BEHAVIOR OF FORMFACTORS OF NUCLEON RESONANCES AND QUARK-HADRON DUALITY

V.V. DAVIDOVSKY, B.V. STRUMINSKY

UDC 539.21:537.1

Institute for Nuclear Research, Nat. Acad. Sci. of Ukraine (47, Nauky Prosp., Kyiv 03028, Ukraine; e-mail: odavi@kinr.kiev.ua)

Nucleon structure functions in the resonance region are investigated. Expressions for resonance production formfactors dependent on photon virtuality Q^2 , which have correct threshold behavior and take into account the available data on resonance decays, are obtained. The resonance part of nucleon structure functions is calculated. The manifestation of the quark-hadron duality in the behavior of the structure function F_2 is studied using the obtained expressions. The relation between the structure functions F_1 and F_2 in the resonance region is derived.

Introduction

Many years ago the duality of resonances at low energies and Regge poles at high energies was found. The term "duality" means here that an amplitude can be described either by resonances or by reggeons.

The Veneziano amplitude was the first example of successful implementation of the hadron-reggeon duality. Another example is a Dual Amplitude with Mandelstam Analyticity (DAMA) [1, 2], in which complex non-linear Regge trajectories with correct thresholds can be introduced.

A little bit later the quark-hadron duality was discovered by Bloom and Gilman [3]. They analyzed the nucleon structure function W_2 and found that the structure function in the nucleon resonance excitation region, averaged over resonances, with good accuracy coincides with the structure function in the deep inelastic region.

The exact mathematical formulation of the duality (both resonance-reggeon and quark-hadron ones) was given on the basis of the finite-energy sum rules formulated independently in [4—6].

The investigation of the quark-hadron duality can improve our understanding of the structure and interaction of hadrons in terms of the quark and gluon degrees of freedom.

The quark and gluon degrees of freedom are a convenient basis for the gauge invariant theory of strong interaction — quantum chromodynamics (QCD). The

hadronic degrees of freedom form other basis. Since all physical quantities must not depend on the choice of a basis, the descriptions of processes in terms of the quark-gluon and hadronic degrees of freedom must be equivalent.

The choice of degrees of freedom for the description of hadron processes depends on specific kinematic conditions. For instance, in lepton-nucleon scattering, the resonance production region is usually described using the hadronic degrees of freedom, whereas the deep inelastic region is naturally described in terms of the quark-gluon degrees of freedom.

Formally, the quark-hadron duality is exact, but practically the necessity to cut the expansion of any Fock state leads to different manifestations of the duality under various kinematic conditions and in various reactions.

The duality offers additional possibilities for the investigation of nucleon structure using the data on the properties of nucleon resonances. For example, the corrections to scaling behavior of the structure function F_2 , calculated in QCD and measured during the last decade, can be extracted from the resonance data [7].

For theoretical description of the duality, it's necessary to construct structure functions in the resonance region (threshold energies and small photon virtualities). For this purpose, the dependence of the resonance production formfactors $\gamma^*N \to R$ on photon virtuality Q^2 must be known. In this article, we construct the formfactors and study the quark-hadron duality.

The experimental data, obtained in the Jefferson LAB [8, 9], were a good incentive for this work.

Formfactors

Lepton-hadron scattering in the lowest order on the electromagnetic coupling constant is described as an exchange of a virtual photon with virtuality $Q^2 = -q^2$ (q is the four-dimensional momentum of a photon) and

energy (in the nucleon rest frame) $\nu = (pq)/m$ (p is the four-dimensional momentum of nucleon, m is the nucleon mass). In this case, q^2 serves the role of squared momentum transferred from a lepton to a nucleon.

The mechanism of the virtual photon and nucleon interaction depends on the quantity of transferred momentum Q^2 and photon energy. At high Q^2 and ν , noncoherent scattering of a virtual photon on nucleon quarks takes place, and a huge amount of hadrons is produced.

At moderate Q^2 ($\sim 1 \text{ GeV}^2$) and ν , internal nucleon spin states are excited, which leads to the resonance production (Fig. 1).

