TWO-FLUID STATIC SPHERICAL CONFIGURATIONS WITH LINEAR MASS FUNCTION IN THE EINSTEIN — CARTAN THEORY

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In the framework of the Einstein — Cartan theory, two-fluid static spherical configurations with linear mass function are considered. One of these modelling anisotropic matter distributions within star and the other fluid is a perfect fluid representing a source of torsion. It is shown that the solutions of the Einstein equations for anisotropic relativistic spheres in General Relativity may generate the solutions in the Einstein — Cartan theory. Some exact solutions are obtained.

Introduction

It is well known that, in the contemporary theory of gravity and astrophysics, the relativistic theories of gravity are applied for the construction of stellar models: General Relativity (GR) and its generalizations. For the investigation of the relativistic distributions of a perfect fluid in case of spherical symmetry, the mass function has often been used. In the framework of GR, the relativistic static spherically symmetric isotropic configurations with the mass function directly proportional to the radius have been studied in detail (see, for example, [1 — 3] and references therein). Lately the anisotropic models [4 — 7], including ones with linear mass function [8 — 10], have aroused much attention.

In this work, two-fluid spherically symmetric configurations with linear mass function are investigated in the Einstein – Cartan theory (ECT). One of fluids is anisotropic and the other one is isotropic, which induces the torsion. It should be noted that such models in ECT have not been studied up to now.

1. The Model

The Lagrangian Ł of the model is chosen as the sum of Lagrangians: gravitational — $Ł_g$ and fluids — $Ł_{fl(1)}$ and $Ł_{fl(2)}$:

$$Ł = Ł_g + Ł_{fl(1)} + Ł_{fl(2)} \ , \tag{1}$$

where

$$Ł_g = -R(\Gamma)/2æ \ , \tag{2}$$

$$Ł_{fl(1)} = -\rho(c^2 + \Pi(\rho, e)) + k \overset{\Gamma}{\nabla}_i (\rho u^i) +$$
$$+ k_1(u_i u^i - 1) + k_2 u^i \partial_i X + k_3 u^i \partial_i e \ . \tag{3}$$

Here, $R(\Gamma)$ is the curvature scalar obtained from the full connection $\Gamma^k_{ij} = \{^{\,k}_{ij}\} + S_{ij\cdot}^{\ \ k} + S^k_{\cdot ij} + S^k_{\cdot ji}$; $\{^{\,k}_{ij}\}$ are Christoffel symbols of the second kind; $S_{ij\cdot}^{\ \ k} = \Gamma^k_{[ij]}$ is the torsion tensor; $æ = 8\pi$ is the Einstein's constant (units are chosen such that $G = c = 1$); ρ is the perfect fluid mass density; $\Pi(\rho, e)$ is its internal energy; k, k_1, k_2, k_3 are the Lagrange multipliers; X is the Lagrangian coordinates of the matter particles; e is the entropy per volume [11, 12] ; u^i is the four-velocity; $\overset{\Gamma}{\nabla}_i$ is the covariant derivative of the Riemann-Cartan space. The Lagrangian $Ł_{fl(2)}$ for the anisotropic fluid is not indicated since there is no torsion vector for it in the derivative of the term, which regulates the conservation of the number of particles.

The metric g_{ik} has signature (– , – , – , +), the Riemann and Ricci tensors are defined as $R_{ijk\cdot}^{\ \ \ m} = \Gamma^m_{jk,i} - \Gamma^m_{ik,j} + \Gamma^m_{ip}\Gamma^p_{jk} - \Gamma^m_{jp}\Gamma^p_{ik}$, $R_{jk} = R_{ijk\cdot}^{\ \ \ i}$. As shown in [12, 13], it follows from (3) that the torsion can interact with a perfect fluid only through its trace: $S_i = S_{ik\cdot}^{\ \ k}$ (the vector of torsion). Hence, the curvature scalar can be presented in the form:

$$R(\Gamma) = R(\{\}) + 4S^k_{;k} - \frac{8}{3} S_k S^k \ , \tag{4}$$

where $R(\{\})$ is the Riemannian part of the curvature built from Christoffel symbols; the semicolon denotes a covariant derivative of the Riemannian space.

