L}(\boldsymbol{k}) = \frac{1}{2}\int_0^1\frac{d\tau}{2\pi}\int_0^1\frac{d\rho}{2\pi}\,\frac{\boldsymbol{k}^2(1-2\tau\,\tau')(1-2\,\rho\,\rho')}{\boldsymbol{k}^2\,\rho\,\rho' + Q^2\rho\,\tau\,\tau'}, \qquad (3)$$

where ρ, τ are the Sudakov variables associated to the momenta at the photon vertex and $\tau' \equiv (1-\tau)$ and $\rho' \equiv (1-\rho)$.

Gauge invariance requires the proton impact factor to vanish at $\boldsymbol{k}$ going to zero, and it is modeled in a simple way,

$$\Phi_p(\boldsymbol{k'}) = \mathcal{N}_p\,\frac{\boldsymbol{k'}^2}{\boldsymbol{k'}^2+\mu^2}, \qquad (4)$$

where $\mathcal{N}_p$ is the unknown normalization of the proton impact factor and μ^2 is a scale which is typical of the non-perturbative dynamics. These scales will be considered adjustable parameters in the analysis. Considering the electroproduction process and summing the first orders in perturbation theory, we can write the expression for the inclusive structure function,

$$F_2^{\rm hard}(x,Q^2) = \frac{8}{3}\,\frac{\alpha_s^2}{\pi^2}\sum_f e_f^2\mathcal{N}_p\Big[\,I^{(0)}(Q^2,\mu^2) +$$

$$+\frac{3\,\alpha_s}{\pi}\,\ln\frac{x_0}{x}\,I^{(1)}(Q^2,\mu^2) +$$

$$+\frac{1}{2}\left(\frac{3\,\alpha_s}{\pi}\,\ln\frac{x_0}{x}\right)^2\,I^{(2)}(Q^2,\mu^2)\Big], \qquad (5)$$

where the functions $I^{(n)}(Q^2,\mu^2)$ correspond to the n-rung gluon ladder contribution. The quantity x_0 gives the scale normalizing the logarithms on energy for the LLA BFKL approach, which is arbitrary and enters as an additional parameter. The contributions are written explicitly as

$$I^{(0)} = \frac{1}{2}\ln^2\left(\frac{Q^2}{\mu^2}\right) + \frac{7}{6}\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{77}{18}, \qquad (6)$$

$$I^{(1)} = \frac{1}{6}\ln^3\left(\frac{Q^2}{\mu^2}\right) + \frac{7}{12}\ln^2\left(\frac{Q^2}{\mu^2}\right) +$$

$$+\frac{77}{18}\ln\left(\frac{Q^2}{\mu^2}\right)+\frac{131}{27}+2\,\zeta(3)\,, \quad (7)$$

$$I^{(2)}=\frac{1}{24}\ln^4\left(\frac{Q^2}{\mu^2}\right)+\frac{7}{36}\ln^3\left(\frac{Q^2}{\mu^2}\right)+\frac{77}{36}\ln^2\left(\frac{Q^2}{\mu^2}\right)+$$

$$+\left(\frac{131}{27}+4\zeta(3)\right)\ln\left(\frac{Q^2}{\mu^2}\right)+\frac{1396}{81}-\frac{\pi^4}{15}+\frac{14}{3}\zeta(3)\,, \quad (8)$$

where $\zeta(3) \approx 1.202$. The main result in [10] is in a good agreement, in terms of a χ^2/dof test, for the inclusive structure function in the range $0.045 \leq Q^2 \leq 1500$ GeV^2, once considering the restricted kinematical constraint $x \leq 0.025$. The non-perturbative contribution (from the soft dynamics), initially considered as a background, was found to be not negligible. We have estimated that such effects introduce a correction of the same order to the overall normalization. In the next section, we perform a global analysis in lines of our previous work [10], extending the range on x fitted by adding the non-singlet contribution modeled through the usual Regge parametrizations.