Below we introduce main notations and define necessary quantities.

The four-dimensional momenta of the nucleon, photon, and resonance are denoted as p, q, and P, respectively. In the resonance rest frame, components of these vectors read:

$$p = (\frac{M^2 + m^2 + Q^2}{2M}; 0, 0, -\frac{\sqrt{(M^2 - m^2 - Q^2)^2 + 4M^2Q^2}}{2M}),$$

$$q = (\frac{M^2 - m^2 - Q^2}{2M}; 0, 0, \frac{\sqrt{(M^2 - m^2 - Q^2)^2 + 4M^2Q^2}}{2M}),$$
(2)

$$q = \left(\frac{M^2 - m^2 - Q^2}{2M}; 0, 0, \frac{\sqrt{(M^2 - m^2 - Q^2)^2 + 4M^2 Q^2}}{2M}\right),\tag{2}$$

$$P = (M; 0, 0, 0), \tag{3}$$

where $p^2 = m^2$, $P^2 = M^2$, and M is the mass of a resonance.

The vertex of virtual photon absorption by a nucleon $\gamma^* N \to R$ is described by three independent formfactors $G_{+,0}(Q^2)$ (or by two, when the spin of a resonance equals to 1/2, and $G_{-}(Q^{2}) \equiv 0$), which are (in the resonance rest frame) helicity amplitudes of the transition $\gamma^* N \to$

$$G_{\lambda_{\gamma}} = \frac{1}{2m} \langle R, \lambda_R = \lambda_N - \lambda_{\gamma} | J(0) | N, \lambda_N \rangle, \tag{4}$$

where λ_R , λ_N and λ_{γ} are the helicities of resonance, nucleon, and photon, respectively; J(0) is the current operator; λ_{γ} takes the values of -1, 0, +1.

Nucleon structure functions can be expressed in terms of formfactors (4) as follows [10]:

$$F_1(x,Q^2) = m^2 \, \delta(W^2 - M^2) \, \left[|G_+(Q^2)|^2 + |G_-(Q^2)|^2 \right], \tag{5}$$

$$\left(1 + \frac{\nu^2}{Q^2}\right) F_2(x, Q^2) = m \nu \delta(W^2 - M^2) \times$$

$$\times \left[|G_{+}(Q^{2})|^{2} + 2|G_{0}(Q^{2})|^{2} + |G_{-}(Q^{2})|^{2} \right] , \qquad (6)$$

$$\left(1+\frac{Q^2}{\nu^2}\right)\,g_1(x,Q^2) = m^2\,\delta(W^2-M^2)[|G_+(Q^2)|^2 -$$

$$-|G_{-}(Q^{2})|^{2} + (-1)^{J-1/2} \eta \frac{Q\sqrt{2}}{\nu} G_{0}^{*}(Q^{2}) G_{+}(Q^{2})], \quad (7)$$

$$\left(1 + \frac{Q^2}{\nu^2}\right) \, g_2(x,Q^2) = -m^2 \, \delta(W^2 - M^2) \, [|G_+(Q^2)|^2 -$$

$$-|G_{-}(Q^{2})|^{2} - (-1)^{J-1/2} \eta \frac{\nu\sqrt{2}}{Q} G_{0}^{*}(Q^{2}) G_{+}(Q^{2})], \quad (8)$$

where J, η are the spin and parity of a resonance, respectively; $W^2 = (p+q)^2$ is the square of the total energy of a photon and a nucleon in the c.m. frame; x is the Bjorken variable. Just to recall, the structure functions F_1 and F_2 are related to W_1 and W_2 by the relations $F_1(x,Q^2) = mW_1(\nu,Q^2)$ and $F_2(x,Q^2) =$ $\nu W_2(\nu, Q^2)$.

