TO THE PROBLEM OF INFLUENCE OF CHARGING PROCESSES ON THE GRAIN SCREENING IN PLASMAS

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Effects of charging processes on the effective screened field around a charged grain in plasma are studied within a linearized model of collisionless plasmas. It is shown that the above effects give rise to small contributions to the effective screened field predicted by the equilibrium linear screening theory within the rather wide range of plasma parameters typical of dusty plasmas. A considerable deviation from the linear screening theory is expected for larger grain sizes, of the order of the Debye length. The asymptotic behavior of effective potentials is considered.

1. Introduction

One of the important problems of dusty plasma (DP) theory is the calculation of the effective electric potentials of highly charged grains with due regards to charging processes. Such potentials produce most important contributions to the grain-grain interactions and thus determine collective properties of the grain component in DP. Until now collective effects, such as the formation of liquid and crystalline phases in the grain subsystem and phase transitions in DP, have been discussed mainly within the so-called Yukawa model [1-4] based on the linearized Poisson-Boltzmann theory of grain screening [5,6]. The main drawback of this approach in the context of DP is that the system is assumed to be in thermodynamical equilibrium. Actually, even in the case of a stationary state of the system, the grain charge is maintained by plasma currents to the grain surface. Therefore, we have the example of an open system, which is far from thermodynamic equilibrium. As a result, the equilibrium Boltzmann distribution is invalid for plasma particles in the vicinity of grains, and the effective electric potential may deviate from the Debye screened potential. The influence of grain charging on the effective potentials has been already discussed in a number of papers. In particular, in [7,8], it was mentioned that the effective potential within the linear collisionless model decreases at large distances inversely proportionally to the square of the distance. Recently, this result was recovered in [9], in which numerical solutions of the nonlinear problem in some particular cases were also presented. Grain screening in collisional plasmas was studied numerically in [10-12] on the basis of the drift-diffusion model. One of the important conclusions of these studies is that the effective potential has the Coulomb-like asymptotic behavior [12]. Thus, the theory and simulations predict the possibility of considerable deviations of screened potentials from the equilibrium Debye potential under various conditions. On the other hand, the results of recent experimental studies of effective grain-grain interactions have demonstrated the Yukawa-type form of these latter [13]. Thus, the question arises: in what cases and to what extent the deviatons from the Debye form are important for potentials in DP. The purpose of this paper is to answer the above question on the basis of a more detailed treatment of the linear collisionless model and a numerical analysis of the effective potentials, calculated within such a model, for the parameters typical of DP experiments.

2. Formulation of the Problem and Numerical Results

Let us consider the problem of grain screening within a linearized collisionless model [7,8]. Following these works, we write the linearized expressions for the electron and ion densities around a single spherical grain in the form

$$n_e(r) = n_{0e}\left[1+\frac{e\varphi(r)}{k_{\rm B}T_e}\right] \quad (1)$$

$$n_i(r) = n_{0i}\left[1-\frac{eZ_i\varphi(r)}{k_{\rm B}T_i}\right] - n_{0i}\frac{a^2}{4r^2}\left[1+\frac{2eZ_i(|\phi_0|+\varphi(r))}{k_{\rm B}T_i}\right] \quad (2)$$

where a is the grain radius, e is the absolute value of electron charge, $k_{\rm B}$ is the Boltzmann's constant, Z_i

is the ion charge number, T_e and T_i are the electron and ion temperatures, n_{0e} and n_{0i} are the electron and ion densities at infinity. The quantity $\varphi(r)$ is the self-consistent effective potential around the grain and ϕ_0 is the value of this potential at the grain surface. By obtaining relations (1),(2), the velocity distribution is assumed to be Maxwellian for fast electrons, so that the deviation from the Maxwellian distribution is taken into account only for ions.

The effective screened potential around the grain satisfies the Poisson equation

$$\Delta\varphi(r) = -4\pi e(Z_i n_i(r) - n_e(r)) \tag{3}$$

with the right-hand side determined by relations (1),(2). The relevant boundary conditions for the potential φ have the form:

$$\varphi(a) = \phi_0, \tag{4}$$

$$\varphi(\infty) = 0. \tag{5}$$

For the purpose of numerical solution of Eq.(3), condition (5) should be replaced by

$$\varphi(r_{\max}) = \varphi_{\text{asymp}}(r_{\max}). \tag{6}$$

Let us discuss the unknown quantities entering these relations. In order to find the potential ϕ_0 at the grain surface maintained by plasma currents, we use the well-known relation (see, for instance, [14])

$$\frac{\omega_{pe}^2}{s_e}e^{-u} = \frac{\omega_{pi}^2}{s_i}(t+u). \tag{7}$$

Here $\omega_{p\sigma}^2 = 4\pi e_\sigma^2 n_\sigma/m_\sigma$, $s_\sigma = (k_{\rm B}T_\sigma/m_\sigma)^{1/2}$, $t = T_i/T_e Z_i$, n_σ is the particle density of σ species at infinity, and $u = e|\phi_0|/k_{\rm B}T_e$ is the sought-for dimensionless potential at the grain surface.

