ACCELERATION OF ELECTRONS BY WAKE FIELDS OF A REGULAR TRAIN OF BUNCHES IN A DIELECTRIC WAVEGUIDE OF FINITE LENGTH

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The excitation of wake fields by a regular train of relativistic electron bunches in a cylindrical waveguide, which is partially filled with a dielectric, is considered. The expressions for all components of the electromagnetic field excited in a dielectric waveguide are obtained. The waveguide's finiteness along the longitudinal direction is accounted by the introduction of the wake field's trailing edge, which propagates with the velocity equal to the group velocity of the resonance wave. The numerical simulation of the self-consistent dynamics of particles of bunches in the wake fields, which the bunches excite themselves, is carried out. It is shown that, in the cases of relativistic bunches with small charge or ultra-relativistic intense bunches, taking the dynamics into account does not influence essentially the excited field. High radial stability of such bunches is found. Acceleration of electrons in a wake field of a train of bunches is investigated. Numerical calculations are carried out for the systems with parameters which are close to those of the experiments carried out in NSC "KhIPT", Ukraine and in the National Laboratory of High Energy Physics (NLHEP), Japan. The calculated accelerating fields are 100 kV/m and 18 MV/m, respectively.

Introduction

An electromagnetic field is excited when a charged bunch moves in a slow-wave medium with a higher velocity than the phase velocity of eigenwaves of the medium. This field is bound with the Cherenkov radiation of a bulk of charged particles. Inasmuch as the electromagnetic field excited by a charged bunch fills the region behind the bunch, it was called the wake field. The waveguide, which is partially filled with dielectric, can serve as a slow-wave medium.

Recently a number of articles has appeared, where it has been proposed to use intense wake fields excited in a dielectric waveguide for intense acceleration of charged particles [1, 2] and for generation of high-power pulsed radiation [3]. Besides the theoretical investigations, the experiments demonstrating the availability of this method have been carried out [4, 5]. In [5, 6], it has been proposed to use the regular train of identical relativistic bunches with relatively small charge for obtaining an intense wake field. High-energy bunches, whose velocity scarcely varies along the system's length, radiate almost identically, despite each of them is in the retarding field of the preceding bunches. As a result, the electromagnetic field excited by the train of such "rigid" bunches increases in direct proportion to, and the energy losses grow in proportion to the second power of the number of bunches in the train. The essential increase of the wake field's amplitude can be reached by multimode excitation of eigenwaves of a dielectric waveguide [1].

The waveguides, which are usually considered in the most of theoretical articles, are infinite along the longitudinal axis z. The essential effects related to the system's finite length become apparent only under complete numerical simulation of the system [2]. In present paper, the process of excitation of wake fields by a train of relativistic bunches in a finite length dielectric waveguide is investigated theoretically. In Section 2, the expressions describing the wake field of a thin annular bunch in an infinite waveguide are obtained. In Section 3, the conversion to the fields of a regular train of electron bunches with the rectangular profile of charge density is accomplished. In Section 4, the difference between the finite-length system and the infinite system is discussed. The concept of a trailing edge of a wake field in a semiinfinite waveguide is introduced and the expressions describing the wake field are modified in a proper way. In Section 5, the results of the investigation of self-consistent excitation of the wake field by a train of bunches and acceleration of electrons by this field are represented. In Conclusion, the main theoretical results are briefly resumed, and the recommendations for the future experiments planned in NSC "KhIPT" are given.

1. Excitation of the Field by a Thin Annular Bunch

We consider a round metal waveguide of radius b, which is filled with a dielectric with the relative permittivity ε. The round vacuum channel of radius a, in which the electron bunches will move, is made inside the dielectric. This channel is coaxial with the waveguide. A charged bunch, which has the shape of an infinitely thin and short ring, propagates along this channel. The charge density of this bunch is

$$\rho = -\frac{q}{2\pi r_{\rm L} v_{\rm L}}\delta(r - r_{\rm L})\delta(t - t_{\rm L}) \ , \qquad (1)$$

where $-q$ is the charge, $r_{\rm L}$, $t_{\rm L}$, $v_{\rm L}$ are the Lagrangian radius, time, and velocity of the bunch, respectively. We suppose that the bunch moves uniformly and $r_{\rm L}$ = const, $v_{\rm L}$ = const, $t_{\rm L} = t_0 + z/v_{\rm L}$, where t_0 is the moment of time when the bunch passes the conventional beginning of the system $z = 0$ (or the moment of arrival). Let $v_{\rm L} > c/\sqrt{\varepsilon}$, where c is the speed of light in vacuum.

The wave equation for the excited field, which follows from the Maxwell's system, reads:

$$\nabla \times (\nabla \times \mathbf{E}) + \frac{\varepsilon}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t} \ . \qquad (2)$$

Determining the Fourier transformation (FT) $\widetilde{F}$ for an arbitrary function $F(z,t)$ in the form

$$\widetilde{F} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(z,t)\exp(-ikz + i\omega t)\ dz\ dt \ ,$$

for the FT of the longitudinal component of the E-wave field $\widetilde{E}_z$ from (2) with the source (1), we obtain

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\varepsilon\omega^2}{c^2} - k^2\right)\widetilde{E}_z =$$

$$= -\frac{iq}{\pi\varepsilon r_{\rm L} v_{\rm L}}\left[k\exp(ikv_{\rm L}t_0) - \frac{\varepsilon\omega v_{\rm L}}{c^2}\exp(i\omega t_0)\right] \times$$

$$\times\delta(r - r_{\rm L})\delta(k - \frac{\omega}{v_{\rm L}}) \ . \qquad (3)$$

The FT of electric and magnetic components of the E-wave field are connected with each other as follows:

$$\widetilde{E}_r = \frac{ik}{\varepsilon\omega^2/c^2 - k^2}\frac{\partial \widetilde{E}_z}{\partial r} \ , \quad \widetilde{H}_\varphi = \frac{\omega\varepsilon}{kc}\widetilde{E}_r \ . \qquad (4)$$