Now, it's easy to derive the relation between the structure functions F_1 and F_2 from (5) and (6):

$$(1 + \frac{4m^2x^2}{Q^2})F_2 = 2xF_1(1 + R(Q^2)), \qquad (9)$$

where the ratio of longitudinal and transverse cross sections of virtual photon absorption $R(Q^2) =$ $= \sigma_L(Q^2)/\sigma_T(Q^2)$ is expressed in terms of formfactors as follows:

$$R(Q^2) = \frac{2\sum_{R} |G_0(Q^2)|^2}{\sum_{R} |G_+(Q^2)|^2 + |G_-(Q^2)|^2}.$$
 (10)

This relation in the scaling limit $Q^2 \to \infty$ under assumption $R(Q^2 \to \infty) \to 0$ transforms to the well-known Callan-Gross relation valid in the parton model for quarks with spin 1/2.

Formulas (5)—(8) determine a contribution of one infinitely narrow resonance to nucleon structure functions. For a resonance with width Γ in (5)–(8), we change the delta-function $\delta(W^2 - M^2)$ to

$$\frac{1}{\pi} \frac{M\Gamma}{(W^2 - M^2)^2 + M^2\Gamma^2} \,. \tag{11}$$

In principle, this expression is not a unique approximation of the resonance shape. But now the particular choice of the resonance contribution is not very significant. It's worth mentioning here that expression (11) originates from the propagator of a resonance.

The basic idea of this paper is to take contributions of all resonances, whose data are published in the literature [11], into account. If we let $F_{1,2}^R$ and $g_{1,2}^R$ to denote the contribution of a resonance R to spin-independent and spin-dependent structure functions, respectively, then the contribution of only resonances to the structure functions can be written as a sum:

$$F_{1,2} = \sum_{R} F_{1,2}^{R}; \quad g_{1,2} = \sum_{R} g_{1,2}^{R}.$$
 (12)

To calculate the resonance contribution to the structure function, one must construct the formfactors of resonance production as the functions of photon virtuality Q^2 . Note that the dependence of formfactors of the known resonances [11] on Q^2 practically is not studied experimentally. In tables in [11], only their values at $Q^2 = 0$ are listed.

The transitions $\gamma^* N \to R$ may be of two types by parity: normal, i.e.

$$1/2^+ \to 3/2^-, 5/2^+, 7/2^-, \dots$$
 (13)

and abnormal:

$$1/2^+ \to 1/2^-, 3/2^+, 5/2^-, \dots$$
 (14)

About the corresponding formfactors $G_{\pm,0}(Q^2)$, it's known the following: a) formfactor threshold behavior at $|\vec{q}| \to 0$ [12], b) formfactor asymptotic behavior at high Q^2 , c) formfactor value at $Q^2 = 0$ [11].

So, as was shown in [12], the formfactors of production of a resonance with spin J in case of the normal parity transition $\gamma^*N \to R$ (13) have the following threshold behavior:

$$G_{\pm}(Q^2) \sim |\vec{q}|^{J-3/2},$$
 (15)

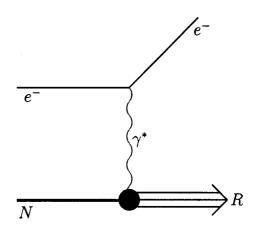


Fig.1. Nucleon resonance production in the lowest order on electromagnetic coupling constant

$$G_0(Q^2) \sim \frac{q_0}{|\vec{q}|} |\vec{q}|^{J-1/2}.$$
 (16)

In case of abnormal parity transitions (14), we have:

$$G_{\pm}(Q^2) \sim |\vec{q}|^{J-1/2},$$
 (17)

$$G_0(Q^2) \sim \frac{q_0}{|\vec{q}|} |\vec{q}|^{J+1/2}.$$
 (18)

A special case is presented by the transitions $1/2^+ \rightarrow 1/2^+$, which are determined by only two formfactors G_+ and G_0 (G_- corresponds to resonance helicity 3/2 and, thus, is absent for resonances with spin 1/2). Their threshold behavior for the transition $1/2^+ \rightarrow 1/2^+$ is as follows:

$$G_{+}(Q^{2}) \sim |\vec{q}|,$$
 (19)

$$G_0(Q^2) \sim \frac{q_0}{|\vec{q}|} |\vec{q}|^2.$$
 (20)

The formfactors of the transition $1/2^+ \rightarrow 1/2^-$ are determined by (17) at J=1/2, i.e.

