THE PROBLEM OF INTERACTION IN A DYNAMICAL THEORY OF PARTICLES (GENERAL QUESTIONS). 2

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We continue the consideration of the interaction problem in the frame of a new field particle theory. Here a new correspondence principle and the connection between bilocal and usual local fields are discussed. The method of second quantization of bilocal fields is formulated and a scattering matrix is built. Explicit form of smearing operators and formfactors is found. Comparison of a new particle field theory with the old (local) axiomatic approach is given.

1. A New Correspondence Principle

In the equations of interacting fields (see formula (15) in [1]), the biolocal fields enter: fundamental $\psi(X, Y)$ and degeneration field $\theta(X, Y')$. As usual, the variables internal Y, Y' of particlies are not fixed in experiment. And there are two reasons for this: 1) particles rapidly go away from the interacting region, i.e. in the case where $|X| >> |Y|$, $|X| >> |Y'|$, and therefore it may be considered that $Y = Y' = 0$ and the interaction 'current“ may be written in the local form $\psi(X)\,\theta(X)$, where $\psi(X) = \psi(X, 0)$, $\theta(X) = \theta(X, 0)$, 2) or particles describing by fields ψ, θ stay together a very long time (for example at the consideration of radiative corrections), so that the correlation between the inner variables Y and Y' may be established and, as a result, the 'current“ $\psi(X, Y)\,\theta(X, Y')$ must be integrated over the inner variables Y, Y' with some measure $d\mu(Y, Y')$. The explicit form of this measure is found proceeding from a new correspondence principle connected with the third fundamental constant *k*.

We now formulate this principle. For this purpose, we introduce new dimensionless variables u_μ, u'_μ putting $Y = u/k$, $Y' = u'/k$. Hence k^{-1} plays the role of fundamental length in the space of inner coordinates $Y, Y' \in \mathbf{R}_{3,1}$. Let us put $d\mu(Y, Y') = \mu(u, u')\, d^4u\, d^4u'$ and require that $\mu(u, u')$ be a relativistic and universal function not depending on the sort of a particle Σ. Then $\mu(u, u')$ must depend on the invariant uu' only (and does not depend on u^2 and u'^2), so that $\mu(u, u') = \mu(uu')$. Considering a typical correlation

$$\int \psi\left(X, \frac{u}{k}\right) \theta\left(X, \frac{u'}{k}\right) \mu(uu')\, d^4u\, d^4u' = \overbrace{\psi(X)\,\theta}(X), \tag{1}$$

we can put $u = k\upsilon$, $u' = \upsilon'/k$ and write

$$\int \psi(X, \upsilon)\, \theta\left(X, \frac{\upsilon'}{k^2}\right) \mu(\upsilon\upsilon')\, d^4\upsilon\, d^4\upsilon'. \tag{2}$$

A new correspondence principle is formulated in such a way: at $k^{-1} \to 0$, (1), (2) must transfer into a local coupling $\psi(X)\,\theta(X)$, i.e. in the limit $k^{-1} = 0$, a new theory transfers into the usual local one. Putting $k^{-1} = 0$ in (1), we come to the condition

$$\int \mu(uu')\, d^4u' = \delta^4(u) \tag{3}$$

and, considering $k^{-1} = 0$ in (2), we get

$$\int \mu(uu')\, d^4u\, d^4u' = 1. \tag{4}$$

If we introduce a new variable $\frac{1}{(2\pi)^4} e^{-iuu'}$, then $\mu(uu')$ will be written in the form $\mu(uu') = I\left(\frac{e^{-iuu'}}{(2\pi)^4}\right)$. The new variable possesses the group property therefore it is natural to assume that I realizes a group homomorphism. Then the well-known theorem about the continuation of a group homomorphism to the group ring homomorphism allows us to write

$$\int I\left(\frac{e^{-iuu'}}{(2\pi)^4}\right) d^4u\, d^4u' =$$

$$= I\,(\int \frac{e^{-iuu'}}{(2\pi)^4}\, d^4u\, d^4u') = I\,(1) = 1$$

and

$$\int I\left(\frac{e^{-iuu'}}{(2\pi)^4}\right) d^4u' =$$

$$= I\,(\int \frac{e^{-iuu'}}{(2\pi)^4}\, d^4u') = I\,(\delta^4(u)) = \delta^4(u).$$