Let us divide the waveguide's inner space into three regions along the radius: $0 \le r \le r_{\rm L}$, $r_{\rm L} \le r \le a$, and $a \le r \le b$. We will label the field in each of these regions with the corresponding indices: I, II, or III. At the boundaries of these regions, the following conditions must be satisfied:

$$\widetilde{E}_z^{I}(r_{\rm L}) = \widetilde{E}_z^{II}(r_{\rm L}) \ ,$$

$$\widetilde{E}_z^{II}(a) = \widetilde{E}_z^{III}(a) \ ,$$

$$\widetilde{E}_z^{III}(b) = 0 \ ,$$

$$\frac{\partial \widetilde{E}_z^{II}(r_{\rm L})}{\partial r} - \frac{\partial \widetilde{E}_z^{I}(r_{\rm L})}{\partial r} =$$

$$= -\frac{iq}{\pi r_{\rm L} v_{\rm L}}\left[k\exp(ikv_{\rm L}t_0) - \frac{\omega v_{\rm L}}{c}\exp(i\omega t_0)\right]\delta(k - \frac{\omega}{v_{\rm L}}) \ ,$$

$$\frac{\varepsilon}{k_d^2}\frac{\partial \widetilde{E}_z^{III}(a)}{\partial r} + \frac{1}{k_v^2}\frac{\partial \widetilde{E}_z^{II}(a)}{\partial r} = 0 \ , \qquad (5)$$

where $k_v^2 \equiv k_v^2(\omega,k) = k^2 - \omega^2/c^2$, $k_d^2 \equiv k_d^2(\omega,k) = \varepsilon\omega^2/c^2 - k^2$.

Due to the form of the left-hand side of (3), $\widetilde{E}_z$ in the regions can be presented as follows:

$$\widetilde{E}_z^{I} = C_1 I_0(k_v r) \ ,$$

$$\widetilde{E}_z^{II} = C_2 I_0(k_v r) + C_3 K_0(k_v r) \ ,$$

$$\widetilde{E}_z^{III} = C_4 J_0(k_d r) + C_5 N_0(k_d r) \ , \qquad (6)$$

where J_0, N_0, I_0, and K_0 are the cylindrical Bessel, Neumann, the modified cylindrical Bessel, and Macdonald functions of zero order, respectively. Substituting (6) into (5), we find the expressions for the constants C_i ($i = 1, \dots, 5$). Fulfilling the reverse FT with accounting (4), we obtain the final expressions for the components of the field of the thin annular bunch (1) in the vacuum channel:

$$E_z(t,r,z,t_0,r_{\rm L},v_{\rm L}) = -\frac{4q}{a^2}\sum_{s=1}^{\infty}\frac{I_0(k_{vs}r_{\rm L})I_0(k_{vs}r)}{I_0^2(k_{vs}a)G_s}\times$$

$$\times\Theta(t-t_{\rm L})\cos[\omega_s(t-t_{\rm L})] \ ,$$

$$E_r(t,r,z,t_0,r_{\rm L},v_{\rm L}) = \frac{4q\gamma_{\rm L}}{a^2}\sum_{s=1}^{\infty}\frac{I_0(k_{vs}r_{\rm L})I_1(k_{vs}r)}{I_0^2(k_{vs}a)G_s}\times$$

$$\times\Theta(t-t_{\rm L})\sin[\omega_s(t-t_{\rm L})] \ ,$$

$$H_\varphi(t,r,z,t_0,r_{\rm L},v_{\rm L}) = \beta_{\rm L}E_r(t,r,z,t_0,r_{\rm L},v_{\rm L}) \ . \qquad (7)$$

Here, $k_{vs}^2 \equiv k_v^2(\omega_s,\omega_s/v_{\rm L}) = \omega_s^2/(v_{\rm L}^2\gamma_{\rm L}^2)$, $k_{ds}^2 \equiv \equiv k_d^2(\omega_s,\omega_s/v_{\rm L}) = \omega_s^2(\varepsilon\beta_{\rm L}^2-1)/v_{\rm L}^2$, $\gamma_{\rm L} = (1-\beta_{\rm L}^2)^{-1/2}$, $\beta_{\rm L} = v_{\rm L}/c$,

$$G_s = \varepsilon - \frac{4\varepsilon}{\pi^2 k_{ds}^2 a^2 A_s^2} - \frac{\varepsilon B_s^2}{A_s^2} - 1 + \frac{I_1^2(k_{vs}a)}{I_0^2(k_{vs}a)} \ ,$$

$A_s \equiv A(k_{ds})$, $B_s \equiv B(k_{ds})$, $A(k_d) = J_0(k_d a)N_0(k_d b) - -N_0(k_d a)J_0(k_d b)$, $B(k_d) = J_1(k_d a)N_0(k_d b) - -N_1(k_d a)J_0(k_d b)$. The resonance frequencies ω_s ($s = = 1, 2, ...$) must be defined from the dispersion equation $D(\omega_s, \omega_s/v_{\rm L}) = 0$, where the dispersion function has the form:

$$D(\omega,k) = \varepsilon k_v\frac{B(k_d)}{A(k_d)} - k_d\frac{I_1(k_v a)}{I_0(k_v a)} \ .$$

The function $\Theta(x)$ is defined as follows:

$$\Theta(x) = \begin{cases} 0 \text{ if } x \le 0 \ , \\ 1 \text{ if } x > 0 \ . \end{cases}$$