$$G_+(Q^2) \sim \text{const},$$
 (21)

$$G_0(Q^2) \sim \frac{q_0}{|\vec{q}|} |\vec{q}|.$$
 (22)

The behavior of formfactors at high Q^2 is determined by quark counting rules [13, 14], according to which

$$G_{+}(Q^{2}) \sim Q^{-3}, \quad G_{0}(Q^{2}) \sim Q^{-4}, \quad G_{-}(Q^{2}) \sim Q^{-5}.$$

$$(23) \quad \left|G_{+}(Q^{2})\right|^{2} = \left|G_{+}(0)\right|^{2} \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \frac{Q_{0}^{'2}}{Q^{2} + Q_{0}^{'2}}\right)^{2} \times \left|G_{+}(Q^{2})\right|^{2}$$

So, we suggest the expressions for formfactors, possessing all the above-mentioned properties, as

$$\left|G_{\pm}(Q^2)\right|^2 = \left|G_{\pm}(0)\right|^2 \times$$

$$\times \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \frac{Q_0^{'2}}{Q^2 + Q_0^{'2}} \right)^{2J-3} \left(\frac{Q_0^2}{Q^2 + Q_0^2} \right)^{m_{\pm}}, \tag{24}$$

$$\left|G_0(Q^2)\right|^2 = C^2 \left(\frac{Q^2}{Q^2 + Q_0^{''2}}\right)^{2a} \frac{q_0^2}{|\vec{q}|^2} \times$$

$$\times \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \frac{Q_0^{'2}}{Q^2 + Q_0^{'2}} \right)^{2J-1} \left(\frac{Q_0^2}{Q^2 + Q_0^2} \right)^{m_0} \tag{25}$$

for normal parity transitions and

$$\left|G_{\pm}(Q^2)\right|^2 = \left|G_{\pm}(0)\right|^2 \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \frac{{Q_0'}^2}{Q^2 + {Q_0'}^2}\right)^{2J-1} \times$$

$$\times \left(\frac{Q_0^2}{Q^2 + Q_0^2}\right)^{m_{\pm}},\tag{26}$$

$$\left|G_0(Q^2)\right|^2 = C^2 \left(\frac{Q^2}{Q^2 + Q_0^{"2}}\right)^{2a} \frac{q_0^2}{|\vec{q}|^2} \times$$

$$\times \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \frac{{Q_0^{'2}}}{Q^2 + {Q_0^{'2}}} \right)^{2J+1} \left(\frac{{Q_0^2}}{Q^2 + {Q_0^2}} \right)^{m_0}$$
 (27)

for abnormal parity transitions, where

$$|\vec{q}| = \frac{\sqrt{(M^2 - m^2 - Q^2)^2 + 4M^2Q^2}}{2M},$$

$$|\vec{q}|_{Q=0} = \frac{M^2 - m^2}{2M}$$
 (28) and $m_+ = 3$, $m_- = 5$, $m_0 = 4$.

Formfactors of the transition $1/2^+ \rightarrow 1/2^+$ are

$$|G_{+}(Q^{2})|^{2} = |G_{+}(0)|^{2} \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \frac{Q_{0}^{'2}}{Q^{2} + Q_{0}^{'2}}\right)^{2} \times$$

$$\times \left(\frac{Q_0^2}{Q^2 + Q_0^2}\right)^{m_+},\tag{29}$$

$$|G_0(Q^2)|^2 = C^2 \left(\frac{Q^2}{Q^2 + Q_0^{"2}}\right)^{2a} \frac{q_0^2}{|\vec{q}|^2} \times$$

$$\times \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \, \frac{Q_0^{'2}}{Q^2 + Q_0^{'2}} \right)^4 \, \left(\frac{Q_0^2}{Q^2 + Q_0^2} \right)^{m_0}. \tag{30}$$

In expressions (24)—(26), the quantities Q_0^2 , $Q_0^{'2}$, $Q_0^{''2}$, and a are free parameters, which could be determined by fitting experimental data. The coefficient C could be determined when the experimental data on the ratio of longitudinal and transverse cross sections of virtual photon absorption $R(Q^2)$ will become available, because of relation (10).

