ON THE CONNECTION OF TOTAL CROSS SECTIONS WITH THE IMAGINARY PART OF THE SCATTERING FORWARD AMPLITUDE AND THE DIFFRACTION CONE SLOPE PARAMETER

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UDC 530.145.539.172
‰ 2002

We derive the formulas connecting the ratio of the total cross-sections σ_{el}/σ_t and the ratio of the total cross-section to the diffraction cone slope parameter σ_t/b with the ratios of the real and imaginary parts of the scattering forward amplitude. The semi-quantitative consideration is done. It is shown that the results become better due to the more precise calculation of the diffraction cone slope parameter and to the influence of the parameter $c > 1$. Comparison with the experimental data on pp- and $p\bar{p}$-interaction shows better agreement in a wide range of energies $\sqrt{s} = 5 \div 546$ GeV. It is important to note that the relations obtained lead to a large value of the ratio of the real part of the scattering forward amplitude to its imaginary part at $\sqrt{s} = 546$ GeV comparing to the values given by many theoretical models and the calculations by the dispersion relations.

1. The important characteristics of hadron-hadron interaction at high energies are the total cross-section of the interaction $\sigma_t(s)$, the total cross-section of elastic scattering $\sigma_{\rm el}(s)$, total inelastic cross-section $\sigma_{\rm in}(s) = \sigma_t(s) - \sigma_{\rm el}(s)$, and the slope parameter $b(s)$ of the differential cross-section of scattering at zero momentum transfer $t = 0$ (s is the energy squared in the center of mass frame). However, along with the use of these parameters, it is also important to introduce their ratios $\sigma_{\rm el}/\sigma_t$, σ_t/b, $\sigma_{\rm el}/b$, $\sigma_{\rm in}/b$. These parameters can play an important role in the choice of the strong interaction regimes at high energies since they are very sensitive to the opaqueness of hadrons. In the given work, we will obtain relations of the ratios with the $\delta(s) \equiv {\rm Re}\,F(s,0)/{\rm Im}\,F(s,0)$ ratio of the real and imaginary parts of the forward scattering amplitude in the frames of the inelastic overlap function (IOF) model. We used the model to describe the experimental data on the differential cross-section $d\sigma/dt$ of the elastic pp-scattering at the energies of ISR and $p\bar{p}$- scattering at the energies of the SPS-collider [1 – 5]. Here, behaviour of $\delta(s)$ plays the crucial role in the description. In particular, it monitors the transition of the 'minimum – secondary maximum" type at the energies of ISR into the structure of the 'shoulder" type at the energies of the SPS-collider [6]. From the results obtained, it is possible to conclude that $\sigma(s)$ also monitors the energetic dependence of the parameters that are of special interest for us.

Basing on [1 – 5], it is possible to obtain the following relations:

$$\frac{\sigma_{\rm el}}{\sigma_t} = \frac{ca}{4}\left[\frac{1 + (ca^2)^{-1}(L\delta^2 + M)}{1 + (4a)^{-1}(L\delta^2 + M)}\right], \quad (1)$$

$$\frac{\sigma_t}{b_1} = 4\pi a + \pi(L\delta^2 + M), \quad (2)$$

$$\frac{\sigma_{\rm el}}{b_1} = \pi ca^2 + \pi(L\delta^2 + M), \quad (3)$$

$$\frac{\sigma_{\rm in}}{b_1} = 4\pi a(1 - ca/4), \quad (4)$$

where

$$L \equiv a^2 B(1 - aB/(0{,}5 + B))/(1 - aB/(1 + B))^2, \quad (5)$$

$$M \equiv 2(c - 1)[a + \ln(1 + a)]. \quad (6)$$

According to the geometric scaling, c, a, B are the parameters independent of energy, and b_1 is almost equal to the slope parameter b_I of the imaginary part of the amplitude square [2]

$$b_I \approx b_1 [1 + (c - 1) a/8]. \quad (7)$$

Taking into account the smallness of δ^2 relatively to the up-to-date energies, Eq. (1) can be expanded in δ^2, and one can obtain the equation similar to (2) and (3):

$$\frac{\sigma_{el}}{\sigma_t} = 4^{-1} ca (1 + L_1 \delta^2 + M_1), \quad (1')$$

where

$$L_1 \equiv L [(ca^2)^{-1} - (4a)^{-1}], \ M_1 \equiv M [(ca^2)^{-1} - (4a)^{-1}]. \quad (8)$$