The right boundary $r_{\max}$ has to be chosen at a sufficiently long distance exceeding the Debye screening length, so that the potential assumes its asymptotic form $\varphi_{\text{asymp}}(r)$. In order to find the latter, we solved Eq.(3) under the assumption that the absolute value of the potential monotonically decreases with distance and, therefore, the inequality $|\phi_0| \gg |\varphi|$ holds at $r \geq r_{\max}$. In this case the quantity $\varphi(r)$ in the second term in the expression for the ion density (2) may be omitted

$$n_i(r) \simeq n_{0i}\left[1 - \frac{eZ_i\varphi(r)}{k_{\rm B}T_i}\right] - n_{0i}\frac{a^2}{4r^2}\left[1 + \frac{2eZ_i|\phi_0|}{k_{\rm B}T_i}\right], \tag{8}$$

and the asymptotic solution of Eq.(3) can be obtained in analytic form:

$$\varphi_{\text{asymp}}(r) = \frac{A}{2k_D r}e^{-k_D r}\times$$

$$\times\left[E_i(k_D r) - E_i(k_D a) - E_1(k_D a)e^{2k_D a}\frac{1-k_D a}{1+k_D a}\right] +$$

$$+\frac{A}{2k_D r}e^{k_D r}E_1(k_D r) + \frac{Q_g exp(k_D a - k_D r)}{r(1+k_D a)}. \tag{9}$$

Here, Q_g is the time-independent grain charge; E_i and E_1 are the conventionally defined integral exponent functions [15]; $k_D = [4\pi e^2(n_{0e}/k_{\rm B}T_e + n_{0i}Z_i/k_{\rm B}T_i)]^{1/2}$ is the inverse Debye length; the expression for A reads

$$A = -\pi e n_{0i}a^2\left(1 + \frac{2eZ_i|\phi_0|}{k_{\rm B}T_i}\right).$$

In obtaining the asymptotic form (9), we imposed the boundary conditions

$$\varphi'(a) = -Q_g/a^2, \tag{10}$$

$$\varphi(\infty) = 0, \tag{11}$$

specifying the electric field on the grain surface and the potential at infinity. The reformulation of the boundary condition at $r = a$ is needed since the approximate distribution (8) is invalid in the vicinity of the grain. The value of the grain charge Q_g consistent with the initial formulation of the problem can be determined by taking into account the fact that relations (4), (7) must hold. Notice, that by using the asymptotic expressions for the functions E_i, E_1 at larger values of arguments, one can verify that the asymptotic behavior of the potential at long distances,

$$\varphi(r) \sim \frac{A}{(k_D r)^2},$$

coincides with that obtained in [7-9].

Thus, we formulated the completely determined bounary value problem (1)-(6). Before solving it numerically, it is expedient to reformulate the problem in dimensionless form. Below we use the following dimensionless quantities and parameters:

i) distance $x = k_D r$;

ii) potential $w = e\varphi/k_{\rm B}T_e$;

iii) ion-to-electron mass ratio $\mu = m_i/m_e$;

iv) ion-to-electron temperature ratio $\tau = T_i/T_e$;

v) grain size $\rho = ak_D$;

vi) electron plasma background coupling $\Gamma_e = e^2(n_{e0})^{1/3}/k_{\rm B}T_e$.