By the way, expression (10) together with (24)—(26)allow one to determine the behavior of R, which may be used for the experimental data analysis.

The values of formfactors at $Q^2 = 0$ are related to the helicity amplitudes of photoproduction $A_{1/2}$ and $A_{3/2}$, listed in [11], as follows [10]:

$$|G_{+,-}(0)| = e^{-1} \sqrt{\frac{M^2 - m^2}{m}} |A_{1/2,3/2}|,$$
 (31)

where $e = \sqrt{4\pi/137}$ is electron charge. Note, that the longitudinal formfactor at $Q^2 = 0$ turns to zero: $G_0(0) = 0.$

Substituting expressions (5)—(8) written for each particular resonance, taking into account parity of the transition, and using the proper expressions for formfactors, to (12), we get the structure functions in the resonance region:

$$F_1(x, Q^2) =$$

$$= \sum_{R} \frac{m^2}{\pi} \frac{M\Gamma}{(m^2 + Q^2(1/x - 1) - M^2)^2 + M^2\Gamma^2} \times$$

$$\times \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \, \frac{Q_0^{'2}}{Q^2 + Q_0^{'2}} \right)^{n_+} \times$$

$$\times \left[|G_{+}(0)|^{2} \left(\frac{Q_{0}^{2}}{Q^{2} + Q_{0}^{2}} \right)^{m_{+}} + |G_{-}(0)|^{2} \left(\frac{Q_{0}^{2}}{Q^{2} + Q_{0}^{2}} \right)^{m_{-}} \right], \quad g_{2}(x, Q^{2}) = -\sum_{R} \frac{1}{1 + 4m^{2}x^{2}/Q^{2}} \frac{m^{2}}{\pi} \times \left[\frac{1}{1 + 4m^{2}x^{2}} + \frac{1}{1 + 4m^{2}$$

$$F_2(x,Q^2) = \sum_{R} \frac{2m^2x}{1 + 4m^2x^2/Q^2} \frac{1}{\pi} \times$$

$$imes rac{M\Gamma}{(m^2 + Q^2(1/x - 1) - M^2)^2 + M^2\Gamma^2} imes$$

$$\times \frac{M\Gamma}{(m^2+Q^2(1/x-1)-M^2)^2+M^2\Gamma^2} \times$$

$$\times \left[\left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \frac{Q_0'^2}{Q^2 + Q_0'^2} \right)^{n_+} \left(|G_+(0)|^2 \left(\frac{Q_0^2}{Q^2 + Q_0^2} \right)^{m_+} + \right] \right]$$

$$+|G_{-}(0)|^{2}\,\left(\frac{Q_{0}^{2}}{Q^{2}+Q_{0}^{2}}\right)^{m_{-}}\right)+2C^{2}\left(\frac{Q^{2}}{Q^{2}+Q_{0}^{\prime\prime2}}\right)^{2a}\times$$

$$\times \frac{q_0^2}{|\vec{q}|^2} \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \frac{Q_0'^2}{Q^2 + Q_0'^2} \right)^{n_0} \left(\frac{Q_0^2}{Q^2 + Q_0^2} \right)^{m_0} \right], \qquad (33) \qquad -(-1)^{J-1/2} \eta \frac{Q}{\sqrt{2} m x} C \left(\frac{Q^2}{Q^2 + Q_0''^2} \right)^a |G_+(0)| \times \frac{Q^2}{|\vec{q}|^2} \left(\frac{|\vec{q}|^2}{|\vec{q}|^2} + \frac{Q_0'^2}{Q^2 + Q_0''^2} \right)^{n_0} \left(\frac{Q_0^2}{Q^2 + Q_0''^2} \right)^{n_0}$$