It follows from here that I is the identical mapping and therefore $\mu\,(uu') = \frac{1}{(2\pi)^4}\, e^{-iuu'}$. Further,since

$$\int \psi\,(X, \frac{u}{k})\, \frac{e^{-iuu'}}{(2\pi)^4}\, d^4u\, d^4u' = \psi\,(X, 0) = \psi\,(X) \qquad (5)$$

so, after integrating (15) from [1] over Y and Y' with the measure $d\,\mu\,(Y, Y')$, we obtain the equation

$$(\Gamma^{\Sigma}_{\mu} \frac{\partial}{\partial X_{\mu}} + M_{\Sigma})\, \Psi^{\Sigma}(X) = J^{\Sigma}(X) \qquad (6)$$

in which the 'current" has the structure

$$J^{\Sigma}(X) = \overbrace{\Psi^{\Sigma}(X)\, \theta\,(X)}. \qquad (7)$$

Generally speaking, the field $\Psi\,(X)$ in (6) should not be confused with the asymptotic fields $\Psi\,(X)$ obtained from $\Psi\,(X, Y)$ at $|X| >> |Y|$. Obviously, the field coupling (7) has local form (in X) but indeed it is one of the kinds of non-local coupling (see further).

2. Closing the Lagrangian Field System

The system of interacting fields $\{\psi, \theta\}$ is Lagrangian: Eqs. (6) may be obtained from the variation principle for the action $A = \int L\,(X)\, d^4X$, where the Lagrangian (as for the hermitization of L_i, see further)

$$L = \overline{\Psi}\left(\Gamma_{\mu} \frac{\partial}{\partial X_{\mu}} + M\right)\Psi +$$

$$+ \frac{i}{2}\left(\overline{\Psi}\,\Gamma_{\mu} \overbrace{\frac{\partial \Phi}{\partial X_{\mu}}}\, \Psi + \overbrace{\overline{\Psi}\,\Gamma_{\mu}} \frac{\partial \Phi}{\partial X_{\mu}}\, \Psi\right) = L_{\Psi} + L_i \qquad (8)$$

is a Hermitian form. Hereby only asymptotic fields (free, local) may be varied. Bilocal fields whose inner variables are integrated are not subjected to the variation as a direct created by the be-Hamiltonian system (the variation principle is not applied to the f and $\dot{f}$ because a bi-Hamiltonian system is not a Lagrangian one).

We see that the equations for 'gluing" fields (degeneration degrees of freedom or interactions) θ do not follow from the base of this scheme. But they may be obtained if we close the system $\{\Psi, \theta\}$ by means of adding the Lagrangian L_{θ} of the free field θ to the Lagrangian $L_{\Psi} + L_i$. In general case, L_{θ} is the unknown function. But, in the limit of weak and slowly changed in space-time fields θ, the Lagrangian $L_{\theta} = -\frac{1}{8\pi}\left(\frac{\partial\theta}{\partial X_{\mu}}\right)^2$. This Lagrangian satisfies (in the frame of gauge formalism) the condition $L_{\theta} = 0$ at $\theta = \text{const}$. Therefore 'bare" quanta of fields θ have zero mass and, in this limit, the free equation is $\Box_X\, \theta\,(X, Y) = 0$.

3. Connection between Bilocal and Local Fields

It is natural to connect a further development of the theory with the Feynman procedure of quantization using the continual integrating over c-number fields. But more effective is the operator formalism. At usage of this formalism, the connection between bilocal and local fields plays the important role. This connection is established proceeding from the explicit form of bilocal fields $\psi\,(X, Y)$, see [1]. The field $\overset{(+)}{\psi}{}^{\Sigma}(X, Y)$ may be written as

$$\overset{(+)}{\psi}{}^{\Sigma}_{\alpha k}(X, Y) = O^{\Sigma}_{\alpha k}\left(-i\frac{\partial}{\partial X}, -i\frac{\partial}{\partial Y}\right) F\left(Y, -i\frac{\partial}{\partial X}; n\right) \times$$

$$\times\ (2\pi)^{3/2}\, i\,\Delta^{(+)}(X; M_{\Sigma}; n)$$

where

$$\Delta^{(+)}(X; M_{\Sigma}; n) =$$

$$= \frac{-i}{(2\pi)^3} \int e^{iPX}\, \theta\,(Pn)\, \delta\,(P^2 + M^2_{\Sigma})\, d^4P$$

