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RARE K-MESON DECAYS

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We present the theoretical consequences obtained with the help of the vector dominance model consistent with the perturbative quantum chromodynamics (QCD) for rare K-meson decays. These results are in qualitative and semi-quantitative agreement with the experimental data.

Introduction

Studying the rare modes of K-meson decays is an important source of information on their inner structure. This allows one to verify qualitatively and quantitatively the theoretical conclusions based on different model approaches concerning the K-meson structure and to understand the dynamics of interaction of the inner components of a K-meson. In 1990s, the intensive studies of rare decays of kaons, $\mathrm{K}\to \pi\nu\bar{\nu}$, $\mathrm{K}\to \pi \mathrm{e}^+\mathrm{e}^-$, etc., have been carried out on various experimental installations (BNL, FNAL, KEK, TRIUMF) [1]. The aim of the present work is to study the K-meson decays with the help of the Hamiltonian of four-quark interaction. Below, we use the effective Lagrangian obtained in [2]. It may be expressed in terms of the four-fermion operators of u-, d-, s-quarks. While obtaining these operators, it was assumed that the distances between quarks and gluons are small and the QCD, being the non-Abelian gauge theory, possesses the property of asymptotic freedom. This allow one to consider the quarks as almost free. The coupling constant α_s decreases with the distance between quarks logarithmically and may be taken as a small expansion parameter. In the given domain, we obtained the effective Lagrangian of weak interactions with the strangeness change $\Delta S = 1$ by using the renornormalization group analysis and perturbation theory in α_s.

In this paper, we propose a method of calculation of the K-meson rare decays that includes the following steps.

1. Use the effective Lagrangian of weak interactions with $\Delta S = 1$ obtained in [2] within the framework of the six-quark model.

2. Pass from the quark fields to the Lagrangians of observable fields by changing the quark currents by the relevant meson currents with the same quantum numbers [3].

At this, the meson currents are found from the Lagrangian within the framework of the model with broken chiral $U_3 \times U_3$ symmetry that takes into account the axial anomaly [4]. It results in the appearance of additional vertices due to the π^0—η—η' mixing that contribute to the rule $\Delta T = 1/2$. The largest contribution to the transition with $\Delta T = 1/2$ is given by the right hadron currents generated on the quark level by the penguin-type diagrams from the gluon field and electromagnetic field as well. It is necessary to note that here we made the analysis of the intermediate vector and axial-vector states that are important in the low-energy hadron physics [5, 6].

Chiral Lagrangian of Weak Interactions

The K-meson decays at small distances are described by the effective Hamiltonian of weak interactions with a change of strangeness $\Delta S = 1$ [4]

$$L_\mathrm{w}(\Delta S = 1) = k \sum_{n=1}^{6} C_n Q_n, \tag{1}$$

where

$$k = -1.86 \cdot 10^{-6}\,\mathrm{GV}^{-2}, \quad C_1 = -0.87 + 0.036\tau,$$

$$C_2 = 1.51 - 0.036\tau, \quad C_3 = -0.021 - 0.012\tau,$$

$$C_4 = 0.39, \quad C_5 = 0.011 + 0.007\tau,$$

$$C_6 = -0.047 - 0.072\tau, \quad \tau = S_2^2 + S_2 C_2 S_3 \frac{e^{i\delta}}{C_1 C_2}, \tag{2}$$

and Q_i are the four-fermion operators that contain a product of the left and right quark currents:

$$Q_1=(\bar{s}d)_{V-A}(\bar{u}u)_{V-A},\ Q_2=(\bar{s}u)_{V-A}(\bar{u}d)_{V-A},$$

$$Q_3=(\bar{s}d)_{V-A}(\bar{q}q)_{V-A},\ Q_4=-Q_1+Q_2+Q_3,$$

$$Q_5=(\bar{s}d)_{V-A}(\bar{u}u)_{V+A},\ Q_6=-8(\bar{s}_{\text{left}}q_{\text{right}})(\bar{q}_{\text{right}}d_{\text{left}}),$$

$$(\bar{q}_iq_j)_{V\pm A}\to\bar{q}_i\gamma_\mu(1\pm\gamma_5)q_j,\ q_{\text{left,right}}=\frac{1}{2}(1\pm\gamma_5)q. \quad (3)$$