$$g_1(x, Q^2) = \sum_R \frac{1}{1 + 4m^2 x^2 / Q^2} \frac{m^2}{\pi} \times$$

$$\times \frac{M\Gamma}{(m^2+Q^2(1/x-1)-M^2)^2+M^2\Gamma^2} \times$$

$$\times \left[\left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \, \frac{{Q_0^{'2}}}{Q^2 + {Q_0^{'2}}} \right)^{n_+} \left(|G_+(0)|^2 \left(\frac{{Q_0^2}}{Q^2 + {Q_0^2}} \right)^{m_+} - \right. \right.$$

$$-|G_{-}(0)|^{2}\left(\frac{Q_{0}^{2}}{Q^{2}+Q_{0}^{2}}\right)^{m_{-}}+$$

$$+(-1)^{J-1/2} \eta \frac{2\sqrt{2} m x}{Q} C \left(\frac{Q^2}{Q^2 + Q_0''^2} \right)^a |G_+(0)| \times$$

$$\times \frac{|q_0|}{|\vec{q}|} \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \frac{{Q_0'}^2}{Q^2 + {Q_0'}^2} \right)^{(n_0 + n_+)/2} \times$$

$$\times \left(\frac{Q_0^2}{Q^2 + Q_0^2} \right)^{(m_0 + m_+)/2} , \tag{34}$$

$$\times \left[\left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \, \frac{{Q_0^{'2}}}{Q^2 + {Q_0^{'2}}} \right)^{n_+} \left(|G_+(0)|^2 \left(\frac{{Q_0^2}}{Q^2 + {Q_0^2}} \right)^{m_+} - \right. \right.$$

$$-|G_{-}(0)|^{2}\left(\frac{Q_{0}^{2}}{Q^{2}+Q_{0}^{2}}\right)^{m_{-}}\right)-$$

$$-(-1)^{J-1/2} \eta \frac{Q}{\sqrt{2} m x} C \left(\frac{Q^2}{Q^2 + Q_0^{\prime\prime 2}} \right)^a |G_+(0)| \times C \left(\frac{Q^2}{Q^2 + Q_0^{\prime\prime 2}} \right)^a$$

$$\times \frac{|q_0|}{|\vec{q}|} \, \left(\frac{|\vec{q}|}{|\vec{q}|_{Q=0}} \, \frac{{Q_0^{'2}}}{Q^2 + {Q_0^{'2}}} \right)^{(n_0 + n_+)/2} \times$$

$$\times \left(\frac{Q_0^2}{Q^2 + Q_0^2}\right)^{(m_0 + m_+)/2} \bigg], \tag{35}$$

where $n_{+} = 2J - 3$, $n_{0} = 2J - 1$ for normal parity transitions and $n_+ = 2J - 1$, $n_0 = 2J + 1$ for abnormal parity transitions, and the sum is over the resonances. We take into account the contributions of the following resonances: N(1440), N(1520), N(1535), N(1650), N(1675), N(1680), N(1700),N(1710), $N(1720), N(1990), \Delta(1232),$ $\Delta(1550),$ $\Delta(1620), \quad \Delta(1700), \quad \Delta(1900),$ $\Delta(1905),$ $\Delta(1920), \Delta(1930), \Delta(1950).$

The above expressions determine the resonance contribution into nucleon structure functions. It's obvious that the production of resonances in electronnucleon scattering is not the only process contributing to structure functions. The production of mesons and other hadrons forms a non-resonant background, which also must be taken into account.

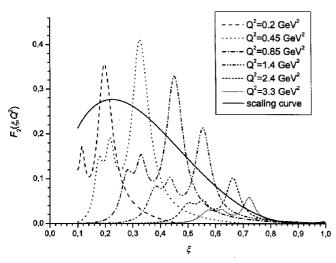


Fig.2. Resonance contribution to the structure function $F_2(\xi,Q^2)$ in resonance region

The non-resonant background could be parameterized [9] as:

$$F_2^{nr}(x,Q^2) = \frac{Q^2}{4\pi^2\alpha} \, \frac{1-x}{1+\frac{4m^2x^2}{Q^2}} \, (1+R(x,Q^2)) \times$$

$$\times \sum_{n=1}^{N} C_n(Q^2) \left[W - W_{\text{th}} \right]^{n-1/2}, \tag{36}$$

where N=3, C_n are fitting coefficients, $W_{\rm th}$ is the threshold energy.