2. The Field of a Train of Bunches

Expressions (7) allow us to find a field of both a finite-size bunch and a train of such bunches. One should substitute $-q$ in (7) by $dt_0 dr_{\rm L} 2\pi r_{\rm L} j_z(r_{\rm L},t_0)$, where $j_z(r_{\rm L},t_0)$ is the electrical current density, and integrate over all values of the ring's radius and arrival time. If the distribution of charge density has the rectangular profile within the bunch boundaries in both longitudinal and radial directions, then $j_z(r_{\rm L},t_0) = -Q_b/(\pi r_b^2 t_b)$, where $-Q_b$, r_b, and t_b are the charge, radius, and duration of the bunch, respectively ($t_b = l_b/v_{\rm L}$, l_b is the bunch length). In the case of a regular train of N bunches with periodicity T_b ($T_b = f_{\rm mod}^{-1}$, $f_{\rm mod}$ is the frequency of movement of bunches) in the vacuum channel, we have

$$E_z^N(t,r,z) = -\frac{8Q_b v_{\rm L}^2\gamma_{\rm L}}{r_b l_b a^2}\sum_{s=1}^{\infty}\frac{I_1(k_{vs}r_b)I_0(k_{vs}r)}{I_0^2(k_{vs}a)\omega_s^2 G_s}\times$$

$$\times\sum_{i=1}^{N}\varphi_{is}^z(t,z) \ ,$$

$$E_r^N(t,r,z) = \frac{8Q_b v_{\rm L}^2\gamma_{\rm L}^2}{r_b l_b a^2}\sum_{s=1}^{\infty}\frac{I_1(k_{vs}r_b)I_1(k_{vs}r)}{I_0^2(k_{vs}a)\omega_s^2 G_s}\times$$

$$\times\sum_{i=1}^{N}\varphi_{is}^r(t,z) \ ,$$

$$H_\varphi^N(t,r,z) = \beta_{\rm L}E_r^N(t,r,z) \ , \qquad (8)$$

where

$$\varphi_{is}^z(t,z) = \Theta[t-(i-1)T_b - \frac{z}{v_{\rm L}}]\times$$

$$\times\sin\{\omega_s[t-(i-1)T_b-\frac{z}{v_{\rm L}}]\} -$$

$$-\Theta[t-t_b-(i-1)T_b-\frac{z}{v_{\rm L}}]\times$$

$$\times\sin\{\omega_s[t-t_b-(i-1)T_b-\frac{z}{v_{\rm L}}]\} \ ,$$

$$\varphi_{is}^r(t,z) = \Theta[t-(i-1)T_b-\frac{z}{v_{\rm L}}]\times$$

$$\times(1-\cos\{\omega_s[t-(i-1)T_b-\frac{z}{v_{\rm L}}]\}) -$$

$$-\Theta[t-t_b-(i-1)T_b-\frac{z}{v_{\rm L}}]\times$$

$$\times(1-\cos\{\omega_s[t-t_b-(i-1)T_b-\frac{z}{v_{\rm L}}]\}) \ . \qquad (9)$$

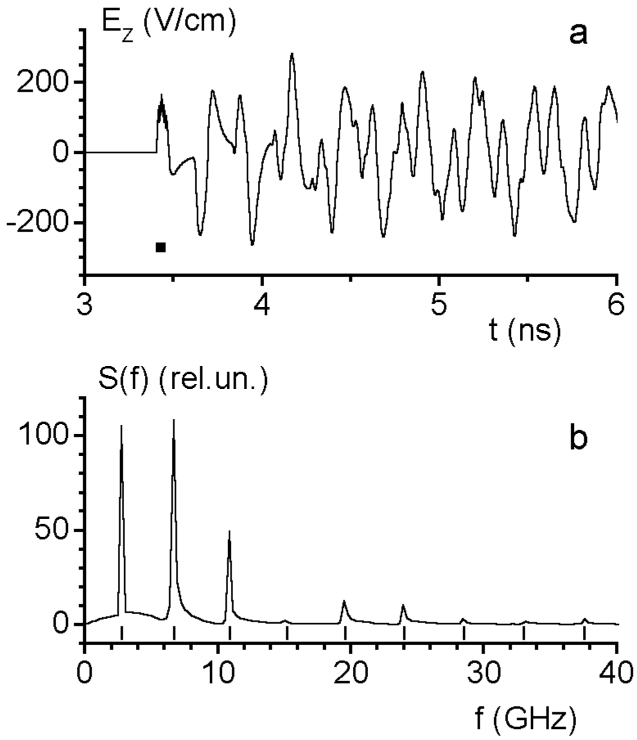

Fig. 1. Characteristics of the wake field of a solitary bunch. a — the dependence of the longitudinal electric field at the point z = 100 cm and $r = 0.5$ cm on time; the black square marks position of the bunch. b — the frequency spectrum of this pattern; the black vertical strokes mark the resonance frequencies. Parameters of the system: $a = 1.05$ cm, $b = 4.23$ cm, $\varepsilon = 2.1$, $Q_b = 0.32$ nCl, the energy of the bunch $W_b = 2$ MeV, $l_b = 1.7$ cm, $r_b = 0.5$ cm. At the moment of time $t = 0$, the head of the bunch passes the cross-section $z = 0$

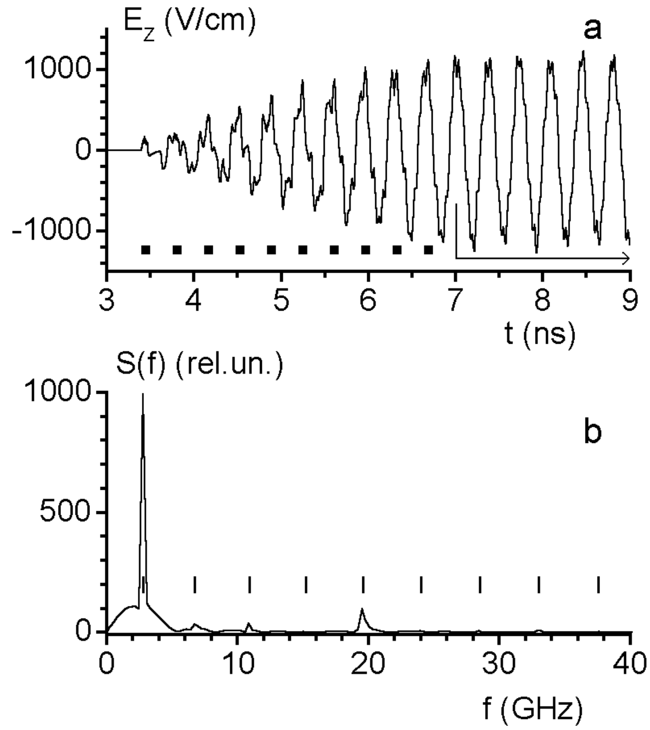

Fig. 2. Characteristics of the wake field of 10 successive bunches. a — the dependence of the longitudinal electric field at the point z = 100 cm and r = 0.5 cm on time; the black squares mark positions of the bunches, the leftmost square denotes the first bunch of the train. b — the frequency spectrum of the signal behind the last bunch (part of the pattern denoted by the arrow); black vertical strokes mark the resonance frequencies. Parameters of the waveguide and of each bunch are the same as in Fig. 1, $f_{\mathrm{mod}} = 2790$ MHz. At the moment of time $t = 0$, the head of the first bunch passes the cross-section $z = 0$