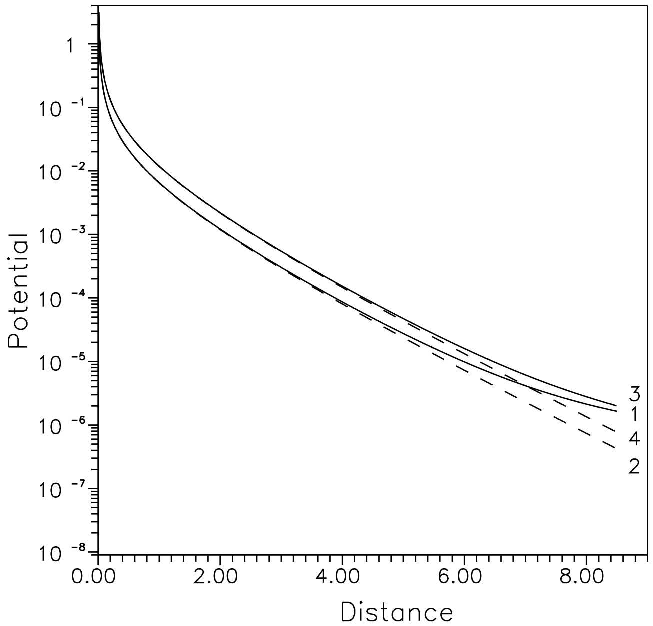

Fig.1. Computed absolute values of potentials (solid lines) compared with predictions of the DLVO theory (dashed lines) for the plasma paramaters $\tau = 0.01$ (the curves 1,2), and $\tau = 1.0$ (the curves 3,4); $\mu = 10^4$; $Z_i = 1$; $\rho = 0.01$. The distance is measured in r_D; the unit of potential is $k_{\rm B}T_e/e$

With these notations, problem (3)-(5) takes the form:

$$w''(x) = -\frac{2}{x}w'(x) + \left(1 + \frac{Z_i}{2x^2}\frac{\rho^2}{(\tau + Z_i)}\right) w(x) + \frac{\rho^2}{4x^2}\frac{\tau + 2Z_i u}{\tau + Z_i} \qquad (12)$$

with the boundary conditions

$$w(\rho) = -u; \qquad (13)$$

$$w(x_{\max}) = \frac{e\varphi_{\rm asymp}(r_{\max})}{k_{\rm B}T_e}. \qquad (14)$$

We solved this problem numerically, by using the shooting methods for two point (boundary) value problems [16]. The results of computations for the range of plasma parameters characteristic of experiments on dusty plasmas are given in Figs.1–6. The relevant values of grain charge number can be estimated from

T a b l e

ρ	τ	Z_g
0.01	0.01	$4.94 \cdot 10^2$
0.01	1.0	$6.4 \cdot 10^3$
0.1	0.01	$5.33 \cdot 10^3$
0.1	1.0	$6.93 \cdot 10^4$
1.5	0.01	$1.46 \cdot 10^5$
1.5	1.0	$2.1 \cdot 10^6$