The closed subsystem of equations corresponding to Lagrangian (1) has the form [13]:

$$G_{ij}(\{\}) = 8\pi(T^{fl(1)}_{ij} + T^{fl(2)}_{ij}) + \Lambda_{ij} \ , \tag{5}$$

$$(\Theta u^i)_{;i} = -(\varepsilon_{fl(1)} + P_{fl(1)}) \ . \tag{6}$$

Here, $T_{ij}^{fl(1)}$ and $T_{ij}^{fl(2)}$ are the energy–momentum tensors for the perfect fluid, which generate the torsion, and anisotropic one, respectively; $\Theta = k\rho$; $\varepsilon_{fl(1)}$ is the perfect fluid energy density; $P_{fl(1)}$ is its pressure;

$$\Lambda_{ij} = \frac{8}{3}S_i S_j - \frac{4}{3}S_k S^k g_{ij} \ , \qquad S^k = 6\pi\Theta u^k \ . \tag{7}$$

In the framework of GR, this subsystem describes the gravitational interaction of two fluids.

The line element for the static spherically symmetric metric may be written in the curvature coordinates as

$$ds^2 = -e^{\lambda(r)}dr^2 - r^2 d\Omega^2 + e^{\nu(r)}dt^2 \ , \tag{8}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$.

As shown in [12] for the static distribution of matter, it follows from (6) that

$$P_{fl(1)} = -\varepsilon_{fl(1)} = -C_1 \ , \qquad C_1 = \text{const} \ . \tag{9}$$

In the comoving frame of reference, the energy–momentum tensors $T_{ij}^{fl(1)}$ and $T_{ij}^{fl(2)}$ have the following components:

$$T_{\beta}^{\beta fl(1)} = -P_{fl(1)} \ , \qquad T_{4}^{4fl(1)} = \varepsilon_{fl(1)} \ , \tag{10}$$

$$T_{1}^{1fl(2)} = -P_r \ , \ \ T_{\gamma}^{\gamma fl(2)} = -P_{\perp} \ , \ \ T_{4}^{4fl(2)} = \varepsilon \ , \tag{11}$$

where $\beta = 1,2,3$; $\gamma = 2,3$; $\varepsilon(r)$ is the anisotropic fluid energy density; $P_r(r)$ and $P_{\perp}(r)$ are the radial and tangential pressures, respectively.

The Einstein's field equations for metric (8) have the following form:

$$e^{-\lambda}\Big(\frac{\lambda'}{r} - \frac{1}{r^2}\Big) + \frac{1}{r^2} = 8\pi(\varepsilon + 2C_1 + \varphi(r)) \ , \tag{12}$$

$$e^{-\lambda}\Big(\frac{\nu'}{r} + \frac{1}{r^2}\Big) - \frac{1}{r^2} = 8\pi(P_r + \varphi(r)) \ , \tag{13}$$

$$\frac{1}{2}e^{-\lambda}\Big(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu'\lambda'}{2}\Big) =$$

$$= 8\pi(P_{\perp} + \varphi(r)) \ , \tag{14}$$

where $\varphi(r) = -C_1 + 6\pi\Theta^2$; the prime denotes the differentiation with respect to r.

Following the paper [10], let us assume that the mass function $m(r) = r(1 - \exp(-\lambda))$ is directly proportional to the radius:

$$m = m_0 r \ , \quad m_0 = \alpha(\alpha+1)^{-1} \ , \quad e^{\lambda} = \alpha + 1 \tag{15}$$

where $\alpha = \text{const}, \alpha > 0$.

Then we obtain from (12):

$$\varepsilon_{\text{eff}} = C_1 + 6\pi\Theta^2 + \varepsilon = \alpha/8\pi(\alpha+1)r^2 \ . \tag{16}$$

It is not difficult to show that, in absence of anisotropic fluid, the set of equations (13), (14), and (16) is inconsistent.

Thus, we have three equations for five unknowns ν, Θ, ε, P_r and $P_{\perp}$ and hence, only if there exist certain connections between the unknowns, solutions may exist.

Below, some examples of exact solutions of equations (13), (14) and (16) are presented under fulfilment of the requirements $\varepsilon > 0$ and $\Theta^2 > 0$.