The pattern of field (8) is determined by both the summation of the infinite number of radial modes (the sum over s) and the summation over bunches (the sum over i). When the amplitudes of several modes are comparable with that of the first mode, the narrow spikes of the field, whose intensity is much higher than the intensity of a single mode, are formed [1]. For the field represented in Fig. 1,a, the modes with numbers 1, 2, and 3 have the maximal values (see the spectrum in Fig. 1,b). As $\Delta\omega_s = \omega_{s+1} - \omega_s \neq \mathrm{const}$ in the considered case, the periodicity of field spikes from the solitary bunch changes with time, and this makes the coherent summation of fields in the regular train of bunches more difficult. In [3, 7], it was proposed to take $f_{\mathrm{mod}} \approx (\omega_{s+1} - \omega_s)/(4\pi)$ as the frequency of modulation. But the essential increase of the wake field as a result of coherent summation of many modes will appear only in the case of large values of permittivity $\varepsilon \geq 10$, when $\omega_s \approx \omega_0(s - 1/2)$. If one uses the dielectric with small value of ε (e.g., fluoroplastic-4, or Teflon, which has $\varepsilon = 2.1$), then one must modulate the train of bunches at the frequency of the first resonance mode. In this case (see Fig. 2,a), the amplitude of oscillations increases in direct proportion to the number of injected bunches, and the pattern of the wake field becomes similar to that of a single mode field. As seen in Fig. 2,b, the spectrum of oscillations behind the last bunch has a peak corresponding to the frequency of train's modulation that essentially exceeds the peaks corresponding to the frequencies of other excited modes.

3. The Wake Field in a Semiinfinite Waveguide

We obtained expressions (8), (9) for the wake field assuming that the waveguide is infinite along its axis. According to (9), in any cross-section of the waveguide, after the bunch passes through it, the wake field oscillations will exist for an infinitely long period (we neglect the damping). Therefore, it is possible to sum the fields from a large number of bunches and to obtain the corresponding high values of the wake field. In reality, the waveguide has a finite length and this model cannot be applied to it.

In [8, 9], the problem of excitation of a wake field by a charged bunch in a semiinfinite waveguide filled with solid dielectric was solved. It was shown that, in this structure, the wake field of a solitary charged bunch has the trailing edge, which follows the bunch with the velocity equal to the group velocity of resonance eigenwaves. A transition radiation smoothing the field envelop at the region of the trailing edge appears as well. Three main conclusions follow from here [9]. First, the wake field bulk following the exciting bunch has finite length; although the length of the field bulk increases with time, but the wake field will disappear in any cross-section of the waveguide in certain interval of time after the bunch passes through it. We call this phenomenon as the effect of field removal. Secondly, when the semiinfinite waveguide is excited by the train of successive bunches, the net field's amplitude grows from the beginning of the system to the position of the trailing edge of the field excited by the first bunch and decreases from this point to the position of the first bunch. Thirdly, the field amplitude in some cross-section experiences a growth while several first bunches passes through it and the following saturation; the saturated field does not depend on the total number of bunches passed through the cross-section, but strongly depends on the distance to the input end of the system.

The problem statement in [8, 9] is the first approximation to the description of a finite-length waveguide without reflections at its ends. If we consider such a waveguide, the maximal number of bunches which contribute to the field amplitude in its output end will be

$$N_{\max} \approx \frac{l_{\mathrm{sys}}(v_{\mathrm{L}} - v_{\mathrm{gr}})}{l_{\mathrm{mod}} v_{\mathrm{gr}}} + 1 \ , \tag{10}$$

where l_{sys} is the waveguide length, l_{mod} is the spatial period of movement of bunches, and v_{gr} is the group velocity.

Let us suppose that, first, our waveguide has finite length and no reflection at its ends takes place. Secondly, in a finite-length dielectric waveguide with vacuum drift channel, the propagation of the wake field occurs in the above-described way. Thirdly, we can neglect the transition radiation. That is why, during the calculation of the wake field (8) excited by the train of identical finite-size bunches, we suppose that each mode of the elementary field (7) is nonzero in the region $z_{\mathrm{gr},s} < z < (t-t_0)v_{\mathrm{L}}$, where $z_{\mathrm{gr},s} = (t-t_0)v_{\mathrm{gr},s}$, $v_{\mathrm{gr},s}$ is the group velocity of the mode with number s. The value of $v_{\mathrm{gr},s}$ is defined as follows:

$$v_{\mathrm{gr},s} = -\frac{D'_k(\omega_s, \omega_s/v_{\mathrm{L}})}{D'_\omega(\omega_s, \omega_s/v_{\mathrm{L}})} \ , \tag{11}$$

where

$$D'_k(\omega, k) = \frac{k k_v a}{k_d}\left[-\varepsilon + \frac{4\varepsilon}{\pi^2 k_d^2 a^2 A^2} - \right.$$

$$\left. -\varepsilon\frac{B^2}{A^2} - \frac{k_d^2}{k_v^2}\left(1 - \frac{I_1^2(k_v a)}{I_0^2(k_v a)}\right)\right] + 2\frac{k}{k_d}\left(\frac{k_d^2}{k_v^2} + 1\right)\frac{I_1(k_v a)}{I_0(k_v a)} \ ,$$

$$D'_\omega(\omega, k) = \frac{\omega\varepsilon}{kc^2}\left\{\frac{k(\varepsilon - 1)}{k_v \varepsilon}\left[2\frac{k_d}{k_v}\frac{I_1(k_v a)}{I_0(k_v a)} - \right.\right.$$

$$\left.\left. -a k_d\left(1 - \frac{I_1^2(k_v a)}{I_0^2(k_v a)}\right)\right] - D'_k(\omega, k)\right\} \ .$$