Duality

So finally we obtained the nucleon structure functions in the resonance region that are dependent on the Bjorken variable and photon virtuality.

Let's consider, following [3], how the expressions for structure functions (32)—(35), which contain resonance terms and have the strong dependence on Q^2 (because of formfactors), change over in the limit of high Q^2 to scaling expressions, which weakly depend on Q^2 .

We change over to the variable $x' = Q^2/(Q^2 + W^2)$, which is related to the Bjorken one x as follows:

$$x' = \frac{x Q^2}{Q^2 + x m^2} \,. \tag{37}$$

Let's consider the case of infinitely narrow resonances, i.e. $\Gamma \to 0$. In this case, a structure function will be nonzero only at those values of x', at which $W^2 = M^2$, i.e. $x' = Q^2/(M^2 + Q^2)$. One may easily

see that the resonance shifts to the region x' = 1 as Q^2 increases. As experimental data indicate, the resonance follows the scaling curve. The form of this curve can be evidently derived from the expressions for structure functions in the resonance region by the substitution $Q^2 = M^2 x'/(1-x')$.

In the case of resonances with finite width, the equality $W^2 = M^2$ is approximate, which leads to corrections to scaling behavior, which are proportional to Γ/M .

In this paper, we restrict the consideration to the structure function F_2 . It's worth mentioning that the structure function g_1 is sensitive, unlike F_2 , to the longitudinal formfactor and relative values of G_+ and G_- , because they are included in (35) as a difference.

In (33), we change over to the variable x' and get at high Q^2 :

$$F_2(x') = \sum_{R} \frac{2m^2x'}{\pi M \Gamma} \left[|G_+(0)|^2 \left(\frac{Q_0^2}{M^2} \right)^{m_+} \left(\frac{1-x'}{x'} \right)^{m_+} + \right]$$

$$+|G_{-}(0)|^{2}\left(rac{Q_{0}^{2}}{M^{2}}
ight)^{m_{-}}\left(rac{1-x'}{x'}
ight)^{m_{-}}+$$

$$+2C^{2}\left(\frac{Q_{0}^{2}}{M^{2}}\right)^{m_{0}}\left(\frac{1-x'}{x'}\right)^{m_{0}}\right].$$
(38)

It follows from (38) that the structure function $F_2(x') \sim (1-x')^{m_+}$ as $x' \to 1$.

In the resonance region, the Nachtmann variable $\xi = 2x/(1+\sqrt{1+4m^2x^2/Q^2})$ is often used (variable $x=Q^2/2m\nu$ is not convenient for the analysis of structure functions in the resonance region, because all resonances are densely situated in the region $x\sim 1$). From the inequality 0< x<1, it's easy to derive that $0<\xi< Q/m(\sqrt{1+Q^2/4m^2}-Q/2m)$.

The dependence of the resonance part of the structure function F_2 on the Nahtmann variable ξ at various values of Q^2 is shown on Fig. 2. Specific values of Q_0 , Q_0' , and Q_0'' used to plot Fig. 2, are not significant at this point. The solid line corresponds to the fit to the deep inelastic data for $F_2(\xi)$ [8].

One can see that $\Delta(1232)$ -isobar gives the largest contribution to the structure function. The non-resonant background in this region appears to be relatively small, unlike the regions corresponding to more massive resonances. One can also see that the $\Delta(1232)$ peak exactly follows the scaling curve as Q^2 changes, which is a manifestation of the duality.

For the comparison with experimental data, the parametrization of the non-resonant background contribution must be carried out. However, now this problem has no unique solution.

Taking the non-resonant background into account must lead to a rise of the structure function in the region of higher resonances where it follows the scaling curve (in varying Q^2) in a broad region of ξ .

Conclusion

The equality, discovered by Bloom and Gilman, of the nucleon structure function F_2 , averaged over a big enough domain of the scaling variable, in the resonance region and the deep inelastic region confirmed the existence of quark-hadron duality, i.e. the possibility of the description of processes using the language of either only hadronic or quark degrees of freedom.