2. Method for Generating Solutions in ECT

Introducing the effective expressions for pressures

$$P_{r(\text{eff})} = P_r + \varphi(r) \ , \qquad P_{\perp(\text{eff})} = P_{\perp} + \varphi(r) \ , \tag{17}$$

one can use the results of paper [10] for attaining exact solutions in ECT. Note that the behaviour of the effective energy density and pressures has been analyzed in [10]. Below we apply the notations, which are used in [10].

2.1. Solutions with $P_{\perp(\text{eff})} = kP_{r(\text{eff})}$

Let us consider some examples showing an application of the method.

1. Under the condition $k(\alpha - k) > 0$, the solution may be written as follows:

$$e^{\nu} = e^{\nu_0}\Big(\frac{r}{r_0}\Big)^{2k}\cos^2\Big(s\ln\frac{r}{r_0}\Big) \ , \qquad \varepsilon = \varepsilon_{\text{eff}} - 2C_1 \ ,$$

$$P_{r(\text{eff})} = \frac{\varepsilon_{\text{eff}}}{\alpha}\Big(2k - \alpha - 2s\tan\Big(s\ln\frac{r}{r_0}\Big)\Big) \ ,$$

$$6\pi\Theta^2 = C_1 \ , \ \ P_r = P_{r(\text{eff})} \ , \ \ P_{\perp} = kP_{r(\text{eff})} \ , \tag{18}$$

where $s = \sqrt{k(\alpha - k)}$; here and below, k, ν_0, and r_0 are constants ($r_0 > 0$).

The analysis of solution (18) shows that, at the points

$$r_s = r_0\exp(\pi(1+2N)/2s) \ , \quad N = 0, \pm1, \pm2, \ldots , \tag{19}$$

it has the singularities ($e^{\nu} \to 0$, $P_r \to \infty$, $P_{\perp} \to \infty$) and it takes place only in the region $r_s < r < \hat{r}$, where $\hat{r} = \big(\frac{\alpha+1}{\alpha}16\pi C_1\big)^{-1/2}$, $r_s < \hat{r}$. Thus, solution (18) may be interpreted locally only because it describes the part of the interior region of a star. Since, the square of the trace of torsion is determined by the expression $S^2 = 36\pi^2\Theta^2$, solution (18) associates with constant torsion.

2. With $k(k-\alpha)=0$, a solution has the form

$$e^{\nu}=e^{\nu_0}r^{2k}\ln^2(\frac{r}{r_0}),\ P_{r(\mathrm{eff})}=\frac{\varepsilon_{\mathrm{eff}}}{\alpha}\left(\frac{2}{\ln(\frac{r}{r_0})}+2k-\alpha\right). \qquad (20)$$

The following cases are possible:

a) for $k=\alpha$, $P_r=0$, the solution is

$$P_{\perp}=(\alpha-1)P_{r(\mathrm{eff})}\ , \qquad 6\pi\Theta^2=P_{r(\mathrm{eff})}+C_1\ ,$$

$$\varepsilon=-2\varepsilon_{\mathrm{eff}}\Big(\alpha\ln(\frac{r}{r_0})\Big)^{-1}-2C_1\ . \qquad (21)$$

Solution (21) describes the interior region of a star: $0<r\le r_b$ ($P_{r(\mathrm{eff})}(r_b)=0$). Under the condition $r_b<\hat{r}$, this solution can be matched [10] with the Schwarzchild exterior metric on the boundary of matter

$$ds^2=-\frac{dr^2}{1-r_g/r}-r^2d\Omega^2+(1-r_g/r)dt^2\ , \qquad (22)$$

where $r_b=r_0\exp(-2/\alpha)$, $r_g=\alpha r_b/(\alpha+1)$.

b) for $k=\alpha$, $P_{\perp}=0$, we obtain

$$P_r=(1-\alpha)P_{r(\mathrm{eff})}\ , \qquad 6\pi\Theta^2=\alpha P_{r(\mathrm{eff})}+C_1\ ,$$

$$\varepsilon=\varepsilon_{\mathrm{eff}}(1-\alpha-2\ln^{-1}(r/r_0))-2C_1\ . \qquad (23)$$