The net field is still determined by expressions (8), where $\varphi_{is}^{z,r}$ have the following form:

$$\varphi_{is}^z(t, z) = \left\{\Theta[t - (i-1)T_b - \frac{z}{v_{\mathrm{L}}}] - \right.$$

$$\left. -\Theta[t - (i-1)T_b - \frac{z}{v_{\mathrm{gr},s}}]\right\} \times$$

$$\times \sin\{\omega_s[t - (i-1)T_b - \frac{z}{v_{\mathrm{L}}}]\} -$$

$$-\left\{\Theta[t - t_b - (i-1)T_b - \frac{z}{v_{\mathrm{L}}}] - \right.$$

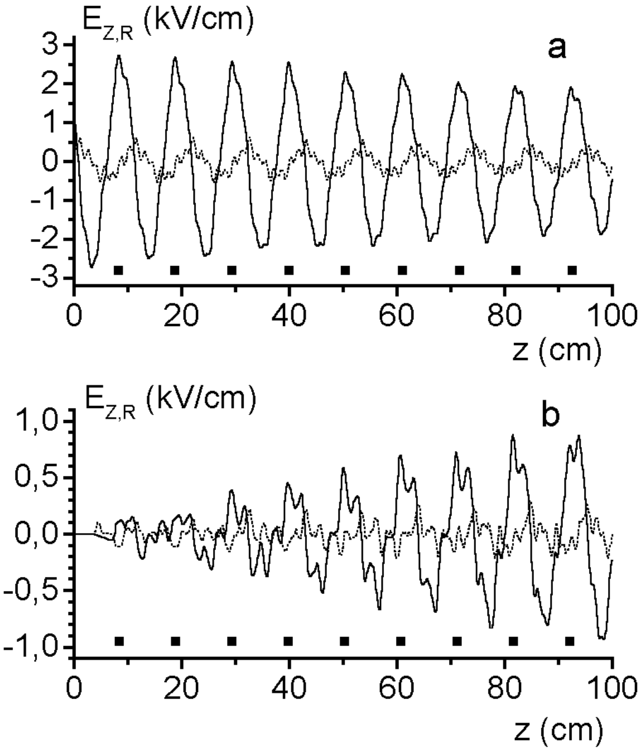

Fig. 3. Longitudinal distribution of electric components of the wake field excited by the train of 28 bunches in the waveguide of length $l_{\rm sys} = 100$ cm at the moment of time $t = 10$ ns for $r = 0.5$ cm. a — the fields are calculated without taking the effect of wake field removal into account. b — the fields are calculated assuming that the trailing edge follows each bunch with the velocity $v_{\rm gr} < v_{\rm L}$. The black squares mark positions of the bunches. The leftmost square denotes the last bunch of the train. The solid lines depict the longitudinal field, and the dashed lines depict the radial field. Parameters of the system are the same as in Fig. 2

$$-\Theta[t - t_b - (i-1)T_b - \frac{z}{v_{{\rm gr},s}}]\Bigg\}\times$$

$$\times\sin\{\omega_s[t - t_b - (i-1)T_b - \frac{z}{v_{\rm L}}]\}+$$

$$+\Bigg\{\Theta[t - (i-1)T_b - \frac{z}{v_{{\rm gr},s}}]-$$

$$-\Theta[t - t_b - (i-1)T_b - \frac{z}{v_{{\rm gr},s}}]\Bigg\}\sin[\omega_s z(\frac{1}{v_{{\rm gr},s}} - \frac{1}{v_{\rm L}})] \ ,$$

$$\varphi^r_{is}(t,z) = \Bigg\{\Theta[t - (i-1)T_b - \frac{z}{v_{\rm L}}]-$$

$$-\Theta[t - (i-1)T_b - \frac{z}{v_{{\rm gr},s}}]\Bigg\}\times$$

$$\times\Bigg(1 - \cos\{\omega_s[t - (i-1)T_b - \frac{z}{v_{\rm L}}]\}\Bigg) -$$

$$-\Bigg\{\Theta[t - t_b - (i-1)T_b - \frac{z}{v_{\rm L}}]-$$

$$-\Theta[t - t_b - (i-1)T_b - \frac{z}{v_{{\rm gr},s}}]\Bigg\}\times$$

$$\times\Bigg(1 - \cos\{\omega_s[t - t_b - (i-1)T_b - \frac{z}{v_{\rm L}}]\}\Bigg) -$$

$$-\Bigg\{\Theta[t - (i-1)T_b - \frac{z}{v_{{\rm gr},s}}]-$$

$$-\Theta[t - t_b - (i-1)T_b - \frac{z}{v_{{\rm gr},s}}]\Bigg\}\times$$

$$\times\cos[\omega_s z(\frac{1}{v_{{\rm gr},s}} - \frac{1}{v_{\rm L}})] \ . \qquad (12)$$

The difference between the fields, which are calculated with the help of Eqs. (8), (9) and (8), (12), can be noticed in Fig. 3. The fields are calculated at the moment of time when 28 bunches have been injected into the waveguide and 19 of them have left the waveguide already. One can see that the neglect of the effect of field removal (Fig. 3,a) results in the considerable overestimation of the excited wake field intensity and in a qualitatively different pattern of the wake field (compare Fig. 3,a and Fig. 3,b). In Fig. 3, the parameters of the waveguide and of the train of bunches are close to those of the wake field acceleration experiment which has been carried out in NSC "KhIPT" [5].