Here we used the contributions of gluons and electromagnetic (EM) interactions [7]. The effective Hamiltonian reads

$$H_{\text{EM}}=(-G_F/\sqrt{2})c_1c_3s_1(C_7Q_7+C_8Q_8), \quad (4)$$

where

$$C_7=(0.037-0.067\tau)\alpha,\ C_8=(0.008-0.11\tau)\alpha,$$

$$\alpha=\frac{1}{137};$$

$$Q_7=(\bar{s}d)_{V-A}\left[(\bar{u}u)_{V+A}-\frac{1}{2}(\bar{d}d)_{V+A}-\frac{1}{2}(\bar{s}s)_{V+A}\right],$$

$$Q_8=(\bar{s}_\alpha d_\beta)_{V-A}\left[(\bar{u}_\beta u_\alpha)_{V+A}-\frac{1}{2}(\bar{d}_\beta d_\alpha)_{V+A}-\right.$$

$$\left.-\frac{1}{2}(\bar{s}_\beta u_\alpha)_{V+A}\right]. \quad (5)$$

The chiral Lagrangian describing the meson processes in the low-energy QCD domain may be presented as [3, 5]:

$$L=L_0+L_{\text{SB}}+L_Q+L_{\text{WZW}}, \quad (6)$$

$$L_0=\frac{F_\pi^2}{8}\text{Sp}(D_\mu UD_\mu U), \quad (7)$$

where L_0 is the kinetic part of the chiral Lagrangian that determines the p^2-th order of the expansion of the amplitudes of meson processes in the momenta of interacting particles; $F_\pi=94$ MeV is the $\pi\to\mu\nu$ decay constant; $U=\exp\left(\frac{2i}{f_\pi}\varphi\right)$ is the matrix of the nonet of pseudoscalar mesons, and $\varphi=\frac{1}{\sqrt{3}}\varphi_0+\frac{1}{\sqrt{2}}\sum_{i=0}^{8}\lambda_i\varphi_i$, $\varphi_i(i=0,...,8)$;

$$D_\mu U=\partial_\mu U+\frac{ig}{2}[V_\mu,U]-\frac{ig}{2}\{A_\mu,U\}, \quad (8)$$

V and A are the nonets of vector and axial-vector meson, respectively.

The gauge invariant Wess—Zumino—Witten generalization in the case of $1^\pm$-mesons takes the form:

$$L_{\text{WZW}}=\frac{1}{48\pi^2}\varepsilon_{\mu\nu\alpha\beta}[Z_{\mu\nu\alpha\beta}(U,V,A)-Z_{\mu\nu\alpha\beta}(I,V,A)]. \quad (9)$$

The explicit expression of $Z_{\alpha\beta\mu\nu}$ is given in [4].

$$L_Q=\frac{1}{32e^2}\text{Sp}[D_\mu UU^+,\partial_\mu UU^+]^2. \quad (10)$$

Relation (10) describes the p^4-th order of the basic Lagrangian L_0. The coefficient e^2 in the QCD low-energy domain is uniquely connected with the number of quark colors, N_c: $e^2=12\pi^2/N_c$.

The total Lagrangian of the system of 0^--, $1^\pm$-mesons must have the kinetic and mass terms along with the gauge-invariant part (7)—(10):

$$L'=-\frac{1}{2}\text{Sp}(F_{\mu\nu}^{\text{left}}F_{\mu\nu}^{\text{left}}+F_{\mu\nu}^{\text{right}}F_{\mu\nu}^{\text{right}})-$$

$$-m_V^2\text{Sp}(V_\mu^{\text{left}}V_\mu^{\text{left}}+V_\mu^{\text{right}}V_\mu^{\text{right}})+$$

$$+\frac{F_\pi^2}{4}\text{Sp}[M(U+U^+)], \quad (11)$$

where $F_{\mu\nu}^{\text{left,right}}=\partial_\mu V_\nu^{\text{left,right}}-\partial_\nu V_\mu^{\text{left,right}}-ig[V_\mu^{\text{left,right}},V_\nu^{\text{left,right}}]$, $V_\mu^{\text{left,right}}=(\nu_\mu\pm a_\mu)/2$, M is the mass matrix that is chosen in diagonal form, $M=M_i^2\delta_{ij}$.

