TWO-MODE SQUEEZED AND ENTANGLED COLLINEAR GLUON STATES

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We study a non-perturbative evolution of collinear gluon states during small time. Fluctuations of gluons are less than those for coherent states. We show that the gluon entangled states which are closely related with the two-mode squeezed states of gluon fields can appear by analogy with the corresponding photon states in quantum optics.

Introduction

Analogies between multiple high-energy hadron production and photon production in quantum optics (QO) were discussed long ago [1—4]. It was shown, in particular, that the general distribution that characterizes $e^+e^-, p\bar{p}$, neutrino-induced collisions is a k-mode squeezed state distribution [5,6], the multiplicity distribution of pions being explained by the formalism of squeezed isospin states [7].

Squeezed states (SS), introduced by Stoler [8] and named by Hollenhorst [9], provoke great interest in connection with their uncommon properties: they can have both the sub-Poissonian (for coinciding phases) and super-Poissonian (for antiphases) statistics corresponding to the antibunching and bunching of photons [10—13], the characteristic behaviour of the factorial and cumulant moments [14]. Moreover, the oscillatory behaviour of the multiplicity distribution of photon SS [11] is differentiated from Poissonian and negative binomial distributions (NBD). The squeezed light is generated from a coherent one by nonlinear devices and is a pure quantum non-perturbative phenomenon [11—13].

Studying the non-perturbative evolution of gluon states prepared by the perturbative cascade stage in jets [15], we have proved within Quantum chromodynamics (QCD) that this stage of jet evolution can be a source of gluon SS by analogy with nonlinear devices in QO for photons [16—19]. There we investigated the single-mode squeezing effect for virtual gluons with defined colour and vector components during small temporal evolution. Using the local parton hadron duality, it is easy to show that, in this case, the behaviour of the hadron multiplicity distribution in jet events differs from that of the negative binomial one, which is confirmed by experiments on $pp, p\bar{p}$-collisions [20—22].

At the same time, two-mode photon SS [12] in the limit of infinite squeezing are isomorphic to the Bell states [23] which have been introduced in relation to the Einstein—Podolsky—Rosen (EPR) paradox [24] and are one of the examples of the entangled states for two polarizations [25]. The states

$$|\Phi^+\rangle = (|\updownarrow\rangle_1|\updownarrow\rangle_2 + |\leftrightarrow\rangle_1|\leftrightarrow\rangle_2)/\sqrt{2},$$

$$|\Phi^-\rangle = (|\updownarrow\rangle_1|\updownarrow\rangle_2 - |\leftrightarrow\rangle_1|\leftrightarrow\rangle_2)/\sqrt{2},$$

$$|\Psi^+\rangle = (|\updownarrow\rangle_1|\leftrightarrow\rangle_2 + |\leftrightarrow\rangle_1|\updownarrow\rangle_2)/\sqrt{2},$$

$$0|\Psi^-\rangle = (|\updownarrow\rangle_1|\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1|\updownarrow\rangle_2)/\sqrt{2} \quad (1)$$

are the basis of the Bell states. Each of these entangled states, for example, $|\Phi^\pm\rangle$, has a uncommon property: if one photon is registered with defined polarization (for example, with polarization $\updownarrow$), the other photon immediately becomes opposite polarized (longitudinal polarization). Thus, a measurement over one particle have an instantaneous effect on the other, possibly located at a large distance. In particular, it was shown recently that it is possible for two photons to travel a total of 600 meters through free space and still to remain "entangled" [26].