From the results of (1) – (4), one can see that the ratios σ_{el}/σ_t, σ_t/b_1, σ_{el}/b_1, σ_{in}/b_1 can be represented as a sum of two terms: independent of energy and depending on energy as $\delta^2(s)$, and the σ_{in}/b_1 ratio does not depend on energy at all (geometric scaling). It can be derived from the energy dependence of $\delta(s)$ that the ratios σ_{el}/σ_t, σ_t/b_1, σ_{el}/b_1 obtained from our equations decrease with increase of energy, reach a smooth minimum at the energies of ISR and then increase again. These results are in good qualitative agreement with the experimental data within the wide energy range of $\sqrt{s} = 5 \div 546$ GeV. The asymptotic scattering regime by a rigid elastic object can be also reached in the other models, in particular, in the models based on the existence of the vacuum pole of Pomeranchuk in the t-channel expansion of the generalized reaction matrix [7]. It is of special interest to find the value of $\delta^2(s)$ at which $\sigma_{el}/\sigma_t = 1/2$. This corresponds to the scattering regime by an absolutely dark disc. One can obtain the following expression from (1):

$$\delta^2_{1/2} \approx (2a - ca^2 - M/2)/L. \quad (9)$$

To describe the values of $\frac{d\sigma}{dt}$, some parameters are taken from [3, 4]. We take $a = 0.729$, $c = 1.0192$ and $B \approx 3$.

The ratio σ_{el}/σ_t calculated from Eq. (1) shows the following behaviour: at $\sqrt{s} = 5$ GeV, it is 0.24, it decreases at the energies of ISR to 0.18 – 0.19 and reaches the value of 0.21 at $\sqrt{s} = 545$ GeV. Whereas the experimental behaviour is as follows: at $\sqrt{s} = 5$ GeV, it is 0.22, it decreases at the energies of ISR to 0.17 – 0.18 and reaches the value of 0.215 at $\sqrt{s} = 546$ GeV. The ratio σ_t/b_1 calculated by (2) equals to 10 at $\sqrt{s} = 5$ GeV, falls down to 9 at the energies of ISR, and rises up to 9.5 at $\sqrt{s} = 546$ GeV. The experimental data also give 10 at 5 GeV, decrease to the minimal value of 8.5 and go up to 10.5 at the energies of the SPR-collider. In the calculations at 546 GeV, $\delta(s)$ was taken to be equal to 0.2. Thus, in order that (1) qualitatively describe experimental data in a wide range of energies of $\sqrt{s} = 5 \div 546$ GeV for pp-interactions and at the point of $\sqrt{s} = 546$ GeV, it is necessary to take $\delta(s) = 0.2$, which is a larger value than that used in the other models. Similar overestimated value for δ was obtained from the other more accurate calculations. One can recall that approximately the same value for δ is predicted in the scalar model of quantum field theory [2, 11].

The understimated theoretical value 9.5 instead of the experimentally obtained value 10.5 at 546 GeV can be explained as follows. The values of parameters taken in [5] give overestimated values for the slope parameter $b = 18\,(\mathrm{GeV/c})^{-2}$ [8] instead of a further accurately calculated value $b = 15 \div 16\,(\mathrm{GeV/c})^{-2}$. From (9), one can have $\delta^2_{1/2} \approx 0.3$.

The asymptotic regime at $s \to \infty$ and $\sigma_{el}/\sigma_t \to 1/2$ is also predicted by the hypothesis relating the rise of the total hadron-hadron cross-sections with the resonance production of glueballs (with the mass of M_0) from two gluons existing in every colliding hadron [9]. Such a production leads to the following. For all impact parameters $\rho \le R_0(s) = \ln(s/s_0)$, the condition of total absorption is fulfilled, and the rising part of the total cross-sections corresponds to a dark disc with radius $R_0(S)$ and the mass M_0. In this model for total cross-sections, we have

$$\sigma_{el}(s) = \sigma_{el} + \pi R_0^2(s), \ \sigma_t(s) = \sigma_{t0} + 2\pi R_0^2(s). \quad (11)$$