(here n_{μ} is a unit time-like 4-vector: $n^2_{\mu} = \mathbf{n}^2 - n^2_0 = -1$) and

$$\theta\,(Pn)\, F\,(Y, P; n) = \frac{1}{2\pi}\int e^{iQY}\, \theta\,((P+Q)\,n) \times$$

$$\times\ \theta\,((P-Q)\,n)\, \delta\,(Q^2 + P^2)\, \delta\,(QP)\, d^4Q =$$

$$= \theta\,(P\,n)\, \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dz}{z} \sin\,(z\,P\,n)\, \frac{\sin w\,(z)}{w\,(z)}. \qquad (9)$$

In (9), $w(z) = \sqrt{P_{\mu\nu} N_{\mu\nu}(z)} = \sqrt{z^2 I_1 + 2z\, I_2 + I_3}$, where $P_{\mu\nu} = P_{\mu} P_{\nu} - \delta_{\mu\nu} P^2$, $N_{\mu\nu} = z^2\, n_{\mu}\, n_{\nu} +$

$+ 2zn_\mu Y_\nu + Y_\mu Y_\nu$, and $I_1 = (\mathcal{P}n)^2 - \mathcal{P}^2 n^2$, $I_2 = (\mathcal{P}Y)(\mathcal{P}n) - \mathcal{P}^2 (Yn)$, $I_3 = (\mathcal{P}Y)^2 - \mathcal{P}^2 Y^2$. The result of action of the operator $O^\Sigma(\mathcal{P}, -i\frac{\partial}{\partial Y})$ on $F(Y, \mathcal{P}; n)$ may be represented in the form

$$O^\Sigma_{\alpha k}(\mathcal{P}, -i\frac{\partial}{\partial Y})\, F(Y, \mathcal{P}; n) = F_\Sigma(Y, \mathcal{P}; n)\, a^\Sigma_{\alpha k}(\mathcal{P}, Y; n).$$

The amplitudes $a^\Sigma_{\alpha k}(\mathcal{P}, Y; n)$ contain all information about inner symmetry properties (spin and isospin) of a particle with quantum numbers Σ (indices α, k) and also about its space-time configuration (4-momentum $\mathcal{P}_\mu$). We consider (it is some approximation) that all these properties are identical for point-like and smeared particles. Therefore, we put $Y_\mu = 0$ in $a^\Sigma_{\alpha k}(\mathcal{P}, Y; n)$. It is that simplification which gives the possibility to apply the operator formalism and to understand the approximate character of the given quantization procedure.

Now the field $\overset{(+)}{\psi}{}^\Sigma(X, Y)$ may be written as

$$\overset{(+)}{\psi}{}^\Sigma_{\alpha k}(X, Y) = F_\Sigma(Y, -i\frac{\partial}{\partial X}; n)\, \overset{(+)}{\psi}{}^\Sigma_{\alpha k}(X),$$

where

$$\overset{(+)}{\psi}{}^\Sigma_{\alpha k}(X) = a^\Sigma_{\alpha k}(-i\frac{\partial}{\partial X}; 0; n)(2\pi)^{3/2} i\,\Delta^{(+)}(X; M_\Sigma; n) =$$

$$= \frac{1}{(2\pi)^{3/2}} \int e^{i\mathcal{P}X}\, \theta(\mathcal{P}n)\, \delta(\mathcal{P}^2 + M^2_\Sigma) \times$$

$$\times\, a^\Sigma_{\alpha k}(\mathcal{P}, 0; n)\, d^4\mathcal{P}$$

is a usual local field.

In the interaction process (in particular in the measurement process), the amplitudes $a^\Sigma_{\alpha k}$ change but the function F_Σ does not change. It is a universal characteristic of a smeared particle not depending on interaction and measurement, created by the bi-Hamiltonian dynamical system. Its appearance results from the transition $f \to \dot{f}$, and it contains all information about the inner space-time structure of a particle. In the limit $Y = 0$ (not depending on Σ), we have (see [1])

$$F_\Sigma(0, \mathcal{P}; n) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dz}{z} \sin(z\mathcal{P}n) \frac{\sin z\sqrt{I_1}}{z\sqrt{I_1}} = 1.$$

The function $F_\Sigma(Y, \mathcal{P}; n)$ appears in natural way at the positive-frequency part of the field only, arising in the quantum transition $f \to \dot{f}$. We assume that the negative-frequency part of the field arising as a result of interaction only (see [1]) inherits the structure so that the complete field $\Psi^\Sigma(X, Y)$ is written as

$$\Psi^\Sigma_{\alpha k}(X, Y) = F_\Sigma\left(Y, -i\frac{\partial}{\partial X}; n\right) \Psi^\Sigma_{\alpha k}(X). \tag{10}$$

We look for the connection between bilocal and local fields. A function F_Σ is named as a smearing operator. It has a simple form only for particles with spin 0 and 1/2.