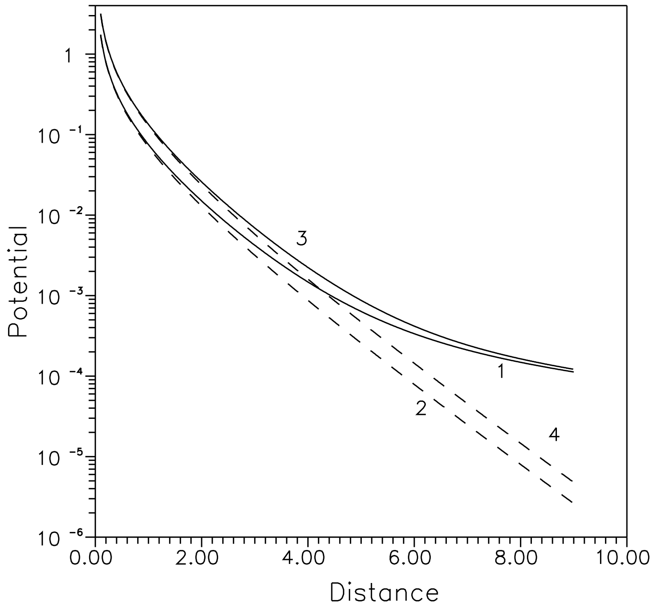

Fig.2. Same as in Fig.1 but for $\rho = 0.1$

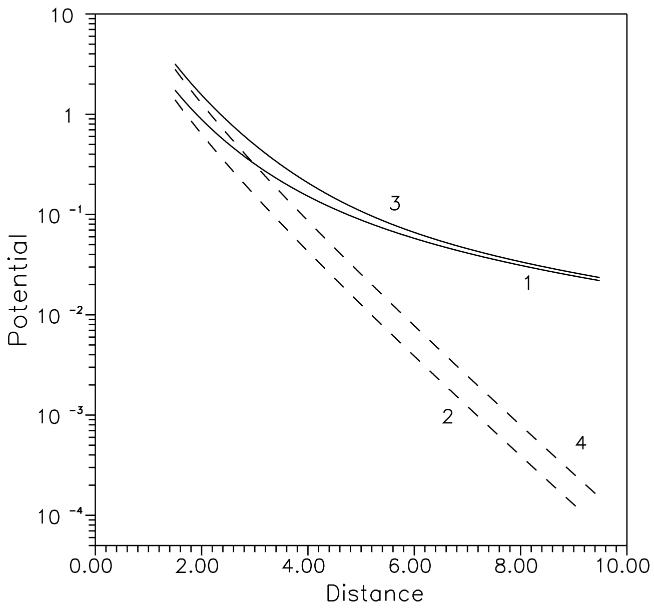

Fig.3. Same as in Fig.1 but for $\rho = 1.5$

the relation

$$Z_g = \frac{Q_g \rho}{\Gamma_e}\left[4\pi\Gamma_e\left(1 + \frac{Z_i}{T_e}\right)\right]^{-1/2}$$

which is the consequence of Eqs.(4) and (7). For the parameters used in our computations ($\mu = 10^4$; $\Gamma_e = 10^{-4}$; $Z_i = 1$) these numbers are given in Table.

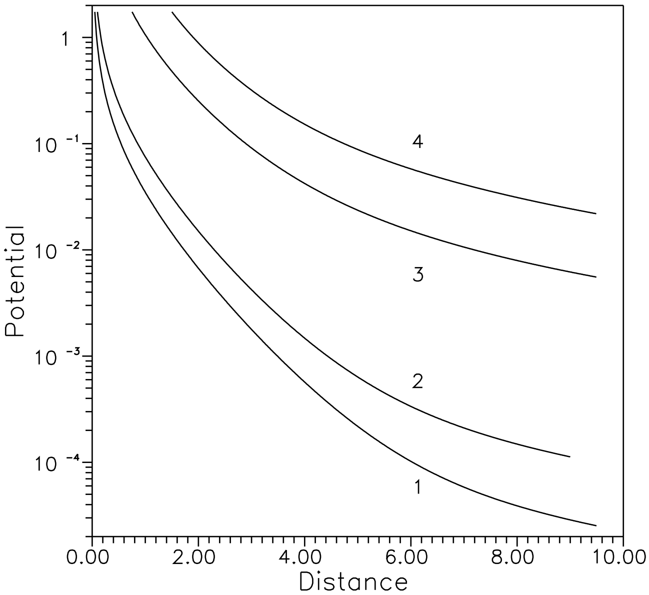

Fig.4. Computed absolute values of potentials for the grain sizes $\rho = 0.05(1); 0.1(2); 0.75(3); 1.5(4)$

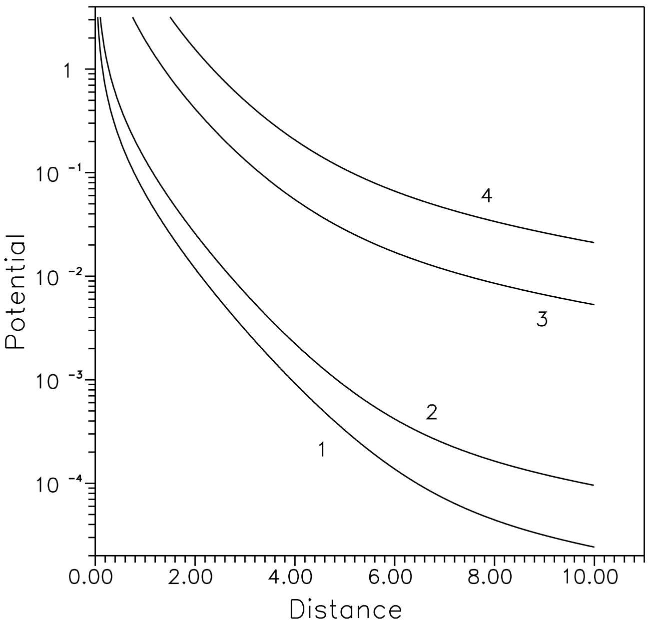

Fig.5. Same as in Fig.4 but for $\tau = 1.0$

Figs.1–5 illustrate the behavior of effective screened potentials for various plasma parameters as compared to the predictions of linear Derjagin—Landau—Verwey—Overbeek (DLVO) [5,6] screening theory.