Lagrangian (1) and (3) is obtained in [8] for the following numerical values of parameters: $\alpha_s(\mu^2)=1$, $\alpha_s(m_z)=0.118$, $m_t=175$ GeV, and $m_b=5$ GeV.

The Lagrangian that describes the $K-\pi$ transitions in the QCD low-energy domain takes the form:

$$L_{\text{w}}=k\left\{C_1(J_\mu^6+iJ_\mu^7)\left(J_\mu^3+\frac{1}{\sqrt{3}}J_\mu^8+\sqrt{\frac{2}{3}}J_\mu^0\right)+\right.$$

$$+(C_3-C_5)\sqrt{6}(J_\mu^6+iJ_\mu^7)J_\mu^0+$$

$$+C_4\left[(J_\mu^6+iJ_\mu^7)\left(-J_\mu^3-\frac{1}{\sqrt{3}}J_\mu^8+\sqrt{\frac{2}{3}}J_\mu^0\right)+\right.$$

$$\left.+\sqrt{\frac{2}{3}}(-\sqrt{2}J_\mu^8+J_\mu^0)(J_\mu^6+iJ_\mu^7)\right]+C_6\frac{1}{m_s+m_d}\times$$

$$\times\left[\frac{1}{m_d}\partial_\mu(J_\mu^6+iJ_\mu^7)\partial_\mu\left(-J_\mu^3+\frac{1}{\sqrt{3}}J_\mu^8+\sqrt{\frac{2}{3}}J_\mu^0\right)\right)+$$

$$\left.\left.+\frac{1}{m_s}\partial_\mu\left(-\frac{2}{\sqrt{3}}J_\mu^8+\sqrt{\frac{2}{3}}J_\mu^0\right)\partial_\mu(J_\mu^6+iJ_\mu^7)\right]\right\}, \quad (12)$$

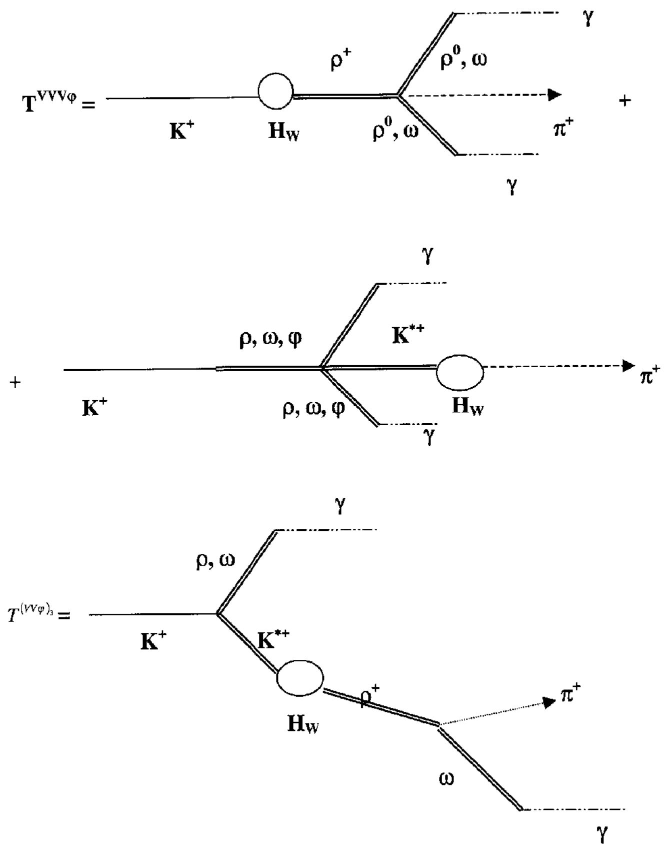

Fig. 1. Pole diagrams of $K^+ \to \pi^+\gamma\gamma$ decay

where J^i_μ $(i = 0, ..., 8)$ are the meson currents that have the conventional $(V - A)$ structure: $J_\mu = J^V_\mu - J^A_\mu$.