Assuming that $2\sigma_{el0}/\sigma_{t0} < 1$, one can have an increase of the value of σ_{el}/σ_t both in the $\pi R_0^2 << \sigma_{el0}$ and $\pi R_0^2 > \sigma_{t0}$ regions,

$$\frac{\sigma_{el}(s)}{\sigma_t(s)} \approx \frac{\sigma_{el0}}{\sigma_{t0}} + \frac{\pi R_0^2}{\sigma_{t0}}\left(1 - 2\frac{\sigma_{el0}}{\sigma_{t0}}\right),$$

$$\frac{\sigma_{el}(s)}{\sigma_t(s)} = \frac{1}{2}\left(1 + \frac{2(\sigma_{el0}/\sigma_{t0}) - 1}{2\pi R_0^2(s)/\sigma_{t0}}\right), \quad (12)$$

the ratio of σ_{el}/σ_t to σ_t/b_1 can be also expressed via the values of t_0 at which one can observe a crevasse in the scattering differential cross-section [2, 10].

2. It was shown in [1 – 4] that the experimental data on the differential cross-section of elastic proton-

proton scattering at the energies of hundreds and thousands GeV in the laboratory frame and the momenta transfer of up to 15 $(\mathrm{GeV/c})^2$ can be described by the approach based on such a general property of the S-matrix as unitarity. Here, simple expressions for the overlapping functions were used. They, phenomenologically, take into account really existing effects, in particular, the absorptive correction factors. The model reconstructs the structure of the differential cross-section of proton-proton scattering, in particular, the existence of a decrease at some values of t_0 which shows itself in the wide range of energies from hundreds GeV up to 2000 GeV (the maximal energy of ISR) and the transfer of the minimum in the shoulder in the region of $0.8 < t < 1.4$ $(\mathrm{GeV/c})^2$ at the energies of $\sqrt{s} = 546$ GeV of the $Sp\bar{p}S$-collider [2, 6]. In the same model, it is possible to obtain expressions connecting ratios of the total cross-sections σ_{el}/σ_t as well as the ratios of the total cross-sections to the slope parameters of diffractional cone σ_t/b and σ_{el}/b together with the ratio of the real part of the scattering forward amplitude to the imaginary part $\delta = \mathrm{Re}\,F(0)/\ /\mathrm{Im}\,F(0)$ [6]. This section is devoted to the discussion of these expressions, in particular, to their quantitative correspondence to devoted experimental data.

From the results of [2] at small δ and C˜1, the following expressions can be derived [2, 6]:

$$\frac{\sigma_{el}}{\sigma_t} \approx \frac{ca}{4}\left[\frac{1 + (ca^2)^{-1}(L\delta^2 + M)}{1 + (4a)^{-1}(L\delta^2 + M)}\right] =$$

$$= \frac{ca}{4}(1 + M_1 + L_1\delta^2), \quad (13)$$

$$\frac{\sigma_t}{b_1} \approx 4\pi a + \pi(L\sigma^2 + M), \quad (14)$$

$$\frac{\sigma_{el}}{b_1} \approx 4\pi a(1 - ca/4), \quad (15)$$

where

$$L \equiv a^2 B[1 - aB/(0{,}5 + B)]/[1 - aB/(1 + B)]^2, \quad (16)$$

$$M \equiv 2(a - 1)(a + a_2(1 - a)), \quad (17)$$

$$L_1/L = M_1/M = (ca^2)^{-1} - (4a)^{-1}, \quad (18)$$

where a, B, c are the model parameters independent of energy, and the parameter b_1 is almost equal to the slope parameter b_1 of the imaginary part of the amplitude squared. If one compares expressions (13) ¯ (15) with the experimental data on pp-interactions, one can observe a good qualitative agreement in the wide range of energies $\sqrt{s} = 5 \div 546$ GeV. However, a quantitative agreement occurs for expression (13) only. We show here that it is possible to reach a better quantitative concordance of (14) and (15) with experiment by taking into account the contribution of the real part of the amplitude to the slope parameter b.