4. Self-consistent Excitation and Acceleration

The $2D$ equations of motion of an electron ring in the field of an E-wave are [10]:

$$\frac{dt_{\rm L}}{dz} = \frac{1}{v_{z{\rm L}}}\ , \quad \frac{dr_{\rm L}}{dz} = \frac{v_{r{\rm L}}}{v_{z{\rm L}}}\ ,$$

$$\frac{dv_{r{\rm L}}}{dz} = -\frac{e}{mv_{z{\rm L}}\gamma_{\rm L}}\times$$

$$\times\left[E_r - \frac{v_{z{\rm L}}}{c}H_\varphi - \frac{v_{r{\rm L}}}{c^2}(v_{r{\rm L}}E_r + v_{z{\rm L}}E_z)\right]\ ,$$

$$\frac{dv_{z{\rm L}}}{dz} = -\frac{e}{mv_{z{\rm L}}\gamma_{\rm L}}\times$$

$$\times\left[E_z + \frac{v_{r{\rm L}}}{c}H_\varphi - \frac{v_{z{\rm L}}}{c^2}(v_{r{\rm L}}E_r + v_{z{\rm L}}E_z)\right]\ , \qquad (13)$$

where $-e$ and m are the charge and the mass of an electron, respectively. We can present finite-size bunches as a set of many macroparticles (rings) with corresponding values of charge, radius, and moment of arrival into the system. The self-consistent excitation of the wake field is described by the equations of motion (13) for each macroparticle in conjunction with the expressions for the field. As the latter, we use (7) assuming that the velocities of particles and the resonance frequencies change slowly. The field, which influences a single particle, is defined as the sum of the fields of other macroparticles.

In order to investigate the self-consistent dynamics, we took the train of 10 bunches, each of them was represented by 50 particles (10 layers along z and 5 layers along r). We restricted ourselves to 10 bunches, because this number follows from (10) for $l_{\rm sys} = 100$ cm, $l_{\rm mod} = 10.52$ cm, $\beta_{\rm L} = 0.979$, $v_{\rm gr}/c \approx 0.5$. We did not take into account the effect of field removal, because it would decrease the field and therefore the probability of the appearance of nonlinear dynamics. The results of the numerical simulation are the following. First, the nonlinear effects (trapping of the particles, etc.) do not appear under the used parameters. Secondly, the net field is very similar to the field calculated by the linear expressions (8), (9). We chose the kinetic energy distribution function and the transverse momentum distribution function as the characteristics of the dynamics of particles forming the train of bunches. At the input end ($z = 0$), all particles have the same

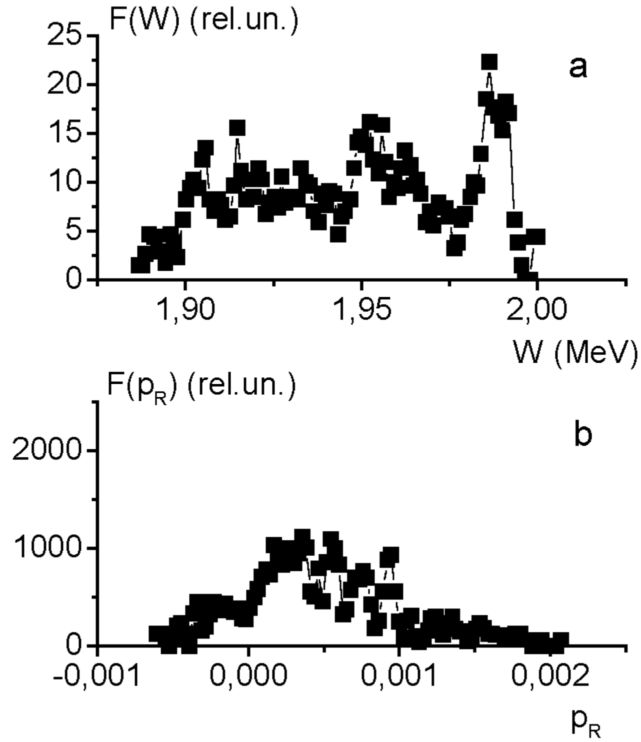

Fig. 4. Characteristics of the self-consistent dynamics of the leading train of 10 bunches calculated in the cross-section $z = 100$ cm. a — the energy distribution function. b — the transverse momentum distribution function. Parameters of the system are the same as in Fig. 2

energy of 2 MeV and zero value of the transverse momentum. At the output end ($z = 100$ cm), the distribution functions become wider (see Fig. 4) — a considerable part of the particles decelerates, the maximal energy losses are near 100 keV, and the insignificant spread of transverse momentum appears. But no particle reaches the dielectric wall and we can consider that the used charge bunches are stable in the system of 1 m length.

As the accounting of the dynamics of small charge bunches does not affect the excited field, we examined the following theoretical model of the dielectric wake field accelerator of electrons. We supposed that the wake field is excited in the finite length waveguide by a regular train of bunches. We neglected the dynamics of bunches but we took into account the effect of wake field removal, therefore the accelerating field was calculated with the help of expressions (8), (12). The low-current nonmodulated beam of electrons, which would be accelerated, was injected simultaneously with the train of bunches. We neglected the influence of these

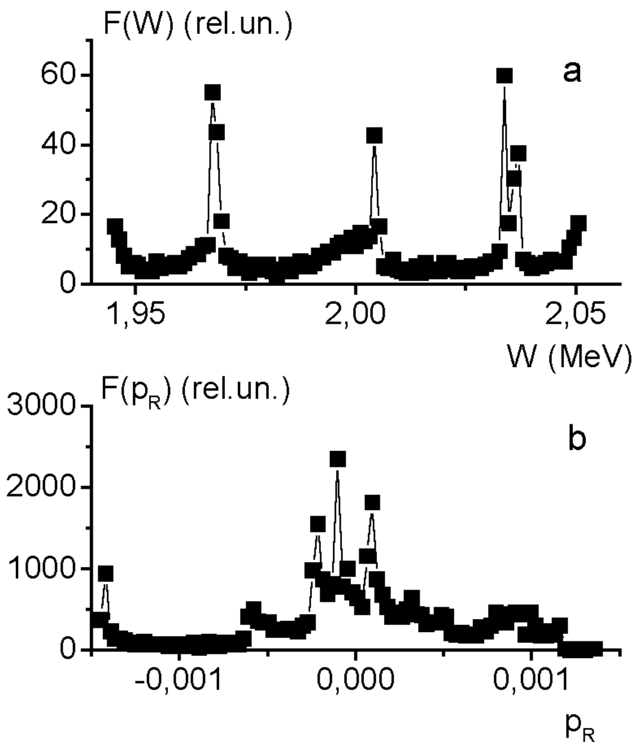

Fig. 5. Characteristics of the dynamics of the accelerated low-current electron beam calculated in the cross-section $z = 100$ cm. a — the energy distribution function. b — the transverse momentum distribution function. Parameters of the waveguide and of the leading train are the same as in Fig. 2. The energy of the beam at the input end of the system ($z = 0$) is 2 MeV