At finite squeezed r, an entangled state of continuous variables is known from quantum optics as a two-mode squeezed state [12, 25]:

$$|\mathrm{f}\rangle = S(r)|0\rangle_1|0\rangle_2 = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n\,|n\rangle_1|n\rangle_2, \quad (2)$$

where $S(r) = \exp\{r(a_1^+ a_2^+ - a_1 a_2)\}$ is the operator of two-mode squeezing. Rewriting (2) at a small squeezing parameter r as

$$|\mathrm{f}\rangle = |0\rangle_1|0\rangle_2 + r\,|1\rangle_1|1\rangle_2, \tag{3}$$

it is easy to show that the state vector $|\mathrm{f}\rangle$ describes the entangled state. Indeed, the entanglement condition of the considered photon state can be verified by investigation of the conditional probability $P\left(Y_j/X_i\right)(i,j=\overline{1,2})$

$$P\left(Y_j/X_i\right) = \frac{\|\langle Y_j|\,\langle X_i\,|\mathrm{f}\rangle\|^2}{\langle \mathrm{f}|\,X_i\rangle\,\langle X_i\,|\mathrm{f}\rangle} = \begin{cases} \delta_{ij} \\ \text{or} \\ 1-\delta_{ij}, \end{cases} \tag{4}$$

where $|X_1\rangle = |0\rangle_1, |X_2\rangle = |1\rangle_1, |Y_1\rangle = |0\rangle_2, |Y_2\rangle = |1\rangle_2$.

Two-mode gluon states with two different colours can lead to $q\bar{q}$-entangled states. Interaction of the quark entangled states with the stochastic vacuum (quantum measurement) has a remarkable property, namely, as soon as some measurement projects one quark onto a state with definite colour, the other quark also immediately obtains opposite colour that leads to the coupling of a quark-antiquark pair, string tension inside it, and free propagation of colourless hadrons. Therefore, the investigation of the gluon entangled states is an issue of the day.

1. Two-mode Squeezed Gluon States

In order to verify whether the gluon state vector describes the two-mode squeezed states on colours h and g, it is necessary to introduce the phase-sensitive Hermitian operators $(X_\lambda^{h,g})_1 = \left[b_\lambda^h + b_\lambda^g + b_\lambda^{h+} + b_\lambda^{g+}\right]/(2\sqrt{2})$ and $(X_\lambda^{h,g})_2 = \left[b_\lambda^h + b_\lambda^g - b_\lambda^{h+} - b_\lambda^{g+}\right]/(2i\sqrt{2})$ by analogy with quantum optics [12] and to establish the conditions under which the variance of one of them can be less than the variance of a coherent state. Here, $b_{(\lambda)}^h, b_\lambda^g (b_\lambda^{h+}, b_\lambda^{g+})$ are the operators annihilating (creating) gluons with colours $h,g=\overline{1,8}$ and polarization index $\lambda=\overline{1,3}$.

Mathematically, the condition of two-mode squeezing on colours h,g is expressed in the form of the inequality

$$\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle < 0. \tag{5}$$

Here, N is the normal-ordering operator such as

$$\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle = \pm\frac{1}{8}\Bigg\{\left\langle\left(b_\lambda^h\right)^2\right\rangle - \left\langle b_\lambda^h\right\rangle^2 +$$

$$+\left\langle\left(b_\lambda^g\right)^2\right\rangle - \left\langle b_\lambda^g\right\rangle^2 + \left\langle\left(b_\lambda^{h+}\right)^2\right\rangle - \left\langle b_\lambda^{h+}\right\rangle^2 +$$

$$+\left\langle\left(b_\lambda^{g+}\right)^2\right\rangle - \left\langle b_\lambda^{g+}\right\rangle^2 \pm 2\Big[\left\langle b_\lambda^{h+} b_\lambda^h\right\rangle - \left\langle b_\lambda^{h+}\right\rangle\left\langle b_\lambda^h\right\rangle +$$

$$+\left\langle b_\lambda^{g+} b_\lambda^g\right\rangle - \left\langle b_\lambda^{g+}\right\rangle\left\langle b_\lambda^g\right\rangle + \left\langle b_\lambda^{h+} b_\lambda^g\right\rangle - \left\langle b_\lambda^{h+}\right\rangle\left\langle b_\lambda^g\right\rangle +$$

$$+\left\langle b_\lambda^{g+} b_\lambda^h\right\rangle - \left\langle b_\lambda^{g+}\right\rangle\left\langle b_\lambda^h\right\rangle\Big] + 2\Big[\left\langle b_\lambda^h b_\lambda^g\right\rangle - \left\langle b_\lambda^h\right\rangle\left\langle b_\lambda^g\right\rangle +$$

$$+\left\langle b_\lambda^{h+} b_\lambda^{g+}\right\rangle - \left\langle b_\lambda^{g+}\right\rangle\left\langle b_\lambda^{g+}\right\rangle\Big]\Bigg\}. \tag{6}$$