The slope of the scattering differential cross-section at the transfer momentum t can be defined by the logarithmic derivative of the differential cross-section

$$b(t) = \frac{d}{dt}\left(\ln\frac{d\sigma}{dt}\right) = 2\frac{F_1(t)F_I' + F_R(t)F_R'(t)}{F_I^2(t) + F_R^2(t)}, \quad (19)$$

where the prime denotes the differention with respect to t; indices R and I represent the real and imaginary parts of the scattering amplitude $F(t)$. It is useful to present a number of expressions for $b(t)$. Substituting the value of the differential cross-section via $\delta(t)$ and $F_I(t)$

$$d\sigma/dt = F_I^2(t)(1 + s^2(t)) \quad (20)$$

to (19), one can obtain

$$b(t) = 2\left(\frac{F_I'(t)}{F_I(t)} + \frac{\delta(t)\delta'(t)}{1 + \delta^2(t)}\right) = b_I(t) + b_R(t). \quad (21)$$

From (21) at $\delta^2(t) << 1$, we have

$$b(t) = b_1(t) + 2\delta'(t)s(t) = b_I(t) + (F_R^2(t))/F_I^2(t), \quad (22)$$

and, in the case of $\delta^2(t) >> 1$,

$$b(t) \approx b_I(t) + 2\delta'(t)/\delta(t) \approx$$

$$\approx 2F_R(t)/F_R(t) = \frac{(F_R^2(t))'}{F_R^2(t)}. \quad (23)$$

If the differential cross-section $d\sigma/dt$ weakly depends on $t\,(b(t) \approx 0)$ in some interval of large momenta transfer and if the both terms in (23) are small, it is obvious that $F_R(t)$ and $F_I(t)$ also weakly depend on t. In general, solving the equation

$$F_I'(t,s)/F_I(t,s) + \delta'(t,s)/\delta(t,s) = 0, \quad (24)$$

one can have

$$F_I(t,s) \approx c(s)/\delta(t,s), \quad (25)$$

i.e., $F_R(t, s)$ weakly depends on t, where $C(s)$ is some arbitrary function of energy $\sqrt{s}$.

3. Now one can calculate the forward slope parameter $b(0) \equiv b$ in our model. At small transfer momenta, it is possible to use the terms with the first order in δ and to neglect the contribution from the expression in $c-1$. Then, one can derive for the scattering amplitude the following expression [1, 2]:

$$F(t) \approx \sqrt{\pi}\, b_1 a \left[i e^{b_1 t/2} + \frac{\delta}{1 - aB/(1+B)} \times \right.$$
$$\left. \times \left(e^{b_1 t/(2B)} \frac{aB}{B+1} e^{b_1 t/(2B+1)} \right) \right]. \qquad (26)$$

By using (17) and (26), one can obtain in this approximation

$$\frac{b}{b_1} \approx \left[1 + \delta^2 \left(1 - \frac{aB}{B+1}\right)^{-2} \times \right.$$
$$\left. \times \left(\frac{1}{B} + \frac{a^2 B^2}{(B+1)^3} - \frac{a(2B+1)}{(B+1)^2} \right) \right] / (1 + \delta^2). \qquad (27)$$

If one takes into account the terms linear in $c-1$ along with linear terms in δ, one can have the following expressions for the scattering amplitude:

$$F_I(t) \approx \sqrt{\pi}\, b_1 a \left(e^{b_1 t/2} - \frac{c-1}{2a} \times \right.$$
$$\left. \times \sum_{m=2}^{\infty} \frac{a^m}{m+B} \exp \frac{b_1 t}{2(m+B)} \right), \qquad (28)$$

$$F_R(t) \approx \sqrt{\pi}\, b_1 \delta A \left(1 - \frac{aB}{1+B} \right)^{-1} \times$$
$$\times \left[e^{b_1 t/2} - \frac{aB}{1+B} \exp \frac{b_1 t}{2(1+B)} + \right.$$
$$\left. + \frac{c-1}{2} B \sum_{m=2}^{\infty} \frac{a^m}{m+B} \exp \frac{b_1 t}{2B+m} \right], \qquad (29)$$

where, in the linear approximation in $c-1$,

$$A \approx 1 - \frac{c-1}{2a} \left[\Sigma_I + aB \left(1 - \frac{aB}{1+aB} \right)^{-1} \Sigma_R \right]. \qquad (30)$$