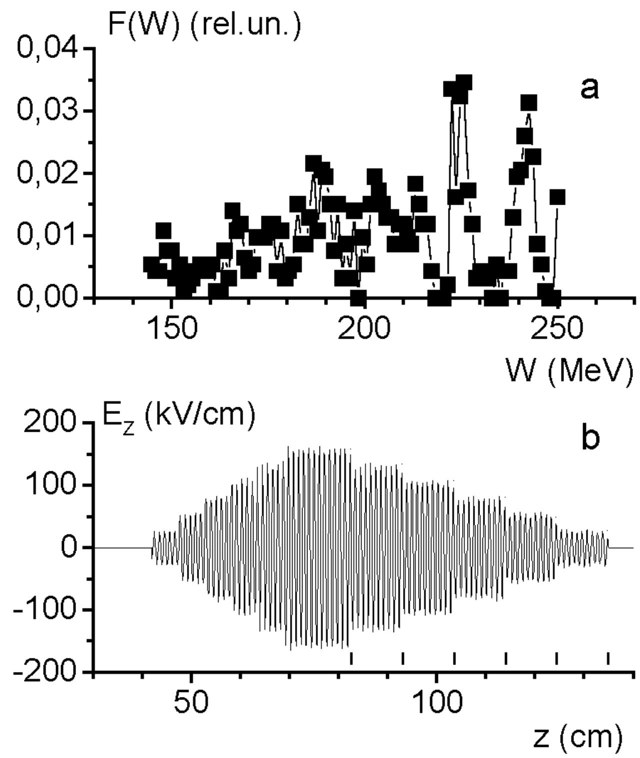

Fig. 6. a — the energy distribution function of the leading train of 6 bunches calculated in the cross-section $z = 100$ cm. b — the longitudinal distribution of the wake field of the train of 6 bunches in the semiinfinite waveguide at the moment of time $t = 4.5$ ns for $r = 0.075$ cm. Parameters of the system: $a = 0.275$ cm, $b = 0.5$ cm, $\varepsilon = 2.1$, $Q_b = 1.67$ nCl, $W_b = 250$ MeV, $f_{\text{mod}} = 2856$ MHz, $l_b = 0.3$ cm, $r_b = 0.075$ cm. The black vertical strokes mark positions of the bunches. At the moment of time $t = 0$, the head of the first bunch passes the cross-section $z = 0$

electrons on one another and on the electrodynamics of the system. The motion of accelerated electrons was described by system (13).

We took the accelerating train of 50 bunches with the parameters close to the experimental ones and the monoenergetic accelerated beam with the initial energy of 2 MeV and duration of 18 ns. The typical picture of the longitudinal distribution of the accelerating field is represented in Fig. 3,b. During the simulation, the accelerated beam was represented by 2000 particles, which were uniformly distributed along the phases of the accelerating field. The results of calculations show that the energy distribution function of the accelerated beam become wider (Fig. 5,a) and accelerated electrons appear. The maximal energy growth does not exceed 50 keV. At the output end of the system, the energy distribution function of accelerated particles becomes slightly asymmetric relatively to the initial energy. In addition, a small spread of radial momenta appears (Fig. 5,b), but accelerated electrons do not reach the dielectric in the 1 m length system.

The obtained analytical results and the numerical calculations carried out demonstrate the possibility, in principle, of creation of an accelerator on the basis of excitation of wake fields by the train of electron bunches in a dielectric waveguide. At the same time, several factors, which limit the efficiency of acceleration under conditions of the carried out [5] and planned experiments, are found. The major of them is the effect of wake field removal, which results in the limiting of the effective number of bunches contributing to the maximal value of the accelerating field. That is why, in order to obtain the accelerating fields of high intensity, it is necessary to use short bunches with high values of charge density and kinetic energy. To illustrate such a possibility, we considered the train

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of 6 bunches with the energy of 250 MeV and the total charge of 10 nCl. The frequency of modulation of the train of 2856 MHz corresponds to the spatial interval between the bunches of 10.5 cm. Each bunch was 3 mm long and 1.5 mm in diameter. This train was used in the experimental investigations of excitation of wake fields in plasma, which were carried out in the NLHEP, Japan. The maximal accelerating field obtained in this experiment [11] was 60 MV/m at the length of 30 cm. At the same time, the calculation with the help of expressions (8), (12) showed that it is possible to obtain the accelerating field near 18 MV/m in the dielectric waveguide (see Fig. 6). We chose the parameters of the waveguide in such a way that $f_1 = 10 f_{\rm mod}$, where f_1 is the frequency of the first resonance mode. The investigation of the self-consistent dynamics of exciting bunches showed that nonlinear effects do not appear due to the high initial energy of bunches. That is why it is possible to use the linear expressions (8), (12) for the calculation of the accelerating field, as was done before. The electron with the initial energy of 250 MeV was accelerated to 262 MeV at the length of 1 m. Despite the strong defocusing fields (near 3 MV/m), the high radial stability is inherent in particles with such an energy.

5. Conclusion

The considered theoretical model of the accelerator of electrons, which uses the wake field of the regular train of relativistic bunches in the dielectric waveguide as the accelerating field, is built on the basis of the following assumptions. First, we suppose that the velocity of charged bunches slowly varies during the time of the flight through the waveguide. This allowed us to use the linear law of changing $t_{\rm L}$. Secondly, we believe that the bunch, which is in the retarding field of the preceding bunches, excites the same field as a solitary bunch. This assumption follows from the previous assumption about the slow changing of the bunch's velocity [6] and is confirmed by the results of numerical simulation of the self-consistent dynamics of excitation of the wake field carried out in Section 5. It should be mentioned that the postulate of the identical radiation of bunches can be violated in some cases [1]. The assumptions mentioned above simplified the calculation of the field of the train of bunches reducing it to the summation of fields of separate bunches. Thirdly, we took into account the finite length of the waveguide and the effect of field removal. Neglecting the reflection at the output end of the waveguide and the transition radiation, we modified the expressions for the field in the infinite waveguide by introducing the trailing edge, which follows the bunch with the velocity equal to the group velocity of the resonance wave [9].