It is interesting to note that if the *smearing* field $\psi^\Sigma(X, Y)$ is written in the form of a *smoothed* field $\psi^\Sigma(X, Y) = \int \psi^\Sigma(X')\, f_\Sigma(X', X; Y)\, d^4X'$ (such fields are used in the axiomatic theory [2]), so f_Σ is

$$f_\Sigma\left(X', X; Y\right) = F_\Sigma\left(Y, -i\frac{\partial}{\partial X}; n\right) \delta^4(X - X'), \tag{11}$$

that is a *non-local generalized function*. In the axiomatic approach, only *fit functions* are used. In this, there is a principal difference between the suggested and axiomatic approaches (see below).

4. Operator Formalism

The transition from the c-number field theory to the q-number one is realized by means of replacement of classical Poisson brackets by quantum ones accounting the connection between spin and statistics. Hereby the problem of constructing the Fock space of state vectors of the system is arisen. For this purpose, it is natural to use the algebra of asymptotic fields $\hat{\overline{\Psi}}{}^\Sigma(X)$, $\hat{\Psi}^\Sigma(X)$, $\hat{\theta}(X)$ (or amplitudes $\hat{a}^\Sigma(\mathcal{P})$, $\hat{b}^\Sigma(\mathcal{P})$, $\hat{a}^{\Sigma +}(\mathcal{P})$, $\hat{b}^{\Sigma +}(\mathcal{P})$, $\hat{c}(k)$, $\hat{c}^+(k)$, further hats over operators will be omitted).

The spaces $\mathcal{H}_{\text{in}}$ and $\mathcal{H}_{\text{out}}$ of incoming and outgoing state vectors may be obtained by means of the same algebra $U[\Psi^\Sigma, \overline{\Psi}{}^\Sigma, \theta]$ (or its some closure)acting on the circle vector $|0\rangle$. Therefore it may be written $\mathcal{H}_{\text{in}} = \mathcal{H}_{\text{out}} = \mathcal{H}$.

A Hermitian definite scalar product on $\mathcal{H}$ is labeled $\langle | \rangle$. In the suggested theory, the zero Wightman's axiom [3] is fulfilled. However, beside the algebra of asymptotic field operators, we have the algebra of bilocal fields. Both these algebras are connected with each other in some way: the permutation relations for bilocal fields are obtained from the permutation relations for local fields by means of the formula

$$[\Psi^\Sigma(X, Y), \overline{\Psi}{}^\Sigma(X', Y')]_\pm = F_\Sigma\left(Y, -i\frac{\partial}{\partial X}; n\right) \times$$

$$\times\, F_\Sigma\left(Y', -i\frac{\partial}{\partial X}; n\right) [\Psi^\Sigma(X), \overline{\Psi}{}^\Sigma(X')]_\pm, \tag{12}$$

where $[\Psi^{\Sigma}(X), \overline{\Psi}^{\Sigma}(X')]_{\pm}$ are the canonical permutation relations in the algebra of local fields operators. The connection between spin and statistics for bilocal fields is determined by the corresponding connection for local fields.

As $F_{\Sigma}(Y, P; n)$ is a non-local generalized function, so the right-hand side in (12) is not equal to zero at space-like $X - X'$ $((X - X')^2 < 0)$. This means that the fourth Wightman's axiom concerning local commutativity (microcausality) is not fulfilled in the theory of bilocal fields. However, at $|X - X'| >> k^{-1}$, the right-hand side in (12) makes zero. It means that macrocausality takes place in a new theory. It is interesting to note that microcausality is restored if (12) is integrated over Y and Y' with measure $d\mu(Y, Y')$: integration gives $\frac{\sin M_{\Sigma}^2}{M_{\Sigma}^2}[\Psi^{\Sigma}(X), \overline{\Psi}^{\Sigma}(X')]_{\pm}$.

On account of (12), the commutator

$$[H_i(X), H_i(X')] \neq 0 \tag{13}$$

is not equal to zero at space-like $X - X'$ but it vanishes like (12) in the asymptotic limit $|X - X'| >> k^{-1}$ (or in the limit $k^{-1} \to 0$). In (13), we have $H_i(X) = -L_i(X)$.