Substituting (28) and (29) in (27), it is possible to have the following expression for the slope of the diffractional cone at $t = 0$ in the linear approximation in $c-1$:

$$\frac{b}{b_1} \approx \left(1 + \frac{c-1}{2a} (\Sigma_I + \Sigma'_I) \right) \left[1 + \delta^2 (1 + (c-1) \times \right.$$
$$\left. \times \left(\frac{B \Sigma_R}{1 - aB/(B+1)} + \frac{\Sigma_I}{a} \right) \right) \Bigg]^{-1} \times$$
$$\times \left\{ 1 + \delta^2 \frac{(1 - aB^2/(1+B)^2)}{B(1 - aB/(1+B))} \left[1 - \frac{c-1}{2a} \times \right. \right.$$
$$\left. \left. \left(\frac{aB^2 \Sigma'_R}{1 - aB^2/(B+1)^2} + \Sigma'_I + \Sigma_I + \frac{aB \Sigma_R}{1 - aB/(B+1)} \right) \right] \right\}, \qquad (31)$$

where

$$\Sigma_I \equiv \sum_{m=2}^{\infty} \frac{a^m}{m}, \quad \Sigma_R(B) \equiv \sum_{m=2}^{\infty} \frac{a^m}{m+B},$$

$$-\Sigma'_I \equiv \frac{a^m}{m^2}, \quad \Sigma'_R(B) = \sum_{m=2}^{\infty} \frac{a^m}{(m+B)}. \qquad (32)$$

These sums can be easily calculated by the methods described in [1 – 6]. Then (27) and (31) for the ratio b/b_1 correspond to $c = 1$ and $c > 1$, respectively. At $c = 1$, formula (31) passes in (27). In [1, 2], the theoretical ratio of δ_t/b_1 was compared to the experimental results on δ_t/b, i.e. supposing that $b \approx b_1$. Formulas (27) and (31) obtained here allow one to take into account the difference between b and b_1 occuring as result of $\delta^2 \neq 0$ and $c > 1$, respectively. It was shown in [1, 2] that the theoretical and experimental values of the σ_{el}/σ_t ratio are in good agreement (both qualitatively and quantitatively) within the wide range of energies $\sqrt{s} = 5 \div 546$ GeV, i.e. the range from the energies at JINR, Dubna to that reached at the $Sp\bar{p}S$-collider. However, even if the results gave a good qualitative agreement for the ratio of the total cross-sections and the slope parameter of the deffractional cone, the quantitative agreements were not reached. As one can see from the (12), (13), (27), and (31), the ratios σ_{el}/σ_t and σ_e/b at $\delta << 1$ can be presented as the sum of two terms: independent of energy and depending on energy as $\delta^2(s)$. If one does not take into account (27) and (31), then the constant terms in σ_t / b are larger and the coefficients of $\delta^2(s)$ are smaller than the experimentally observed values [1, 2]. At the values of parameters of $a \approx 0.729$, $B \approx 3$ describing well the experimental data on pp and $p\bar{p}$ differential scattering cross-sections in the wide range of momenta transfer [3, 5] not considering b_R at $c - 1 = 0$, one can have

$$\delta_t / b \approx 9(1 + \delta^2). \quad (33)$$

Taking into account b_R (formula (27)) leads to the increase of the coefficient at δ^2 by more than one and half times

$$\delta_t / b \approx 9(1 + 1.6\,\delta^2) \quad (34)$$

and, therefore, leads to a better agreement with experiment.

Now let us consider the case $c > 1$ (formula (31)). Since $\Sigma_I > -\Sigma'_I$, the factor of the ratio in (31) becomes less that 1 at $c > 1$. This leads to the decrease of the constant term and, therefore, to a better agreement with experiment. Expanding (31) in $c - 1$ and leaving the linear term only, one can prove that the coefficient of $c - 1$ is positive. This also leads to a better agreement with experiment. At the values of $c - 1 = 0.1$, one can have the following expression instead of (34) corresconding to the case with $c = 1$:

$$\delta_t / b \approx 8.6(1 + 1.9\,\delta^2), \quad (35)$$

Table 1

Interaction	$\sqrt{s}$, GeV	σ_t, mb	σ_{in}, mb	σ_{el}, mb	C	$\delta = \frac{\mathrm{Re}\,F(0)}{\mathrm{Im}\,F(0)}$
pp	6.2	38.0	30.8	7.2	0.86	˜ 0.17
	13.8	37.9	30.3	7.3	0.99	0.03
	19.4	37.7	30.6	7.1	1.01	0.04
	31	392	31.3	7.9	1.01	0.02
	53	41.7	34.3	7.1	1.05	0.06
	62	52.4	35.3	7.1	1.07	0.08
$p\bar{p}$	4.6	54.7	43.1	11.6	0.94	$-2\cdot 10^{-8}$
	7.6	44.8	36.1	8.7	1.08	0.09
	9.8	45.0	35.0	10.0	1.003	$7\cdot 10^{-3}$
	53	42.6	35	7.6	1.05	0.06
	546	62	49	13	1.22	0.25
	630	60.6	49.7	10.9	1.04	0.20
	1800	70.5	52.5	18	1.10	0.3