The quark currents entering into (1)—(5) are expressed through the corresponding meson currents that are proportional to $1^\pm$-, 0^--mesons in the vector dominance model:

$$(J^\nu_\mu)_{ab} = -\frac{\delta L}{\delta \partial_\mu \omega_{ab}} = \frac{m^2_\nu}{g}(V_\mu)_{ab},$$

$$(J^A_\mu)_{ab} = -\frac{\delta L}{\delta \partial_\mu B_{ab}} = \frac{m^2_A}{g}(A_\mu)_{ab} + F_\pi \partial_\mu P_{ab} -$$

$$-\frac{N_c}{24\pi^2}\varepsilon_{\alpha\beta\gamma}(U\partial_\alpha U^+ U\partial_\beta U^+ U\partial_\gamma U^+)_{ab} + ... \quad (13)$$

$K^+ \to \pi^+\gamma\gamma$ Decay

The $K^+ \to \pi^+\gamma\gamma$ decay is a purely structural transition. Due to the gauge invariance of electromagnetic interactions, all the bremsstrahlung diagrams are mutually compensated, and there is no bremsstrahlung $K^+ \to \pi^+\gamma\gamma$ decay.

According to formulas (12), (13), the weak transitions $K \to \pi$, $K^* \to \rho$, and $K^{*0} \to V^0$ are described by the following parts of the Lagrangian:

$$L_{K\to\pi} = F^2_\pi k\left(1 + \frac{C_6}{(m_s + m_u)(m_u + m_d)}\right)\times$$

$$\times[(\partial_\mu\pi^+\partial_\mu K^- + \partial_\mu\pi^-\partial_\mu K^+) - \partial_\mu(K^0 + \bar{K}^0)\partial_\mu\pi^0],$$

$$L_{K^*\to\rho} = \left(\frac{m_V}{g_V}\right)^2 m^2_V k\Bigg((C_1 + C_3 + C_5)(K^{*-}_\mu\rho^+_\mu +$$

$$+K^{*+}_\mu\rho^-_\mu - \bar{K}^{*0}_\mu + K^{*0}_\mu\left(\rho^0_\mu + \frac{1}{\sqrt{3}}\omega_\mu - \frac{2}{\sqrt{3}}\varphi_\mu\right)\Bigg). \quad (14)$$

Within the framework of the idea of vector dominance, the electromagnetic interaction is introduced in (6) by using the substitution [9]

$$V^k_\mu \to V^k_\mu - \frac{e}{g_V}\left(\delta_{k3} + \frac{1}{\sqrt{3}}\delta_{k8}\right)A_\mu, \quad k = 0, ..., 8, \quad (15)$$

and is described by the Lagrangian

$$L_{\rm EM} = \frac{\sqrt{2}m^2_V}{g_V}eA_\mu\left(\rho^0_\mu + \frac{1}{3}\omega_\mu - \frac{\sqrt{2}}{3}\varphi_\mu\right). \quad (16)$$

Now we separate those effective Lagrangians in (6) that describe different vertices of the strong interaction of vector mesons with pseudoscalar fields φ. The minimal interaction $V\varphi\varphi$ is described by the Lagrangian

$$L_{V\varphi\varphi} = \frac{ig_V}{2}{\rm Sp}[V_\mu(\varphi\partial_\mu\varphi - \partial_\mu\varphi\varphi)], \quad (17)$$

that allows us to connect the phenomenological gauge constant g_V with the $\rho \to \pi\pi$ decay constant,

$$g_V = \sqrt{2}g_{\rho\pi\pi}, \quad \frac{g^2_{\rho\pi\pi}}{4\pi} = 3.2. \quad (18)$$