Turning commutator (13) to zero at $(X - X')^2 < 0$ is the necessary condition for integrating the Tomonaga – Schwinger equations. In the suggested theory (like in anyone non-local theory), these equations make no sense. In [4], Heisenberg had emphasized that the impossibility of space-time description of evolution vector states by means of Schrodinger-type equations is the fundamental fact like the impossibility to describe the behaviour of atomic electrons by means of trajectories. He saw the solution of this situation in usage of the S-matrix formalism. The S-matrix approach to the description of quantum systems is more general (that the equation approach) and exists not only in local theory (by the way, due to the well-known Haag's theorem in local theory, the S-matrix is trivial) but in non-local ones too.

5. Scattering Matrix

In the suggested theory, the interaction Lagrangian $\overline{\Psi}(X)J(X)$, where $J(X)$ is given by (7), is hermitized: $\frac{1}{2}(\overline{\Psi}(X)J(X) + \overline{J}(X)\Psi(X))$. In the case of the electromagnetic and gravitational interactions (due to the zero mass of photons and gravitons and properties of the corresponding formfactor, see (19)), it is happened automatically. In the case of strong interaction, the essential role is played again by the involution operation (see [1]). Not only L_i but other dynamical variables of the system are Hermitian. Therefore, a unitary representation of the Lorentz group is realized in the space H (the second Wightman's axiom is valid). Having the Hermitian $L_i(X)$, we can build the unitary operator, the scattering matrix

$$S = T\exp(i\int L_i(X)\, d^4X), \tag{14}$$

where T is the Wick chronological operator.

If we would precede from the non-Hermitian Lagrangian $\overline{\Psi}(X)J(X)$, we would come, using formula (14), to the non-unitary operator $\mathbf{T}$ [5] that might be represented in the form $\mathbf{T} = \mathbf{SR}$, where $\mathbf{S}$ is an unitary operator and $\mathbf{R} = (\mathbf{T}^{+}\mathbf{T})^{1/2}$ is a Hermitian one. It turns out it is impossible to work with the operator $\mathbf{S}$. Therefore, the Heisenberg's way of S-matrix introduction is not acceptable for us. In connection with this, we can say that evolution (in time) and involution (complex conjugation) do not commute one with other.

Meanwhile it is needed indeed to introduce a generally speaking non-Hermitian operator $\mathbf{R}$ with the property: $\mathbf{R}^{-1}H(-\infty) = H_{\text{in}}$, where $H(-\infty)$ is the state vector space obtained from $|0\rangle$ by means of the algebra of field operators carrying an *a priori* normalization N_{Σ} produced by a relativistic bi-Hamiltonian system (see [1]). The operator $\mathbf{R}^{-1}$ has no kernel and may be diagonalized (compare with [6]). It works the states $N_{\Sigma}|\Psi^{\Sigma}\rangle$ into usual states $|\Psi^{\Sigma}\rangle = \hat{\Psi}^{\Sigma}|0\rangle$ (normalized in usual way: one particle in the whole space-time). If all $|N_{\Sigma}| << 1$, so $\mathbf{R}$ is a squeeze operator. And it is natural to assume that it is a Hilbert – Schmidt operator, i.e. $\mathrm{Tr}\,\mathbf{RR}^{+} < \infty$. Then it may be normalized so that $\mathrm{Tr}\,\mathbf{RR}^{+} = 1$ (compare with [6, Capture 6, n.2]). Obviously, we can write $\mathbf{R} = \sum_{\Sigma} N_{\Sigma}|\Psi^{\Sigma}\rangle\langle\Psi^{\Sigma}|$ with $\langle\Psi^{\Sigma}|\Psi^{\Sigma'}\rangle = \delta_{\Sigma\Sigma'}$. Our assumption means that the sum of all *a priori* probabilities is equal to unity: $\sum_{\Sigma}|N_{\Sigma}|^2 = 1$. This condition was used already in [1] for determining the parameters z_k.

Under reducing the S-matrix to the normal form and the calculation of S-matrix elements in the framework of perturbation theory, chronological and usual pairings of field operators entering the states $|\text{in}\rangle$ and $\langle\text{out}|$ are appeared. It is a very simple part of the theory, and we restrict ourselves to the writting of the final result. So we have for chronological pairings:

$$T(\Psi_{\alpha}^{\Sigma}(X, Y)\,\overline{\Psi}_{\alpha'}^{\Sigma}(X') =$$

$$= d^{\Sigma}_{\alpha\alpha'}\left(\frac{\partial}{\partial X}\right)\frac{1}{i}\Delta^{c}(X - X'; M_{\Sigma}; Y),$$