Table 2

C	δ^2	σ_{el}/σ_t (theor.)	σ_{el}/σ_t (exp.)	$\frac{\sigma_t}{b}$ (theor.)	$\frac{\sigma_t}{b}$ (exp.)	$\frac{\sigma_{in}}{b}$ (theor.)	$\frac{\sigma_{in}}{b}$ (exp.)	$\frac{\sigma_t}{\sigma_{in}}$ (theor.)	$\frac{\sigma_t}{\sigma_{in}}$ (exp.)
1	0.04	0.215	0.215 ± 0.005	9.4	10.5 ± 0.3	7.66	8.2 ± 0.3	1.23	1.28 ± 0.06
	0.07	0.240		9.5		7.79		1.21	
1.1	0.04	0.202		9.3		7.43		1.21	
	0.07	0.227		9.8		7.60		1.29	

the result of calculations for the ratio σ_{el}/σ_t gives

$$\delta_{el} / \sigma_t \approx 0.18(1 + 4.5\,\delta^2) \quad (36)$$

at $c = 1$ and

$$\delta_{el} / \delta_e \approx 0.17(1 + 5\,\delta^2) \quad (37)$$

at $c - 1 = 0.1$, i.e. the constant term also decreases for this ratio and the coefficient of δ^2 becomes larger in comparison with the $c = 1$ case considered in [2]. It is necessary to state that, according to (15), the ratio σ_{in}/b_1 does not depend on energy. Whereas, the ratio σ_{in}/b_1, according to our results (27) and (31), depends on energy, which is observed in experiments. The relation is as follows:

$$\delta_{in}/b \approx 7.5(1 + 0.6\,\delta^2),\ \text{at}$$

$$C = 1;\ \delta_{in}/b \approx 7.2(1 + 0.9\,\delta^2)\ \text{at}\ C = 1.1. \quad (38)$$

According to the obtained evaluations, the strongest dependence on δ^2 must be in σ_{el}/σ_t, weaker dependence in σ_t/b, and the weakest in σ_{in}/b. Qualitatively, the picture does not contradict the experimental data [14].

The calculated values of the total cross-sections σ_{el}, σ_t, σ_{in} and $\sigma(0) \equiv \mathrm{Re}\,F(0)/\mathrm{Im}\,F(0)$ for pp- and $p\bar{p}$-interactions for the different values of energies and the parameter c are given in Tab. 1.

In Tab. 2, the theoretical and experimental values of the ratios σ_{el}/σ_t, σ_t/b, σ_{in}/b, σ_t/σ_{in} at $\sqrt{s} = 546$ GeV [see 14] are given.

As seen from Tab. 2, the theoretical and experimental ratios of the total cross-sections are in good agreement with each other. The calculated values of the ratios of the total cross-sections to the slope parameter coincide within two standard deviations. In the range of energies of ISR, these ratios are almost constant and are equal to 0.175, 8.5, 7.0 and 1.2, which is in good agreement with the theory. Thus, we have shown that the expressions obtained in the IOF model [1 – 4] showing the relations between the total cross-sections and the ratios between the total cross-sections and the slope parameter of the diffractional cone via the ratio of the real and imaginary parts of the forward scattering amplitude are in good agreement with the experimental data on pp-scattering in the wide range of energies $\sqrt{s} = 5 \div 546$ GeV. The ratios show a linear behaviour at small δ^2. They become complex if δ^2 is not small. Once again we underline that the large values of $\delta \approx 0.20 \div 0.26$ were obtained in [1 – 4] proceding the experimental data taken from the *SpS*-collider. The experiment gives the value of the ratio $\delta \approx 0.24 \pm 0.04$ [14].

This work supported by the grants of STCU Uzb-32 (j)R.

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Received 26.03.02