The expectation values of the creation and annihilation operators for gluons with specified colour and polarization index are taken over the vector $|\mathrm{f}\rangle$ which describes the evolution of the virtual gluon field during a small time interval Δt within the interaction representation:

$$|\mathrm{f}\rangle \simeq |\mathrm{in}\rangle - i\,\Delta t\, H_{\mathrm{I}}(t_0)\,|\mathrm{in}\rangle. \tag{7}$$

Here, $H_{\mathrm{I}}(t_0) = H_{\mathrm{I}}^{(3)}(t_0) + H_{\mathrm{I}}^{(4)}(t_0)$ is the Hamiltonian of three-gluon ($H_{\mathrm{I}}^{(3)}$) and four-gluon ($H_{\mathrm{I}}^{(4)}$) self-interactions, the explicit forms of which are given in Appendix in the momentum representation, $|\mathrm{in}\rangle$ is an initial state vector of the virtual gluon field, $\Delta t = t - t_0$ (below we will assume $t_0 = 0$ and consequently $\Delta t = t$).

We choose the product of coherent states of the gluons with different colour and polarization indices as the initial state vector, $|\mathrm{in}\rangle \equiv |\,\alpha\rangle = \prod_{\lambda=1}^{3}\prod_{b=1}^{8}|\alpha_\lambda^b\rangle$, because any vector may be decomposed in these basis vectors and coherent states are widely used in QO [12, 13]. By analogy with QO, a gluon coherent state vector $|\alpha_\lambda^b\rangle$ is the eigenvector of the corresponding annihilation operator b_λ^b with eigenvalue $\alpha_{\lambda_1}^b$ which can be written in terms of the gluon coherent field amplitude $|\alpha_{\lambda_1}^b|$ and phase γ_λ^b of the given gluon field $\alpha_{\lambda_1}^b = |\alpha_{\lambda_1}^b|e^{i\,\gamma_{\lambda_1}^b}$. In each gluon coherent state $|\alpha_\lambda^b\rangle$, the gluon number with fixed colour b and polarization λ is arbitrary (the average multiplicity of a given gluon is equal to the square of the gluon coherent field amplitude $\langle n_\lambda^b\rangle = |\alpha_\lambda^b|^2$) and the phase of the considered state γ_λ^b is fixed.

Averaging the annihilation and creation operators $b_\lambda^h, b_\lambda^g, b_\lambda^{h+}, b_\lambda^{g+}$ in (6) over the evolved vector $|\mathrm{f}\rangle$ which is defined according to (7) with taking into account the chosen initial state vector, we write a two-mode squeezing condition in the form

$$\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle = \pm\frac{i\,t}{8}\Big\{\langle\alpha|\big[[H_\mathrm{I}(0),b_\lambda^{h+}],b_\lambda^{h+}\big]|\alpha\rangle - -\langle\alpha|\big[b_\lambda^h,[b_\lambda^h,H_\mathrm{I}(0)]\big]|\alpha\rangle + \langle\alpha|\big[[H_\mathrm{I}(0),b_\lambda^{g+}],b_\lambda^{g+}\big]|\alpha\rangle - -\langle\alpha|\big[b_\lambda^h,[b_\lambda^h,H_\mathrm{I}(0)]\big]|\alpha\rangle + 2\langle\alpha|\big[[H_\mathrm{I}(0),b_\lambda^{h+}],b_\lambda^{g+}\big]|\alpha\rangle - -2\langle\alpha|\big[b_\lambda^g,[b_\lambda^h,H_\mathrm{I}(0)]\big]|\alpha\rangle\Big\} < 0. \qquad (8)$$