$$T(\theta(X, Y)\,\theta(X', Y')) = \frac{1}{i}\Delta^{c}(X - X'; Y, Y')$$

(operator d^{Σ} was determined in [1]). Usual pairings are

$$N(\theta(X, Y)\,\theta(X')) = \frac{1}{i}\Delta^{(-)}(X - X'; Y),$$

$$N(\Psi^{\Sigma}_{\alpha}(X, Y)\,\overline{\Psi}^{\Sigma}_{\alpha'}(X')) =$$

$$= d^{\Sigma}_{\alpha\alpha'}\left(\frac{\partial}{\partial X}\right)\frac{1}{i}\Delta^{(-)}(X - X'; M_{\Sigma}; Y),$$

$$N(\Psi^{\Sigma}_{\alpha}(X)\,\overline{\Psi}^{\Sigma}_{\alpha'}(X')) =$$

$$= d^{\Sigma}_{\alpha\alpha'}\left(\frac{\partial}{\partial X}\right)\frac{1}{i}\Delta^{(-)}(X - X'; M_{\Sigma}).$$

In these formulas,

$$\Delta^{a}(X; Y, Y') = F_{\Sigma}\left(Y, -i\frac{\partial}{\partial X}; n\right)\times$$

$$\times F_{\Sigma}\left(Y', -i\frac{\partial}{\partial X}; n\right)\Delta^{a}(X; M_{\Sigma})$$

and

$$\Delta^{a}(X; M_{\Sigma}; Y) = F_{\Sigma}\left(Y, -i\frac{\partial}{\partial X}; n\right)\Delta^{a}(X; M_{\Sigma}),$$

hereat

$$\Delta^{c}(X; M_{\Sigma}) = \frac{-i}{(2\pi)^4}\int\frac{e^{iPx}}{P^2 + M^2_{\Sigma} - i\varepsilon}\,d^4P,$$

$$\Delta^{(-)}(X; M_{\Sigma}) = \frac{-i}{(2\pi)^3}\int e^{iPX}\,\theta(P_0)\,\delta(P^2 + M^2_{\Sigma})\,d^4P.$$

The function $\Delta^{c}(X; Y, Y')$ obeys the inhomogeneous equation with smearing δ-function on the right-hand side: $F_{\Sigma}\left(Y, -i\frac{\partial}{\partial X}; n\right)F_{\Sigma}\left(Y', -i\frac{\partial}{\partial X}; n\right)\delta^4(X)$.

In specific calculations, it is very convenient to use one simplification: as $F_{\Sigma}(Y, P; n)$ is essential in only that case where $|Pn| >> |n|$ (in opposite case $F_{\Sigma}(0, P; n) = 1$), so one may neglect the 4-vector n_{μ} in F_{Σ} and use $F_{\Sigma}(Y, P; 0) = F_{\Sigma}(Y, P)$ instead of $F_{\Sigma}(Y, P; n)$.

Forestalling we would like to note that $\lim\limits_{X\to 0}\dot{\Delta}^{(-)}(X, 0) = \lim\limits_{X\to 0}\frac{1}{2}\delta^3(X) = \infty$, where $\dot{\Delta}^{(-)}(X, Y) = \frac{\partial}{\partial t}\Delta^{(-)}(X, Y)$, but $\lim\limits_{Y\to 0}\dot{\Delta}^{(-)}(0, Y) = 0$, i.e. the limits on X and Y are not permutable (see further).

6. Explicit Form of Smearing Operators and Form-factors

In the most important case of massive particles with spin 0 and 1/2, a smearing operator $F_{\Sigma}(Y, P)$ is determined by the formula (see [1])

$$F(Y, P) = \frac{1}{2\pi}\int e^{iQY}\,\delta(Q^2 + P^2)\,\delta(QP)\,d^4Q. \qquad (15)$$

To find the explicit expression for $F(Y, P)$, we have to use the well-known representation of the δ-function

$$\delta(X) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\alpha X}\,d\alpha.$$

Taking the integral in (15) with respect to Q_{μ}, we obtain

$$F(Y, P) = \frac{1}{(2\pi)^3}\frac{\pi^2}{i}\int_{-\infty}^{\infty}\frac{d\alpha}{\alpha^2}\,d\beta\times$$

$$\times\exp i\left(\alpha P^2 - \frac{Y^2 + 2\beta YP + \beta^2P^2}{4\alpha}\right).$$