This solution exists for $1<\alpha<2$ in the region $r_1<r\le r_b$, where r_1 is defined by the following inequality:

$$1-2\ln^{-1}(r_1/r_0)-\alpha>(r_1/\hat{r})^2\ . \qquad (24)$$

The matching with the Schwarzchild exterior metric is possible under the same restriction on the parameter r_b, which has been written out for the above case.

c) for $k=0$, $P_{\perp}=0$, the solution is

$$P_r=P_{r(\mathrm{eff})}\ , \quad 6\pi\Theta^2=C_1\ , \quad \varepsilon=\varepsilon_{\mathrm{eff}}-2C_1\ . \qquad (25)$$

The solution exists under the condition $r<\hat{r}$.

The analysis of solutions (20), (21) and (25) shows that they have the singularities at the center ($e^{\nu}\to\infty$, $\varepsilon\to\infty$). For solutions (21) and (25), it is characteristic that $P_{\perp}\to\infty$, $\Theta\to\infty$ and $P_r\to\infty$, $\Theta=\mathrm{const}$ at the center, respectively.

3. Under the condition $k(\alpha-k)<0$, the solution may be written as follows:

$$e^{\nu}=\left(Ar^{k-n}-Br^{k+n}\right)^2\ ,$$

$$P_{r(\mathrm{eff})}=\frac{\varepsilon_{\mathrm{eff}}}{k\alpha}\frac{A(k-n)^2-B(k+n)^2r^{2n}}{A-Br^{2n}}\ , \qquad (26)$$

where A,B are the integration constants; $n=\sqrt{k(k-\alpha)}$.

The following cases are possible:

a) for $k>\alpha$, $P_r=0$, we obtain:

$$P_{\perp}=(k-1)P_{r(\mathrm{eff})}\ , \quad 6\pi\Theta^2=P_{r(\mathrm{eff})}+C_1\ ,$$

$$\varepsilon=\varepsilon_{\mathrm{eff}}-P_{r(\mathrm{eff})}-2C_1\ . \qquad (27)$$

This solution exists for $AB>0$, $r\le r_b$. Under the condition $r_b<\hat{r}$, solution (27) can be matched [10] with the Schwarzchild exterior metric on the boundary of matter: $r_b=(\sqrt{A/B}(k-n)/(k+n))^{1/n}$, $r_g=\alpha r_b/(\alpha+1)$.

b) for $k>\alpha$, $P_{\perp}=0$, we have

$$P_r=(1-k)P_{r(\mathrm{eff})}\ , \qquad 6\pi\Theta^2=kP_{r(\mathrm{eff})}+C_1\ ,$$

$$\varepsilon=\varepsilon_{\mathrm{eff}}-kP_{r(\mathrm{eff})}-2C_1\ . \qquad (28)$$

This solution is applicable under the condition $r<r_3$, where r_3 is defined by the inequality

$$\frac{1}{\alpha}\frac{A(k-n)^2-B(k+n)^2r_3^{2n}}{A-Br_3^{2n}}<1-(r_3/\hat{r})^2\ . \qquad (29)$$

The analysis of solutions (26), (27), and (28) shows that they have the singularities at the center ($\varepsilon\to\infty$, $P\to\infty$, $\Theta\to\infty$) and at the points $r_s^{2n}=A/B$ ($P\to\infty$, $\Theta\to\infty$). The energy density and pressures are monotonously decreasing functions of the radial coordinate in the region $r_s<r\le r_b$ for solution (27) and in the region $r_s<r<r_3$ for solution (28).

4. Under the condition $k(\alpha-k)\le 0$ (this corresponds to the case $Z=\mathrm{const}$ in paper [10]), the solution is

$$e^{\nu}=e^{\nu_0}r^{2(k+\beta n)}\ ,\ P_{r(\mathrm{eff})}=(k+\beta n)^2(k\alpha)^{-1}\varepsilon_{\mathrm{eff}}, \qquad (30)$$

where $\beta=\pm1$, $n=\sqrt{k(k-\alpha)}$.

a) for $k>\alpha$, $P_r=0$, we obtain

$$P_{\perp}=(k-1)P_{r(\mathrm{eff})}\ , \qquad 6\pi\Theta^2=P_{r(\mathrm{eff})}+C_1\ ,$$

$$\varepsilon=2(\varepsilon_{\mathrm{eff}}/\alpha)(\alpha-\beta n-k)-2C_1\ . \qquad (31)$$