The anomalous interactions of vector and pseudoscalar mesons take the form

$$L_{\nu\nu\varphi} = -g_{\nu\nu\varphi}\varepsilon_{\mu\nu\alpha\beta}\,{\rm Sp}(\partial_\mu V_\nu\partial_\alpha V_\beta\varphi),$$

$$g_{\nu\nu\varphi} = \frac{3g^2_V}{16\sqrt{2}\pi^2 F_\pi}; \quad (19)$$

$$L_{\nu\varphi\varphi\varphi} = ih\varepsilon_{\mu\nu\alpha\beta}\,{\rm Sp}(V_\mu\partial_\nu\varphi\partial_\alpha\phi\partial_\beta\varphi),$$

$$h = -\frac{g_V}{4\sqrt{2}\pi^2 F^3_\pi}\left[1 - \frac{3}{2}\left(\frac{g_V F_\pi}{m_V}\right)^2 + \frac{3}{8}\left(\frac{g_V F_\pi}{m_V}\right)^4\right]; \quad (20)$$

$$L_{\nu\nu\nu\varphi} = i\bar{h}\varepsilon_{\mu\nu\alpha\beta}\{\mathrm{Sp}(V_\mu V_\nu V_\alpha \partial_\beta \varphi) -$$

$$-\mathrm{Sp}\lfloor(\partial_\mu V_\nu V_\alpha + V_\mu \partial_\nu V_\alpha)(\varphi V_\beta - V_\beta \varphi)\rfloor\},$$

$$\bar{h} = -\frac{g_V^3}{32\sqrt{2}\pi^2 F_\pi}. \tag{21}$$

The $K^+ \to \pi^+\gamma\gamma$ decay amplitude is determined by the pole diagrams presented in Fig. 1.

The contribution of chiral anomalies in the $K^+ \to \pi^+\gamma\gamma$ decay amplitude is

$$T_{K+\to\pi^+\gamma\gamma} = \varepsilon_{\alpha\beta\mu_1\mu_2}\varepsilon^1_{\mu_1}\varepsilon^2_{\mu_2}(q_1 - q_2)_\beta T_\alpha^{\nu\nu\nu\varphi} +$$

$$+ i\varepsilon_{\sigma\mu_1\alpha\beta}\varepsilon_{\sigma\mu\alpha^1\beta^1}\varepsilon^1_{\mu_1}\varepsilon^2_{\mu_2} q_{1\alpha} q_{2\alpha^1} T^{\nu\nu\varphi}_{\beta\beta^1}; \tag{22}$$

$$T_\alpha^{\nu\nu\nu\phi} = -2\sqrt{2}e^2\rangle\pi^+|H_{\mathrm{w}}^{3/2}|K^+\langle F_\pi \frac{m_V^2}{g_V^3}\times$$

$$\times\left(\frac{4}{3}\frac{1}{m_\rho^2 - m_K^2}K_\alpha + \frac{26}{27}\frac{1}{m_{K^*}^2 - m_\pi^2}P_\alpha\right); \tag{23}$$

$$T^{(\nu\nu\varphi)_1}_{\beta_1\beta^1} = -e^2\rangle\pi^+|H_{\mathrm{w}}^{3/2}|K^+\langle\left(\frac{4\sqrt{2}}{9}F_\pi^2\frac{m_K^2}{m_K^2 - m_\pi^2} -\right.$$

$$\left.-\frac{8\sqrt{2}}{9}\left(\frac{m_V}{g_V}\right)^2\right)\left[\frac{(p+q_1)_\beta(p+q_1)_{\beta^1}}{m_p^2 - (p+q_1)^2} +\right.$$

$$\left.+\frac{(p+q_2)_\beta(p+q_2)_{\beta^1}}{m_p^2 - (p+q_2)^2}\right]; \tag{24}$$

$$T^{(\nu\nu\varphi)_2}_{\beta\beta^1} = e^2\rangle\pi^+|H_{\mathrm{w}}^{3/2}|K^+\langle\frac{g^2_{\nu\nu\varphi}}{g_V^2}\left[\frac{4\sqrt{2}}{9}F_\pi^2\frac{m_\pi^2}{m_K^2 - m_\pi^2} +\right.$$