It turned out that the considered model has the following virtue. The dynamics of small-charge relativistic bunches or ultrarelativistic dense bunches scarcely affects the field excited by them. In the latter case, which is the most interesting, this means that intense bunches with high initial energy are "rigidly" anchored to the corresponding phase of the train's wake field, hence the accelerating field of high intensity can be excited in a rather lengthy system (several meters long). This is one of the reasons of using the wake field in a dielectric rather than that in plasma for the purposes of acceleration.

The experiments [4, 5], which had been carried out, demonstrated the possibility, in principle, of creation of a dielectric wake field accelerator. But the results of the present theoretical investigation show that the use of a train with the great number of small charge bunches for excitation of accelerating wake fields in the dielectric waveguide is complicated. The effect of wake field removal restricts the accelerating field to the value, which does not depend on the number of bunches injected into the waveguide. In order to illustrate this phenomenon, we considered the system with parameters close to those of the experiment on dielectric wake field acceleration [5], where the maximal number of bunches had run up to 6000. According to the theoretical calculations, the intensity of the longitudinal electric field excited by the solitary bunch in the paraxial region is near 200 V/cm. In the case of the train of successive bunches, the excited longitudinal electric field is restricted to the value of 1 kV/cm due to the effect of wake field removal. At any moment of time, only 10 bunches contribute to the net accelerating field. Let us notice that the field of the solitary bunch is amplified less than 10 times, because the applied dielectric with low value of permittivity did not allow one to implement the multimode property of the excited field, and the summation of only the first mode, whose frequency was equal to that of the train's modulation, took place.

In order to increase the intensity of the accelerating field, it is necessary to increase the charge and the energy of each bunch. In this case, the maximal number of bunches which effectively excite the wake field is still determined by expression (10). Another method of increasing the intensity of accelerating fields lies in the use of a dielectric with a considerably higher value of permittivity than that of [5]. In this case, the effective excitation of many eigenwaves of slow-wave structure takes place and the amplitude of the net field will be several times as much as that of the dielectric with a low value of permittivity [1].

Let us mention that our consideration does not take into account the reflection of the excited wave from the output and input ends of the waveguide. This reflection always takes place because the matching of the waveguide with the outer space is imperfect usually. The accounting of this effect probably will result in an increase of both the number of bunches, which contribute to the wake field, and the amplitude of the excited field.

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ПРИСКОРЕННЯ ЕЛЕКТРОНІВ КІЛЬВАТЕРНИМИ ПОЛЯМИ РЕГУЛЯРНОЇ ПОСЛІДОВНОСТІ ЗГУСТКІВ У ДІЕЛЕКТРИЧНОМУ ХВИЛЕВОДІ СКІНЧЕННОЇ ДОВЖИНИ

М.І. Онищенко, Д.Ю. Сидоренко, Г.В. Сотніков

Р е з ю м е

Досліджено збудження кільватерних полів регулярною послідовністю релятивістських електронних згустків у циліндричному хвилеводі, частково заповненому діелектриком. У лінійному наближенні отримано вирази для усіх компонентів електромагнітного поля, що збуджується у діелектричному хвилеводі. Обмеженість хвилеводу у поздовжньому напрямку врахована введенням заднього фронту кільватерної хвилі, який поширюється зі швидкістю, що дорівнює груповій швидкості резонансної хвилі. Проведено чисельне моделювання самоузгодженої динаміки частинок згустків у власних кільватерних полях. Продемонстровано, що у випадках релятивістських згустків з малим зарядом та ультрарелятивістських інтенсивних згустків врахування динаміки згустків не впливає суттєво на збуджене ними поле. Знайдена висока радіальна стійкість таких згустків. Досліджено прискорення електронів у кільватерних полях послідовності згустків. Проведено чисельні розрахунки для систем з параметрами, близькими до параметрів експериментів, що проводилися у ННЦ "ХФТІ" (Україна) та Національній лабораторії фізики високих енергій (Японія). Відповідні розраховані напруженості полів дорівнюють 110 к/Вм та 18 МВ/м.

УСКОРЕНИЕ ЭЛЕКТРОНОВ КИЛЬВАТЕРНЫМИ ПОЛЯМИ РЕГУЛЯРНОЙ ПОСЛЕДОВАТЕЛЬНОСТИ СГУСТКОВ В ДИЭЛЕКТРИЧЕСКОМ ВОЛНОВОДЕ КОНЕЧНОЙ ДЛИНЫ

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Р е з ю м е

Рассмотрено возбуждение кильватерного поля регулярной последовательностью релятивистских электронных сгустков в волноводе, частично заполненном диэлектриком. В линейном приближении получены аналитические выражения для компонент электромагнитного поля, возбуждаемого в диэлектрическом волноводе. Конечная протяженность волновода в продольном направлении учтена введением заднего фронта кильватерного поля, который распространяется со скоростью, равной групповой скорости резонансной волны. Проведено компьютерное моделирование самосогласованной динамики частиц сгустков в возбуждаемом ими кильватерном поле. Показано, что в случае слабых релятивистских или сильноточных ультрарелятивистских сгустков учет динамики сгустков не оказывает существенного влияния на возбуждаемое поле. Обнаружена высокая радиальная устойчивость таких сгустков. Исследовано ускорение электронов в поле последовательности сгустков. Численные расчеты выполнены для систем с параметрами, близкими к параметрам экспериментов, проводившихся в Харьковском физико-техническом институте (Украина) и в Национальной лаборатории физики высоких энергий (Япония). Расчетные значения напряженности ускоряющих полей составили соответственно 100 к/Вм и 18 МВ/м.