This solution exists for $\beta=-1$ under the condition $r<\sqrt{n/(k+n)}\hat{r}$.

b) for $k\ge\alpha$, $P_{\perp}=0$, we have

$$P_r=(1-k)P_{r(\mathrm{eff})}\ , \qquad 6\pi\Theta^2=kP_{r(\mathrm{eff})}+C_1\ ,$$

$$\varepsilon=\varepsilon_{\mathrm{eff}}-kP_{r(\mathrm{eff})}-2C_1\ . \qquad (32)$$

Solution (32) is applicable for $\beta=+1$ under the condition $(r/\hat{r})^2<1-k(k+n)/(k-n)$ and $k-n>$

$k(k+n)$, for $\beta = -1$ under the conditions $(r/\hat{r})^2 < 1 - k(k-n)/(k+n)$ and $k+n > k(k-n)$.

With $k = \alpha$, we obtain $P_{r(\mathrm{eff})} = \varepsilon_{\mathrm{eff}}$, $P_{\perp(\mathrm{eff})} = \alpha\varepsilon_{\mathrm{eff}}$, $6\pi\Theta^2 = \alpha\varepsilon_{\mathrm{eff}} + C_1$, $\varepsilon = \varepsilon_{\mathrm{eff}}(1 - \alpha - r^2/\hat{r}^2)$. This solution exists under the condition $r < \sqrt{1-\alpha}\hat{r}$ and $\alpha < 1$.

c) the case $k = 0$, $P_\perp = 0$ describes the sphere with effective tangential pressures, which equal to zero:

$$e^\nu = (\alpha+1)^{-1} , \quad P_{r(\mathrm{eff})} = -\varepsilon_{\mathrm{eff}} , \quad P_{\perp(\mathrm{eff})} = 0 ,$$

$$P_r = P_{r(\mathrm{eff})} , \quad \varepsilon = \varepsilon_{\mathrm{eff}} - 2C_1 , \quad 6\pi\Theta^2 = C_1 . \tag{33}$$

This solution exists under the condition $r < \sqrt{\alpha}\hat{r}$.

The analysis of the solutions obtained under the condition $k(\alpha - k) \le 0$ shows that they have the same properties at the center as the above solutions.

2.2. Solutions with $P_{r(\mathrm{eff})} = \gamma\varepsilon_{\mathrm{eff}}$

In this case, the solutions may be written as follows:

$$e^\nu = e^{\nu_0} r^{\alpha(\gamma+1)} , \quad P_{\perp(\mathrm{eff})} = \frac{\alpha}{4}(\gamma+1)^2\varepsilon_{\mathrm{eff}} , \tag{34}$$

where $\gamma = \mathrm{const}$.

a) with $P_r = 0$, we obtain

$$P_\perp = (s_1 - \gamma)\varepsilon_{\mathrm{eff}} , \qquad \varepsilon = (1-\gamma)\varepsilon_{\mathrm{eff}} - 2C_1 ,$$

$$6\pi\Theta^2 = C_1 + \gamma\varepsilon_{\mathrm{eff}} , \qquad s_1 = \frac{\alpha}{4}(\gamma+1)^2 . \tag{35}$$

For $\gamma < 0$ ($|\gamma| < 1$), solution (35) takes place only in the region $2|\gamma| < (r/\hat{r})^2 < (1+|\gamma|)$, for $0 < \gamma < 1$ at $r < \sqrt{1-\gamma}\hat{r}$. For $\gamma = 0$, this solution exists under the condition $r < \hat{r}$ and describes the sphere with the effective radial pressure, which is equal to zero:

$$P_{r(\mathrm{eff})} = 0 , \quad P_{\perp(\mathrm{eff})} = \frac{\alpha}{4}\varepsilon_{\mathrm{eff}} , \quad \varepsilon = \varepsilon_{\mathrm{eff}} - 2C_1 ,$$

$$P_\perp = \frac{\alpha}{4}\varepsilon_{\mathrm{eff}} , \qquad 6\pi\Theta^2 = C_1 . \tag{36}$$

b) with $P_\perp = \beta P_r$ ($\beta = \mathrm{const}$), the solution is

$$\varepsilon = -2C_1 + [1 + \beta(\gamma-1) - s_1](1-\beta)^{-1}\varepsilon_{\mathrm{eff}} ,$$

$$P_r = (\gamma - s_1)(1-\beta)^{-1}\varepsilon_{\mathrm{eff}} ,$$

$$6\pi\Theta^2 = C_1 + (s_1 - \beta\gamma)(1-\beta)^{-1}\varepsilon_{\mathrm{eff}} . \tag{37}$$