After the substitution $\alpha P^2 \to \alpha$, $\beta + \frac{PY}{P^2} \to \beta$ and integrating over β, we have

$$F(Y, P) = \frac{1}{(2\pi)^3}\frac{\pi^2}{i}\sqrt{\frac{2\pi}{i}}\times$$

$$\times\int_{-\infty}^{\infty}\frac{d\alpha}{\alpha^{3/2}}\exp i\left(\alpha + \frac{(PY)^2 - P^2Y^2}{4\alpha}\right).$$

If $(PY)^2 - P^2Y^2 > 0$, so the point $\alpha = 0$ in the complex plane (α) must be gone around from below, but if $(PY)^2 - P^2Y^2 < 0$ – from above. Therefore, at the substitution $\alpha \to -\alpha$, we put in the integral $\alpha \to e^{-i\pi}\alpha$ (in the first case) and $\alpha \to e^{i\pi}\alpha$ (in the second case). In result, we obtain

$$F(Y, P) = \frac{1}{(2\pi)^3}\frac{\pi^2}{i}\sqrt{\frac{2\pi}{i}}\times$$

$$\times \int_0^\infty \frac{d\alpha}{\alpha^{3/2}}\left[\exp i\left(\alpha + \frac{(PY)^2 - P^2Y^2}{4\alpha}\right) - \right.$$

$$\left. - i\,\mathrm{sgn}((PY)^2 - P^2Y^2)\exp i\left(-\alpha - \frac{(PY)^2 - P^2Y^2}{4\alpha}\right)\right]$$

and integrating over α gives

$$F(Y, P) = \frac{\sin\sqrt{(PY)^2 - P^2Y^2}}{\sqrt{(PY)^2 - P^2Y^2}}\,\theta((PY)^2 - P^2Y^2), \tag{16}$$

where $\theta(X)$ is the Heaviside function.

For massless particles, we have [1]

$$F_0(Y, k) = e^{iYk}. \tag{17}$$

So the scattering operator is connected in this case with the transition in space-time on the 4-vector Y_μ.

The further integration of a product of two smearing operators with measure $d\mu(u, u')$ (see (1) and the next formulas) gives the formfactor

$$\rho(P, k) = \int F\left(\frac{u}{k}, P\right) F\left(\frac{u'}{k}, k\right) \frac{e^{-inn'}}{(2\pi)^4}\, d^4u\, d^4u' =$$

$$= \int e^{iQq/k^2}\,\delta(Q^2 + P^2)\,\delta(QP)\times$$

$$\times\,\delta(q^2 + k^2)\,\delta(qk)\,\frac{d^4Q}{2\pi}\frac{d^4q}{2\pi}. \tag{18}$$

If a particle with momentum k is massless (like in the case of photon or graviton), so the formfactor is equal

$$\rho(P, k) = \int F\left(\frac{u}{k}, P\right) F_0\left(\frac{u'}{k}, k\right) \frac{e^{-iuu'}}{(2\pi)^4}\, d^4u\, d^4u' =$$

$$= F\left(\frac{k}{k^2}, P\right) = \frac{\sin\sqrt{(Pk)^2 - P^2k^2}}{\sqrt{(Pk)^2 - P^2k^2}}\,\theta((Pk)^2 - P^2k^2). \tag{19}$$

This formfactor does not change at the substitution $P \to P - k$ that quarantees hermiticity of the vertex (see above).

In order to find the explicit expression of formfactor (18), we have to use the previous representation of the δ-function entering (18) and to integrate (18) over Q_μ and q_μ. In result, we came to the integral

$$\rho(P, k) = \frac{-1}{(2\pi)^2}\int_{-\infty}^{\infty} \frac{d\alpha\, d\beta\, d\gamma\, d\delta}{(4\alpha\gamma - 1)^2}\times$$

$$\times \exp i\left[\left(\alpha - \frac{\beta^2\gamma}{4\alpha\gamma - 1}\right)P^2 + \right.$$

$$\left. + \frac{\beta\delta}{4\alpha\gamma - 1}Pk + \left(\gamma - \frac{\delta^2\alpha}{4\alpha\gamma - 1}\right)k^2\right].$$

Integrals over β and δ are Gaussian, and we have

$$\rho(P, k) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{d\alpha\, d\gamma \exp i(\alpha P^2 + \gamma k^2)}{(4d\gamma - 1)\sqrt{(Pk)^2 - 4\alpha\gamma P^2k^2}} =$$

$$= \frac{-1}{16\pi}\int_{-\infty}^{\infty} \frac{dx\, dy\, e^{i(x+y)}}{(xy - B)\sqrt{A - xy}}, \tag{20}$$

where $A = (Pk)^2/4$, $B = P^2k^2/4$. Introducing a new variable $z = x - B/y$ instead of x, taking integral on z, and taking into account that (see [7])

$$\int_{-\infty}^{\infty} \frac{e^{iz}\, dz}{z\sqrt{z - C/y}} = \frac{2\pi i^{3/2}}{\Gamma(3/2)}\,{}_1F_1\left(\frac{1}{2}; \frac{3}{2}; iC/y\right)$$