$$\left.+\frac{4\sqrt{2}}{9}\left(\frac{m_V}{g_V}\right)^2\right]\left[\frac{(p+q_1)_\beta(p+q_1)_{\beta^1}}{m_{K^*}^2 - (p+q_1)^2} +\right.$$

$$\left.+\frac{(p+q_2)_\beta(p+q_2)_{\beta^1}}{m_{K^*}^2 - (p+q_2)^2}\right]; \tag{25}$$

$$T^{(\nu\nu\varphi)_3} = -e^2\rangle\rho^+|H_{\mathrm{w}}^{3/2}|K^{*+}\langle\left(\frac{8\sqrt{2}}{9}\left(\frac{m_V}{g_V}\right)^2 m_\rho^2\times\right.$$

$$\times\frac{(p+q_1)_\beta(p+q_1)_{\beta^1}}{[m_{K^*}^2 - (p+q_1)^2][m_\rho^2 - (p-q_1)^2]} +$$

$$\left.+\frac{(p+q_2)_\beta(p+q_2)_{\beta^1}}{[m_{K^*}^2 - (p+q_2)^2][m_\rho^2 - (p+q_2)^2]}\right), \tag{26}$$

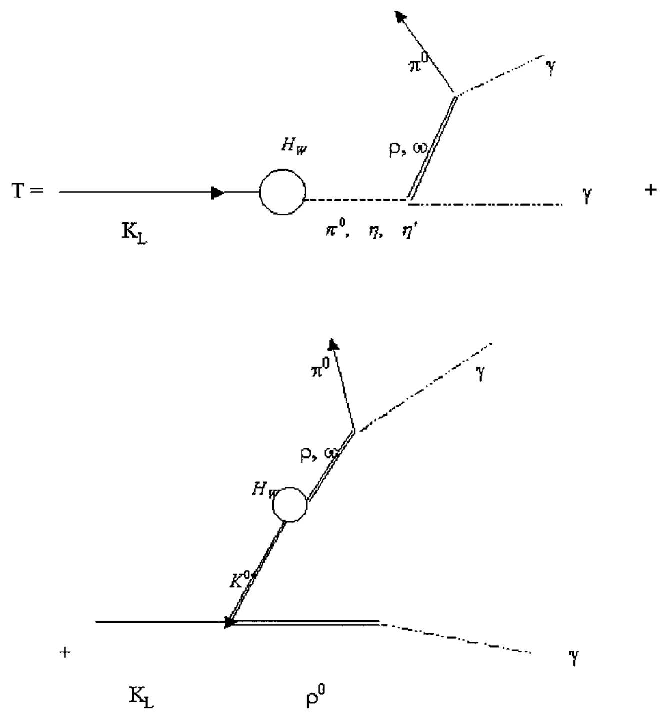

Fig. 2. Pole diagrams of the $K_L \to \pi^0\gamma\gamma$ decay

where k and p are the 4-momenta of K- and π-mesons.

The partial decay probability or the ratio of the partial decay width to the total decay width may be presented as

$$B^{\mathrm{theor}} = B^{VVV\varphi} + B^{VV\varphi} + B^{\mathrm{interf}}.$$

Numerical evaluations showed that the main contribution to the $K^+ \to \pi^+\gamma\gamma$ decay is given by the diagrams conditioned by the $VVV\varphi$-, $VV\varphi$-anomalies with domination of the former:

$$B^{VV\varphi} = 5\cdot 10^{-10}, \quad B^{VVV\varphi} = 2\cdot 10^{-6},$$

$$B^{\mathrm{interf}} = -5.4\cdot 10^{-9}. \tag{27}$$

The total partial probability $B^{\mathrm{theor}}_{K^+\to\pi^+\gamma\gamma} = 0.7\cdot 10^{-6}$ is only 1.5 times lower than the upper experimental limit $B^{\mathrm{exp}}_{K^+\to\pi^+\gamma\gamma} < 1\cdot 10^{-6}$ [10]. In the same manner, we calculated the $K^+ \to \pi^+e^+e^-$, $K^+ \to \pi^+\mu^+\mu^-$ partial decay widths. The results of these calculations are given in Table 1.