It is easy to show that the three-gluon self-interaction (as for the single-mode squeezing of gluons, see [19]) doesn't lead to the squeezing effect since

$$\big[[H_\mathrm{I}^{(3)}(0),b_\lambda^{h+}],b_\lambda^{h+}\big]=0,\quad \big[[H_\mathrm{I}^{(3)}(0),b_\lambda^{g+}],b_\lambda^{g+}\big]=0,$$

$$\big[[H_\mathrm{I}^{(3)}(0),b_\lambda^{h+}],b_\lambda^{g+}\big]=0,\quad \big[b_\lambda^g,[b_\lambda^h,H_\mathrm{I}^{(3)}(0)]\big]=0,$$

$$\big[b_\lambda^h,[b_\lambda^h,H_\mathrm{I}^{(3)}(0)]\big]=0,\quad \big[b_\lambda^g,[b_\lambda^g,H_\mathrm{I}^{(3)}(0)]\big]=0. \qquad (9)$$

Thus, only the four-gluon self-interaction can yield a two-mode squeezing effect. Indeed, the two-mode squeezing condition can be written in explicit form as

$$\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle = \pm\frac{i\,t}{8}g^2(2\pi)^3\int d\tilde{k}_1 d\tilde{k}_2\times$$

$$\times\sum_{\lambda_1,\lambda_2}\Big\{\langle\alpha|b_{\lambda_1}^{b+}(k_1)b_{\lambda_2}^{c+}(k_2) - b_{\lambda_1}^{b}(k_1)b_{\lambda_2}^{c}(k_2)|\alpha\rangle\times$$

$$\times\Big[\delta(2\vec{k}-\vec{k_1}-\vec{k_2})-\delta(2\vec{k}+\vec{k_1}+\vec{k_2})\Big]\Big[(f_{ahb}f_{ahc}+$$

$$+f_{agb}f_{agc}+2f_{ahb}f_{agc})\Big(\varepsilon_\mu^{\lambda_1}(k_1)\,\varepsilon_{\lambda_2}^{\mu}(k_2)\varepsilon_\nu^{\lambda}(k)\,\varepsilon_\lambda^{\nu}(k)-$$

$$-\varepsilon_\mu^{\lambda_1}(k_1)\varepsilon_\lambda^{\mu}(k)\varepsilon_\nu^{\lambda_2}(k_2)\,\varepsilon_\lambda^{\nu}(k)\Big) - 2f_{ahg}f_{abc}\varepsilon_\mu^{\lambda_1}(k_1)\varepsilon_\lambda^{\mu}(k)\times$$

$$\times\varepsilon_\nu^{\lambda_2}(k_2)\varepsilon_\lambda^{\nu}(k)\Big] + 2\langle\alpha|b_{\lambda_1}^{b+}(k_1)b_{\lambda_2}^{c}(k_2)|\alpha\rangle\times$$

$$\times\Big[\delta(2\vec{k}-\vec{k_1}+\vec{k_2})-\delta(2\vec{k}+\vec{k_1}-\vec{k_2})\Big](f_{ahb}f_{ahc}+$$

$$+f_{agb}f_{agc}+f_{ahc}f_{agb}+f_{ahb}f_{agc})\Big(\varepsilon_\mu^{\lambda_1}(k_1)\,\varepsilon_{\lambda_2}^{\mu}(k_2)\times$$

$$\times\varepsilon_\nu^{\lambda}(k)\,\varepsilon_\lambda^{\nu}(k)-\varepsilon_\mu^{\lambda_1}(k_1)\,\varepsilon_\lambda^{\mu}(k)\varepsilon_\nu^{\lambda_2}(k_2)\,\varepsilon_\lambda^{\nu}(k)\Big)\Big\} < 0. \qquad (10)$$

Here, g is a self-interaction constant, $d\tilde{k}=\dfrac{d^3k}{(2\pi)^3 2k_0}$, k_0 is a gluon energy, ε_λ^μ is a polarization vector, f_{ahb} are the structure constants of $\mathrm{SU}_c(3)$ group.