This solution exists for $\beta(\gamma+1) < 1 + s_1$, $\beta < 1$ in the region $2(\beta\gamma - s_1) < (1-\beta)(r/\hat{r})^2 < 1 + \beta(\gamma-1) - s_1$. It should be noted that, for $s_1 = \beta\gamma$ under the condition $r < \hat{r}$, the model with constant torsion is possible:

$$\varepsilon = \varepsilon_{\mathrm{eff}} - 2C_1 , \; P_{\perp(\mathrm{eff})} = \beta\gamma\varepsilon_{\mathrm{eff}} , \; 6\pi\Theta^2 = C_1. \tag{38}$$

c) with $P_r = \lambda\varepsilon$ ($\lambda = \mathrm{const}$), we have

$$\varepsilon = [(\gamma-1)\varepsilon_{\mathrm{eff}} + 2C_1](\lambda-1)^{-1} ,$$

$$P_\perp = [s_1 - (\lambda-\gamma)/(\lambda-1)]\varepsilon_{\mathrm{eff}} + 2\lambda C_1 ,$$

$$6\pi\Theta^2 = [(\lambda-\gamma)\varepsilon_{\mathrm{eff}} - (\lambda+1)C_1](\lambda-1)^{-1} . \tag{39}$$

Solution (39) exists for $\gamma < 1$, $-1 < \lambda < 1$ in the region $2(\lambda-\gamma)/(\lambda+1) < (r/\hat{r})^2 < 1-\gamma$. Note that this solution contains solution (35) for $0 < \gamma < 1$ as a particular case.

d) with $P_\perp = k\varepsilon$ ($k = \mathrm{const}$), the solution is

$$(1-k)\varepsilon = -2C_1 + (1-s_1)\varepsilon_{\mathrm{eff}} ,$$

$$(1-k)P_r = [\gamma + k(1-\gamma) - s_1]\varepsilon_{\mathrm{eff}} - 2kC_1 ,$$

$$(1-k)6\pi\Theta^2 = (1+k)C_1 + (s_1 - k)\varepsilon_{\mathrm{eff}} . \tag{40}$$

This solution is defined only for $k < 1$, $s_1 < 1$ in the region $2(k-s_1)(1+k)^{-1} < (r/\hat{r})^2 < 1 - s_1$.

3. Other Exact Solutions

Now let us list other examples, for which the effective energy density and pressure are not bound up with specific equations of state.

1. With $P_r = \gamma\varepsilon$, $P_\perp = kP_r$, we find a solution in the following form:

$$e^\nu = e^{\nu_0} r^{2(2\alpha-1)} e^{2n_1B_1r} ,$$

$$6\pi\Theta^2 = \frac{1}{\gamma-1}\Big[-(\gamma+1)C_1 + \frac{1}{s_2}\Big(\frac{s_3}{r^2} - \frac{2n_1B_1}{r}\Big)\Big] ,$$

$$\varepsilon = \frac{2}{\gamma-1}\Big[C_1 + \frac{1}{s_2}\Big(\frac{\alpha-1}{r^2} + \frac{n_1B_1}{r}\Big)\Big] . \tag{41}$$

where $n_1 = \pm1$, $B_1 = \sqrt{(4\alpha-3)s_2C_1}$, $s_2 = 8\pi(\alpha+1)$, $s_3 = \alpha(\gamma-3) + 2$, $k = (2\gamma)^{-1}[4\alpha(\gamma-1) + 3 - \gamma]$.