Table 1

Decay	B^{exp} [10]	B^{theor}
$K^+ \to \pi^+e^+e^-$	$(2.74 \pm 0.23)\cdot 10^{-7}$	$1.25\cdot 10^{-7}$
$K^+ \to \pi^+\mu^+\mu^-$	$< 2.3\cdot 10^{-7}$	$0.7\cdot 10^{-7}$
$K^+ \to \pi^+\gamma\gamma$	$< 1\cdot 10^{-6}$	$0.7\cdot 10^{-6}$

$K_L \to \pi^0\gamma\gamma$ Decay

The $K_L \to \pi^0\gamma\gamma$ decay is defined by the transitions $K_L \to$P, P$\to$Vγ, and V$\to \pi^0\gamma$. The decay diagrams are presented in Fig. 2.

The parts of the Lagrangian that describe the vertices of diagrams (Fig.2) are given by (14). The vertex obtained from (19) with regard for (16) is

$$L = g_{VV\varphi}\varepsilon_{\mu\nu\alpha\beta}\partial_\mu A_\nu \partial_\alpha V_\beta \varphi, \quad (28)$$

where $g_{VV\varphi} = \frac{3eg_V}{16\sqrt{2}\pi^2 F_\pi}$.

In the calculations, it is important to take into account the mixing of 0^--, $1^\pm$-neutral fields caused by the mass terms:

$$\pi_0 = 0.95\eta' - 0.31\eta + 0.9\cdot 10^{-2}\pi^0,$$

$$\pi_8 = -0.31\eta' - 0.95\eta + 1.7\cdot 10^{-2}\pi^0,$$

$$\pi_3 = -0.33\cdot 10^{-2}\eta' + 1.9\cdot 10^{-2}\eta + 0.099\pi^0,$$

$$V_8 = \cos\theta_\nu\varphi - \sin\theta_\nu\omega, \quad V_0 = \sin\theta_\nu\varphi + \cos\theta_\nu\omega,$$

$$\theta_\nu = 30^\circ. \quad (29)$$

The CP-preserving amplitude of the $K_L \to \pi^0\gamma\gamma$ decay in the vector dominance model reads

$$A(K_2(k) \to \pi^0(p)\gamma(q_1)\gamma(q_2)) = \varepsilon_{1\mu}\varepsilon_{2\nu}T^{\mu\nu}(q_1,q_2,k), \quad (30)$$

where ε_μ is the photon polarization vector.

$$T^{\mu\nu}(q_1,q_2,k) = -(q_2^\mu q_1^\nu - q_1q_2g_{\mu\nu})\times$$

$$\times\sum_{V=\omega,\rho,\varphi} G_V\left(\frac{k(-k-q_2)}{(k-q_2)^2-m_V^2} + \frac{1}{(k-q_1)^2-m_V^2}\right)+$$

$$+\left(\frac{q_1^2k_1q_2}{q_1q_2}g_{\mu\nu} - \frac{kq_2}{q_1q_2}q_{1\mu}q_{1\nu} + g_{1\mu}k_\nu - \frac{q_1^2}{q_1q_2}q_{2\mu}k_\nu\right)\times$$

$$\times\sum_{V=\rho,\omega,\varphi} G_V\frac{q_1q_2}{(k-q_1)^2-m_V^2}q_2^2\frac{kq_1}{q_1q_2}g_{\mu\nu}, \quad (31)$$

$$G_\rho = g^2_{\rho\pi^0\gamma}\frac{\langle\pi^0|H_{\rm w}|K_L\rangle}{m_K^2}\left(\frac{1}{9}\frac{1}{1-\Delta_\pi^2} + \frac{1}{9}(\sqrt{3}\cos\theta-\right.$$

$$-\sqrt{6}\sin\theta)\frac{1}{\sqrt{3}}(\cos\theta + \frac{4}{\sqrt{6}}\sin\theta)\frac{1}{1-\Delta_\eta^2} + \frac{1}{9}(\sqrt{3}\sin\theta+$$