For visualization, let us investigate the obtained two-mode squeezing condition (10) for a collinear gluon. In this case, the corresponding squeezing condition is

$$\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle = \pm\,t\,\frac{\alpha_s\pi}{4k_0}(f_{ahb}f_{ahc}+$$

$$+f_{agb}f_{agc}+f_{ahb}f_{agc}+f_{agb}f_{ahc})\times$$

$$\times\sum_{\lambda_1\neq\lambda}|\alpha_{\lambda_1}^b||\alpha_{\lambda_1}^c|\sin(\gamma_{\lambda_1}^b+\gamma_{\lambda_1}^c)<0. \qquad (11)$$

Here we have taken into account that $\alpha_{\lambda_1}^b=|\alpha_{\lambda_1}^b|e^{i\gamma_{\lambda_1}^b}$ and $\alpha_{\lambda_1}^c=|\alpha_{\lambda_1}^c|e^{i\gamma_{\lambda_1}^c}$, $\alpha_s=g^2/(4\pi)$ is a coupling constant. The two-mode squeezing condition is fulfilled for any cases apart from $\gamma_{\lambda_1}^b+\gamma_{\lambda_1}^c=0,\pi$. In particular, if all initial gluon coherent fields are real or imaginary, then the two-mode squeezing condition is not fulfilled as in the single-mode case. Obviously, the larger both the amplitudes of the initial gluon coherent fields with different colour and polarization indices and coupling constant, the larger is the two-mode squeezing effect.

Thus, the non-perturbative gluon evolution (at a large coupling constant) is very significant under investigation of the squeezing effect.

2. Entangled Collinear Gluon States

By analogy with QO, we assume that two-mode gluon SS with fixed colours h,g is closely connected with the corresponding entangled states of gluons.

At a small value of the squeeze factor, we have

$$r=2\left|\left\langle N\left(\Delta(X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle\right|. \qquad (12)$$

As initial states we can take a vector including $|0^h_\lambda\rangle\,|0^g_\lambda\rangle$ whose evolution at small time can be written in terms of the squeeze factor as

$$|\mathrm{f}\rangle = |0^h_\lambda\rangle|0^g_\lambda\rangle + r\,|1^h_\lambda\rangle|1^g_\lambda\rangle, \qquad (13)$$

where the squeeze factor for collinear gluons is defined as

$$r = t\,\frac{\alpha_s\pi}{2k_0}(f_{ahb}f_{ahc} + f_{agb}f_{agc} + f_{ahb}f_{agc}+$$

$$+ f_{agb}f_{ahc})\sum_{\lambda_1\neq\lambda}|\alpha^b_{\lambda_1}||\alpha^c_{\lambda_1}|\sin(\gamma^b_{\lambda_1} + \gamma^c_{\lambda_1}). \qquad (14)$$

From the obtained expression (14), it follows that the squeeze factor is not equal to zero for any cases apart from $\gamma^b_{\lambda_1} + \gamma^c_{\lambda_1} = 0, \pi$. Obviously, the larger both the amplitudes of the initial gluon coherent fields with different colour and polarization indices and coupling constant, the larger is the squeezing effect of colour gluons.

The entanglement condition of the considered gluon states with colours h, g and polarization λ can be verified by investigation of the conditional probability $P\,(Y_j/X_i)\,(i,j=\overline{1,2})$ by analogy with the corresponding condition for photons (4) assuming $|X_1\rangle = |0^h_\lambda\rangle$, $|X_2\rangle = |1^h_\lambda\rangle$, $|Y_1\rangle = |0^g_\lambda\rangle$, $|Y_2\rangle = |1^g_\lambda\rangle$. It is not complicated to make sure that condition (4) is fulfilled for the cases as for the two-mode squeezing for the state vector $|\mathrm{f}\rangle$ (13) which describes the non-perturbative evolution of the gluon fields during a small time.

Thus, by analogy with quantum optics as a result of the four-gluon self-interaction, we obtain two-mode squeezed gluon states which are also entangled.