2. Fitting Results and Conclusions

In order to perform the fitting procedure, one uses Eqs. (6)—(8) for the hard piece. For the soft piece, we have selected a model with the most economical number of parameters. For this purpose, the latest version [13] of the CKMT model [14] was selected:

$$F_2^{\rm soft}(x,Q^2)=$$

$$=A\left(\frac{x_1}{x}\right)^{\Delta(Q^2)}\left(\frac{Q^2}{Q^2+a}\right)^{\Delta(Q^2)+1}(1-x)^{n_s(Q^2)}\,, \quad (9)$$

$$\Delta(Q^2)=\Delta_0\left(1+\frac{Q^2\Delta_1}{\Delta_2+Q^2}\right),$$

$$n_s(Q^2)=\frac{7}{2}\left(1+\frac{Q^2}{Q^2+b}\right), \quad (10)$$

where $\Delta(Q^2)$ is the Pomeron intercept. The non-singlet term is taken as

$$F^{\rm ns}(x,Q^2)=$$

$$=A_R\,x^{1-\alpha_R}\left(\frac{Q^2}{Q^2+a_R}\right)^{\alpha_R}(1-x)^{n_{ns}(Q^2)}\,, \quad (11)$$

$$n_{ns}(Q^2)=\frac{3}{2}\left(1+\frac{Q^2}{Q^2+d}\right). \quad (12)$$

Concerning the hard piece, Eq. (7), we selected the overall normalization factor as a free parameter defined as $\mathcal{N}=\frac{8}{3}\frac{\alpha_s^2}{\pi^2}\sum e_f^2\,\mathcal{N}_p$, considering four active flavors. It was also supplemented by a factor $(1-x)^{n_s}$ accounting for the large x effects. For the fitting procedure, we consider the data set containing all available HERA data for the proton structure function F_2 [15] adding only the latest data set of fixed target experiments [16]. For the fit, we have used 1059 experimental points for all available x and Q^2. The best fit parameters for this procedure are presented in Table. In the following, we discuss the relative contribution from the hard and soft pieces. Procedure (I) provides the same quality description $\chi^2/\mathrm{dof}=1.10$ in the whole available interval of the Bjorken variable and virtuality ($0 \leq x \leq 1$ and $0.045 \leq Q^2 \leq 30000$) as in the previous analysis [10]. It is worth to note that the relative contribution of the soft Pomeron remains the same in comparison with this analysis, i.e. the hard and soft pieces are comparable in the considered experimental domain. Concerning the soft Pomeron, the bare intercept comes out quite small in the whole range of Q^2.

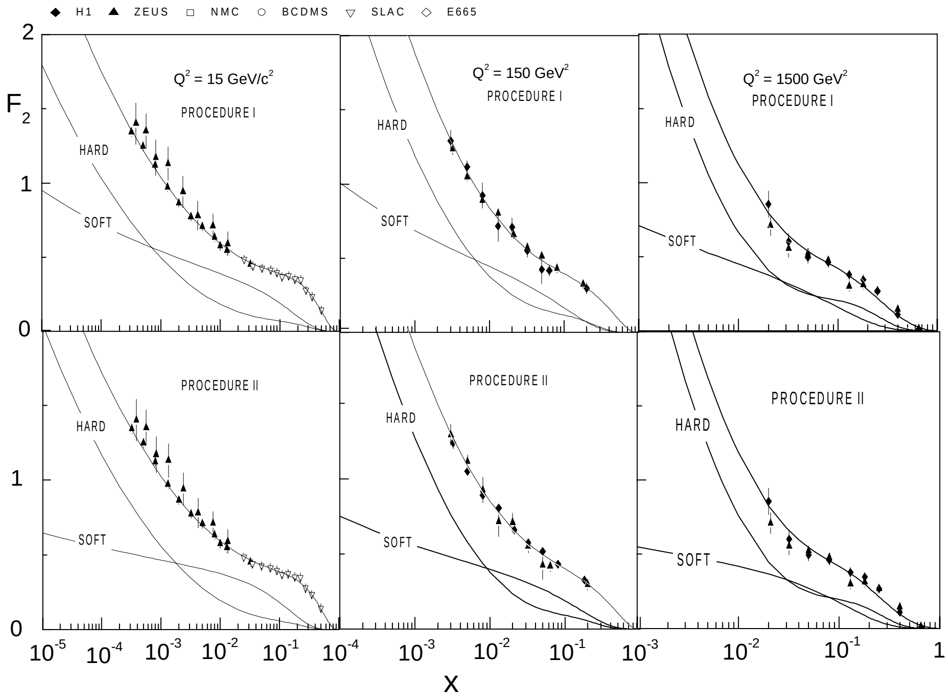
In procedure (II), we have restricted the soft Pomeron intercept through the following way:

$$\Delta(Q^2)=\Delta_0\left(\frac{Q^2}{\Delta_1+Q^2}\right)+\Delta_2\,. \quad (13)$$

The quality is slightly worse than procedure (I)(see Table).

Finally, we succeeded to perform a fit not considering the soft Pomeron contribution, but keeping the non-singlet term (procedure III). A shortcoming in these results is a quite small value for the strong coupling constant, suggesting it being kept fixed. For instance, one can uses $\alpha_s(M_Z)=0.119$. This analysis is not presented in the figures.

Concluding the analysis, in Figure we verify the relative role between the hard and soft Pomerons. We present it explicitly for the virtualities of 15, 150, and 1500 GeV^2, where the contributions from fitting (I) and (II) are shown. The general feature found is that, in procedure (I), the hard and soft pieces are almost equivalent at small x and intermediate Q^2. The soft contribution strongly decreases as the virtualities are large. From procedure (II), the soft piece corresponds to a small contribution in contrast with the analysis (I). In conclusion, we verify that the fitting procedure is equivalent to the model using a two-Pomeron approach [3,5], with the advantage of a clear understanding of the



Results for the inclusive structure function at $Q^2 = 15$, 150, and 1500 GeV^2 virtualities with the contribution of soft and hard pieces for the different procedures

behaviors on x and Q^2 of the corresponding hard content and extending our previous analysis made in [10].

A.L. is grateful to the Organizing Committee, in particular to Prof. L. Jenkovszky, for the possibility to participate in this nice Workshop. M.V.T.M. acknowledges the support of the High Energy Physics Phenomenology Group at the Physics Institute, UFRGS, Brazil.

Parameters of the fit

Procedure		I	II	III
Hard Pomeron	$\mathcal{N}$	0.0237	0.0238	0.0195
	μ^2	1.36	1.40	0.108
	x_0	0.0080	0.0101	0.00398
	α_s	0.202	0.217	0.080
Soft Pomeron	A	0.327	0.366	-
	a	0.754	1.11	-
	Δ_0	0.0064	0.07	-
	Δ_1	25.7	6.00	-
	Δ_2	6.31	0.02	-
	b	33.4	7.35	2.00
Non-singlet	A_R	5.94	6.73	10.3
	a_R	653	306	869
	d	1.14	0.796	2.22
χ^2/d.o.f.		1.10	1.23	1.39

N o t e. Procedure I: hard plus soft terms; Procedure II: similar to analysis I, but restricting soft Pomeron intercept; Procedure III: only hard term and non-singlet contribution.

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ГЛЮОННІ ДРАБИНИ КХД І СТРУКТУРНІ ФУНКЦІЇ HERA

О. Лендел, М.В.Т.Мачадо

Р е з ю м е

Обчислено структурні функції в моделі суми глюонних драбин. В наближенні двох померонів структурна функція складається з сильного внеску моделі, внеску м'якого померона і несинглетної частини. Оцінено роль сильного та м'якого внесків в широкій області змінних.

ГЛЮОННЫЕ ЛЕСТНИЦЫ КХД И СТРУКТУРНЫЕ ФУНКЦИИ HERA

А.Лендел, М.В.Т.Мачадо

Р е з ю м е

Рассчитаны структурные функции в модели суммы глюонных лестниц. В приближении двух померонов структурная функция состоит из сильного вклада модели, вклада мягкого померона и несинглетной части. Выполнена оценка роли сильного и мягкого вкладов в широкой области переменных.