$$\left.+\sqrt{6}\cos\theta)(\frac{1}{\sqrt{3}}\sin\theta - \frac{4}{\sqrt{6}}\cos\theta)\frac{1}{1-\Delta_{\eta^1}^2}\right), \quad (32)$$

$$G_\omega = g^2_{\omega\pi^0\gamma}\frac{\langle\pi^0|H_{\rm w}|K_L\rangle}{m_K^2}\left(\frac{1}{9}\frac{1}{1-\Delta_\pi^2} + \frac{1}{9}(\sqrt{3}\cos\theta-\right.$$

$$-\sqrt{6}\sin\theta)\frac{1}{\sqrt{3}}(\cos\theta + \frac{4}{\sqrt{6}}\sin\theta)\frac{1}{1-\Delta_\eta^2} + \frac{1}{9}(\sqrt{3}\sin\theta+$$

$$\left.+\sqrt{6}\cos\theta)(\frac{1}{\sqrt{3}}\sin\theta - \frac{4}{\sqrt{6}}\cos\theta)\frac{1}{1-\Delta_{\eta^1}^2}\right), \quad (33)$$

$$G_\varphi = g^2_{\varphi\pi^0\gamma}\frac{\langle\pi^0|H_{\rm w}|K_L\rangle}{m_K^2}\left(\frac{1}{9}\frac{1}{1-\Delta_\pi^2} + \frac{1}{9}(\sqrt{3}\cos\theta-\right.$$

$$-\sqrt{6}\sin\theta)\frac{1}{\sqrt{3}}(\cos\theta + \frac{4}{\sqrt{6}}\sin\theta)\frac{1}{1-\Delta_\eta^2} + \frac{1}{9}(\sqrt{3}\sin\theta+$$

$$\left.+\sqrt{6}\cos\theta)(\frac{1}{\sqrt{3}}\sin\theta - \frac{4}{\sqrt{6}}\cos\theta)\frac{1}{1-\Delta_{\eta^1}^2}\right), \quad (34)$$

where $\Delta_p^2 = \left(\frac{m_p}{m_k}\right)^2$.

In the same manner, we calculated the widths of the $K_L \to \pi^0e^+e^-$ and $K_L \to \pi^0\mu^+\mu^-$ decays. The results of calculations are collected in Table 2.

T a b l e 2

Decay	$B^{\rm exp}$ [10]	$B^{\rm theor}$
$K_L \to \pi^0\gamma\gamma$	$(1.7 \pm 0.28)\cdot 10^{-6}$	$0.5\cdot 10^{-8}$
$K_L \to \pi^0e^+e^-$	$< 4.3\cdot 10^{-9}$	$1\cdot 10^{-9}$
$K_L \to \pi^0\mu^+\mu^-$	$< 5.1\cdot 10^{-9}$	$3\cdot 10^{-9}$

Conclusions

In this paper, we have shown that the chiral anomalies give the main contribution to the K-meson rare decays and completely determine the amplitude of structural radiation in these processes. It is worth noting that the account of the chiral anomalies allows one to fit the theoretical data with experimental ones only if weak transitions are considered. Thus, the chiral theory and vector dominance model make it possible to obtain the numerical evaluations of the partial probabilities. In the calculations, we took into account the contributions of both small and large distances. The latter are considered with the help of bosonization.

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РІДКІСНІ РОЗПАДИ КАОНІВ

В.І. Сабов, Т.І. Данило, М.Я. Євич

Р е з ю м е

Одержано теоретичні наслідки з моделі векторної домінантності, сумісної з пертурбативною квантовою хромодинамікою, для рідкісних розпадів каонів. Одержані теоретичні розрахунки якісно та напівкількісно узгоджуються з експериментальними даними.

РЕДКОСТНЫЕ РАСПАДЫ КАОНОВ

В.И. Сабов, Т.И. Данило, М.Я. Евич

Р е з ю м е

Получены теоретические следствия из модели векторной доминантности, совместимой с пертурбативной квантовой хромодинамикой, для редкостных распадов каонов. Полученные теоретические расчеты соответствуют экспериментальным данным.