Conclusion

Investigating the gluon fluctuations, we have proved theoretically the possibility of existence of the gluon two-mode squeezed states. The emergence of such remarkable states becomes possible owing to the four-gluon self-interaction of gluons. The three-gluon self-interaction does not lead to the squeezing effect.

We have shown that QCD evolution leads both to squeezing and entanglement of gluons. It should be noted that the greater both the amplitudes of the initial gluon coherent fields with different colour and polarization indices and coupling constant, the greater are the squeezing and entanglement effects of colour gluons.

Two-mode gluon states with two different colours can lead to $q\bar{q}$-entangled states whose role could be very significant for the explanation of the confinement phenomenon.

APPENDIX

$$H^{(3)}_{\mathrm{I}}(t) = ig(2\pi)^3 f_{abc}\sum_{\lambda_1,\lambda_2,\lambda_3}\int d\tilde{k}_1 d\tilde{k}_2 d\tilde{k}_3\times$$

$$\times\Big\{\vec{k}_1\vec{\varepsilon}_{\lambda_2}(k_2)\varepsilon^\nu_{\lambda_1}(k_1)\varepsilon^{\lambda_3}_\nu(k_3)\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\times$$

$$\times\left[b^a_{\lambda_1}(k_1)b^b_{\lambda_2}(k_2)b^c_{\lambda_3}(k_3)\,e^{-2i(k_{01}+k_{02}+k_{03})t}-\right.$$

$$\left.-\,b^{a+}_{\lambda_1}(k_1)b^{b+}_{\lambda_2}(k_2)b^{c+}_{\lambda_3}(k_3)\,e^{2i(k_{01}+k_{02}+k_{03})t}\right]+$$

$$+\,b^{a+}_{\lambda_1}(k_1)b^b_{\lambda_2}(k_2)b^c_{\lambda_3}(k_3)\,e^{2i(k_{01}-k_{02}-k_{03})t}\times$$

$$\times\Big[\vec{k}_3\vec{\varepsilon}_{\lambda_1}(k_1)\varepsilon^\nu_{\lambda_2}(k_2)\varepsilon^{\lambda_3}_\nu(k_3)+\vec{k}_2\vec{\varepsilon}_{\lambda_3}(k_3)\varepsilon^\nu_{\lambda_1}(k_1)\varepsilon^{\lambda_2}_\nu(k_2)-$$

$$-\vec{k}_1\vec{\varepsilon}_{\lambda_2}(k_2)\varepsilon^\nu_{\lambda_1}(k_1)\varepsilon^{\lambda_3}_\nu(k_3)\Big]\delta(\vec{k}_1-\vec{k}_2-\vec{k}_3)+$$

$$+\,b^{a+}_{\lambda_1}(k_1)b^{b+}_{\lambda_2}(k_2)b^c_{\lambda_3}(k_3)\,e^{2i(k_{01}+k_{02}-k_{03})t}\times$$

$$\times\Big[\vec{k}_1\vec{\varepsilon}_{\lambda_3}(k_3)\varepsilon^\nu_{\lambda_1}(k_1)\varepsilon^{\lambda_2}_\nu(k_2)+\vec{k}_3\vec{\varepsilon}_{\lambda_1}(k_1)\varepsilon^\nu_{\lambda_2}(k_2)\varepsilon^{\lambda_3}_\nu(k_3)-$$

$$-\vec{k}_1\vec{\varepsilon}_{\lambda_2}(k_2)\varepsilon^\nu_{\lambda_1}(k_1)\varepsilon^{\lambda_3}_\nu(k_3)\Big]\Big\}\delta(\vec{k}_1+\vec{k}_2-\vec{k}_3), \qquad (A.1)$$

$$H^{(4)}_{\mathrm{I}}(t) = \frac{g^2}{4}(2\pi)^3\sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\int d\tilde{k}_1 d\tilde{k}_2 d\tilde{k}_3 d\tilde{k}_4\times$$