With $n = +1$, solution (41) exists for $3/4 < \alpha < 1$, $\gamma > 3 - 2/\alpha$ in the region $r_1 < r < r_2$, where r_1 and r_2 are defined as

$$r_1 = \frac{s_4 - \sqrt{4\alpha-3}}{\sqrt{s_2(\gamma+1)^2C_1}} , \qquad r_2 = \frac{1 - \sqrt{4\alpha-3}}{\sqrt{4s_2C_1}} . \tag{42}$$

where $s_4 = \sqrt{\alpha(\gamma-1)^2 + 2\gamma - 1}$.

With $n = -1$, solution (41) exists for $\alpha > 1$, $-1 < \gamma < 1$ in the region $r_3 < r < r_4$, where $r_3 = -r_2$, $r_4 = -r_1$.

2. Under the conditions $P_r = \gamma\varepsilon$, $6\pi\Theta^2 = C_1$, the solution may be written as follows:

$$e^\nu = e^{\nu_0} r^{\alpha(\gamma+1)} e^{-\gamma s_2 C_1 r^2} , \quad \varepsilon = \varepsilon_{\mathrm{eff}} - 2C_1 ,$$

$$P_\perp = s_5 + \frac{\alpha}{4}(\gamma+1)^2\varepsilon_{\mathrm{eff}} + \frac{\alpha}{2}\gamma^2C_1(r/\hat{r})^2 . \tag{43}$$

where $s_5 = -\gamma[2 + \alpha(\gamma+1)]C_1$.

This solution exists for $r < \hat{r}$.

Conclusion

In this article, the possibility of the existence of a static spherical configuration with linear mass function in ECT has been investigated.

It is shown that such configurations cannot exist for one-fluid models, when the torsion is generated by a perfect fluid.

For two-fluid models, which contain isotropic and anisotropic components, the method for generating solutions in ECT from the well-known ones in GR has been proposed. In this case, the effective energy density and pressures are related by the specific equations of state. It is shown that the metric of the space-time may be the same as in GR, but with other restrictions on the parameters of a solution.

The examples of other metrics, for which the effective energy density and pressure are not bound up with any specific equations of state, have been listed.

It is interesting to note that, for five metrics from the seven ones, the solutions with constant torsion, for which $P_{r(\mathrm{eff})} = P_r$ and $P_{\perp(\mathrm{eff})} = P_\perp$, are possible.

It is shown that some solutions can be matched with the Schwarzchild exterior metric.

For all of the above–presented solutions, besides (43), the expressions for ε, P_r, $P_\perp$ are monotonously decreasing functions of the radial coordinate in the regions of their existence.

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ДВОРІДИННІ СТАТИЧНІ СФЕРИЧНІ КОНФІГУРАЦІЇ
З ЛІНІЙНОЮ МАСОВОЮ ФУНКЦІЄЮ
В ТЕОРІЇ ЕЙНШТЕЙНА—КАРТАНА

А.М.Галіахметов

Р е з ю м е

Досліджуються дворідинні статичні сферичні конфігурації з лінійною масовою функцією в теорії Ейнштейна—Картана.Одна з рідин моделює анізотропний розподіл речовини усередині зірки, а друга є ідеальною рідиною, яка виконує роль джерела кручення. Показано, що розв'язки рівнянь Ейнштейна для анізотропних релятивістських сфер у загальній теорії відносності можуть генерувати розв'язки у теорії Ейнштейна—Картана. Знайдено деякі точні розв'язки.

ДВУХЖИДКОСТНЫЕ СТАТИЧЕСКИЕ СФЕРИЧЕСКИЕ КОНФИГУРАЦИИ С ЛИНЕЙНОЙ МАССОВОЙ ФУНКЦИЕЙ В ТЕОРИИ ЭЙНШТЕЙНА—КАРТАНА

А.М.Галиахметов

Р е з ю м е

Исследуются двухжидкостные статические сферические конфигурации с линейной массовой функцией в теории Эйнштейна—Картана. Одна из жидкостей моделирует анизотропное распределение вещества внутри звезды, а другая является идеальной жидкостью, представляющей источник кручения. Показано, что решения уравнений Эйнштейна для анизотропных релятивистских сфер в общей теории относительности могут генерировать решения в рамках теории Эйнштейна—Картана. Приведены некоторые точные решения.