Practically it means that the investigation of nucleon resonances, for instance in terms of formfactors, offers additional information on nucleon properties in the deep inelastic region.

Expressions for the formfactors of nucleon resonance production, obtained in this work taking into account the correct threshold behavior and experimental data on resonance decays, may serve as starting ones for the investigation of nucleon properties in the language of hadronic degrees of freedom.

The obtained expressions for structure functions allow one to show qualitatively the manifestation of the duality with the structure function F_2 as an example.

In future, we plan to investigate the structure function g_1 , which gets an essential contribution from the longitudinal formfactor. A numerical verification of our model is also interesting and will be carried out as new experimental data become available.

The authors thank Laszlo Jenkovszky for helpful discussions.

- Bugrij A.I., Jenkovszky L.L., Kobylinsky N.A. //Nucl. Phys. B.- 1971.- 35.- P.120 - 132.
- Jenkovszky L., Magas V.K., Predazzi E.//Europ. Phys. J. A.– 2001.– 12.– P.361 – 367.
- Bloom E.D., Gilman E.J.//Phys. Rev. D.- 1971.- 4, N9.-P.2901 - 2916.
- Igi K., Matsuda S.//Phys. Rev. Lett. 1967. 18. P.625 627.

- Logunov A., Soloviev L., Tavkhelidze A.//Phys. Lett. B.– 1967.– 24.– P.181 – 182.
- Dolen R., Horn D., Schmid C.//Phys. Rev.- 1968.- 166.-P.1768-1781.
- Carlson C.E., Mukhopadhyay N.C.//Phys. Rev. Lett.- 1995.-74, N8.- P.1288 - 1291.
- Niculescu I., Armstrong C.S., Arrington J. et al.//Ibid.-2000.- 85, N6.- P.1186 - 1189.
- 9. Niculescu I. Ph. D. Thesis, Hampton University. 1999.
- Carlson C.E., Mukhopadhyay N.C.//Phys. Rev. D.- 1998.-58.- P.094029.
- 11. Particle Data Group//Phys. Lett. B.- 1988.- 204.- P.379.
- Bjorken J.D., Walecka J.D.//Ann. phys.- 1966.- 38.- P.35 62.
- 13. $Brodsky\ S.J.$, $Farrar\ G.//Phys.\ Rev.\ Lett.-\ 1973.-\ 31.-P.1153-1156.$
- Matveev V.A., Muradian R.M., Tavkhelidze A.N.//Lett. nuovo cim.- 1973.- 7.- P.719 - 723.

Received 22.05.02

ПОВЕДІНКА ФОРМ-ФАКТОРІВ НУКЛОННИХ РЕЗОНАНСІВ І КВАРК-АДРОННА ДУАЛЬНІСТЬ

В.В. Давидовський, Б.В. Струмінський

Резюме

Досліджено структурні функції нуклона в резонансній області. Одержано залежні від віртуальності фотона Q^2 вирази для форм-факторів народження резонансів, які мають правильну порогову поведінку і враховують наявні експериментальні дані з розпадів резонансів. Обчислено резонансні частини структурних функцій нуклона. Використовуючи одержані вирази, досліджено прояв кварк-адронної дуальності у поведінці структурної функції F_2 . Виведено співвідношення, яке зв'язує структурні функції F_1 і F_2 в резонансній області.

ПОВЕДЕНИЕ ФОРМ-ФАКТОРОВ НУКЛОННЫХ РЕЗОНАНСОВ И КВАРК-АДРОННАЯ ДУАЛЬНОСТЬ

В.В. Давидовский, Б.В. Струминский

Резюме

Исследуются структурные функции нуклона в резонансной области. Получены зависящие от виртуальности фотона Q^2 выражения для форм-факторов рождения резонансов, которые имеют правильное пороговое поведение и учитывают имеющиеся экспериментальные данные по распадам резонансов. Вычислены резонансные части структурных функций нуклона. Используя полученные выражения, исследовано проявление кварк-адронной дуальности в поведении структурной функции F_2 . Выведено соотношение, связывающее структурные функции F_1 и F_2 в резонансной области.