In order to estimate the intensity of screening and the screening length, it is convenient to use the charge distribution function $Q(r) = r^2 E$ (with E being the

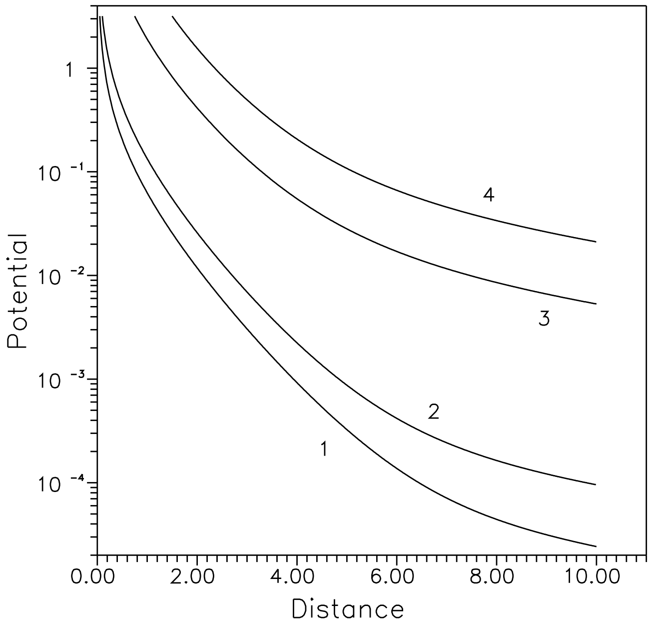

Fig.6. Relative charge distribution Q(r) around a grain in a plasma for the grain sizes $\rho = 1.5$ (the curves 1,2) and 0.01 (the curves 3,4). Solid lines: present computations; dashed lines: DLVO theory. The plasma paramaters are: $\mu = 10^4$; $Z_i = 1$; $\tau = 0.01$. The distance is measured in r_D

electric field) defined as a total charge (including the induced plasma charge) around the grain residing within a sphere of radius r. Typical behavior of this quantity is shown in Fig.6.

The most important conclusion, which follows from the results presented, is that the deviation from the linear screening theory up to the distances $r \simeq 2-3r_D$ is essential only in the case of larger grain sizes, $a \geq 0.5r_D$. This clarifies the recent experimental evidence in favour of the Yukawa-type effective grain-grain interactions in DP [13]. Notice that the typical experimental values for grain sizes are $a/r_D = 0.005 - 0.05$. At longer distances, the deviation becomes considerable due to the difference in asympotic behavior.

Conclusions

To conclude, we studied the effects of grain charging on the effective screened field around a charged impurity within a linearized approach for collisionless plasmas. It was shown that the grain charging gives rise to small contributions to the linear DLVO theory of screening within wide range of plasma parameters characteristic of experiments at the distances up to several Debye lengths. At longer distances, the asymptotic behavior of the screened potential $\varphi \sim r^{-2}$ coincides with that found in [7-9]. Let us point out that this conclusion fairly agrees with the recent experimental studies of effective grain-grain interactions in dusty plasmas [13], where the Yukawa-like character of interactions was observed. A considerable deviation from the DLVO theory would be expected, to our estimates, at larger grain sizes, of the order of the Debye screening length.

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ДО ПРОБЛЕМИ ВПЛИВУ ПРОЦЕСІВ ЗАРЯДКИ
НА ЕКРАНУВАННЯ МАКРОЧАСТИНОК В ПЛАЗМІ

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Р е з ю м е

В рамках лінеарізованої моделі беззіткнювальної плазми досліджено вплив процесів зарядки на ефективне екрановане поле навколо зарядженої макрочастинки. Показано, що в широкому діапазоні параметрів плазми процеси зарядки суттєво не змінюють величину ефективного екранованого поля, яка передбачена рівноважною лінійною теорією екранування. Істотні відхилення від лінійної теорії екранування можна очікувати у випадку великих розмірів макрочастинок – порядку довжини Дебая. Розглянуто асимптотичну поведінку ефективних потенціалів.

К ПРОБЛЕМЕ ВЛИЯНИЯ ПРОЦЕССОВ ЗАРЯДКИ
НА ЭКРАНИРОВАНИЕ МАКРОЧАСТИЦ В ПЛАЗМЕ

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Р е з ю м е

В рамках линеаризированной модели бесстолкновительной плазмы исследовано влияние процессов зарядки на эффективное экранированное поле вокруг заряженной макрочастицы. Показано, что в широком диапазоне параметров плазмы процессы зарядки приводят лишь к небольшим изменениям величины эффективного экранированного поля, которая была предсказана равновесной линейной теорией экранирования. Существенные отклонения от линейной теории экранирования можно ожидать в случае больших размеров макрочастиц – порядка длины Дебая. Рассмотрено поведение асимптотики эффективных потенциалов.