$$\times\Big\{\varepsilon^\mu_{\lambda_1}(k_1)\varepsilon^{\lambda_3}_\mu(k_3)\varepsilon^\nu_{\lambda_2}(k_2)\varepsilon^{\lambda_4}_\nu(k_4)f_{abc}f_{ade}\times$$

$$\times\Big[\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4)\times$$

$$\times\Big(b^b_{\lambda_1}(k_1)b^c_{\lambda_2}(k_2)b^d_{\lambda_3}(k_3)b^e_{\lambda_4}(k_4)\,e^{-2i(k_{01}+k_{02}+k_{03}+k_{04})t}+$$

$$+\,b^{b+}_{\lambda_1}(k_1)b^{c+}_{\lambda_2}(k_2)b^{d+}_{\lambda_3}(k_3)b^{e+}_{\lambda_4}(k_4)\,e^{2i(k_{01}+k_{02}+k_{03}+k_{04})t}\Big)+$$

$$+\,4b^{b+}_{\lambda_1}(k_1)b^c_{\lambda_2}(k_2)b^d_{\lambda_3}(k_3)b^e_{\lambda_4}(k_4)\,e^{2i(k_{01}-k_{02}-k_{03}-k_{04})t}\times$$

$$\times\,\delta(\vec{k}_1-\vec{k}_2-\vec{k}_3-\vec{k}_4)+$$

$$+4b^{b+}_{\lambda_1}(k_1)b^{c+}_{\lambda_2}(k_2)b^{d+}_{\lambda_3}(k_3)b^{e}_{\lambda_4}(k_4)\,e^{2i(k_{01}+k_{02}+k_{03}-k_{04})t}\times$$

$$\times\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3-\vec{k}_4)\Big]+$$

$$+2b^{b+}_{\lambda_1}(k_1)b^{c+}_{\lambda_2}(k_2)b^{d}_{\lambda_3}(k_3)b^{e}_{\lambda_4}(k_4)\,e^{2i(k_{01}+k_{02}-k_{03}-k_{04})t}\times$$

$$\times\,\delta(\vec{k}_1+\vec{k}_2-\vec{k}_3-\vec{k}_4)\times$$

$$\times\Big[\varepsilon^{\mu}_{\lambda_1}(k_1)\varepsilon^{\lambda_3}_{\mu}(k_3)\varepsilon^{\nu}_{\lambda_2}(k_2)\varepsilon^{\lambda_4}_{\nu}(k_4)(f_{abc}f_{ade}+f_{abe}f_{adc})+$$

$$+\varepsilon^{\mu}_{\lambda_1}(k_1)\varepsilon^{\lambda_2}_{\mu}(k_2)\varepsilon^{\nu}_{\lambda_3}(k_3)\varepsilon^{\lambda_4}_{\nu}(k_4)f_{abd}f_{ace}\Big]\Big\}. \qquad (A.2)$$

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ДВОМОДОВІ СТИСНЕНІ ТА ЗЧЕПЛЕНІ КОЛІНЕАРНІ ГЛЮОННІ СТАНИ

В. Кувшинов, В. Шапаро

Р е з ю м е

Досліджено непертурбативну еволюцію колінеарних глюонних станів за короткий проміжок часу. Виявляється, що флуктуації глюонів є меншими порівняно з флуктуаціями в когерентних станах. Показано, що зчеплені глюонні стани, тісно пов'язані з двомодовими стисненими станами глюонних полів, можуть з'являтися аналогічно відповідним фотонним станам в квантовій оптиці.

ДВУХМОДОВЫЕ СЖАТЫЕ И СЦЕПЛЕННЫЕ КОЛЛИНЕАРНЫЕ ГЛЮОННЫЕ СОСТОЯНИЯ

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Изучается непертурбативная эволюция коллинеарных глюонных состояний за короткий период времени. Оказывается, что флуктуации глюонов меньше по сравнению с флуктуациями в когерентных состояниях. Показано, что сцепленные глюонные состояния, тесно связанные с двухмодовыми сжатыми состояниями глюонных полей, могут появляться аналогично соответствующим фотонным состояниям в квантовой оптике.