(here ${}_1F_1$ is the degenerate hypergeometric function and Γ is the gamma Euler function), we get

$$\rho(P, k) = -\frac{1}{4}\sqrt{\frac{i}{\pi}}\int_{-\infty}^{\infty} \frac{dy}{y^{3/2}}\, e^{i(y + B/y)}\,{}_1F_1\left(\frac{1}{2}; \frac{3}{2}; i\frac{C}{y}\right). \tag{21}$$

where $C = A - B = \dfrac{(Pk)^2 - P^2k^2}{4}$. Using the well-known identity ${}_1F_1\left(\frac{1}{2}; \frac{3}{2}; i\frac{c}{y}\right) = e^{iC/y}\,{}_1F_1\left(1; \frac{3}{2}; -i\frac{C}{y}\right)$, we can write

$$\rho(P, k) = -\frac{1}{4}\sqrt{\frac{i}{\pi}}\int_{-\infty}^{\infty} \frac{dy}{y^{3/2}}\, e^{i(y + A/y)}\,{}_1F_1\left(1; \frac{3}{2}; -i\frac{C}{y}\right). \tag{22}$$

The expression for ρ may be obtained in the form of a series, if we decompose ${}_1F_1$ in the series in C/y and use the formula

$$\int_{-\infty}^{\infty} \frac{e^{i(y + z/y)}}{y^{n+3/2}}\, dy = \frac{2\pi\, i^{n+3/2}}{(\sqrt{z})^{n+1/2}}\, J_{n+1/2}(2\sqrt{z}),$$

where J_n are the Bessel functions (at $z = 0$, we have $\int_{-\infty}^{\infty} \frac{e^{iy}}{y^{n+3/2}}\, dy = \frac{2\pi\, i^{n+3/2}}{\Gamma(n + \frac{3}{2})}$). Then, instead of (21), we get

$$\rho(P, k) = \frac{\sqrt{\pi}}{4B^{1/4}}\sum_{n=0}^{\infty} \frac{(-C/\sqrt{B})^n}{n!\,(n + \frac{1}{2})}\, J_{n+\frac{1}{2}}(2\sqrt{B}) \tag{23}$$

and, instead of (22), we obtain

$$\rho(P,k) = \frac{\pi}{4A^{1/4}} \sum_{n=0}^{\infty} \frac{(C/\sqrt{A})^n}{\Gamma(n+\frac{3}{2})} J_{n+\frac{1}{2}}(2\sqrt{A}). \tag{24}$$

It follows from these formulas that, at $A = B = 0$ (or at $k = \infty$ — the limit of local theory), we have $\rho(0,0) = 1$. If $B = 0$ and $C = A$, so

$$\rho(P,k) = \frac{\sqrt{\pi}}{4} \sum_{n=0}^{\infty} \frac{(-A)^n}{n!\,\Gamma(n+\frac{3}{2})(n+\frac{1}{2})} =$$

$$= {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -A\right) = \frac{1}{2\sqrt{A}} \int_0^1 \frac{\sin 2\sqrt{A}\,x}{x} dx =$$

$$= \frac{1}{2\sqrt{A}} \left(\frac{\pi}{2} + \mathrm{si}\,(2\sqrt{A})\right) \tag{25}$$

and if $A = 0$ $(C = -B)$, so

$$\rho(P,k) = \frac{\pi}{4} \sum_{n=0}^{\infty} \frac{(-B)^n}{\Gamma^2(n+\frac{3}{2})} =$$

$$= {}_1F_2\left(1; \frac{3}{2}, \frac{3}{2}; -B\right) = \frac{1}{2\sqrt{B}} \int_0^1 \frac{\sin 2\sqrt{B}\,x}{\sqrt{1-x^2}} dx. \tag{26}$$

At $C = 0$ $(A = B \neq 0,\ k_\mu = aP_\mu)$, we have

$$\rho(P, aP) = \frac{\pi}{4A^{1/4}\,\Gamma\left(\frac{3}{2}\right)} J_{\frac{1}{2}}(2\sqrt{A}) =$$

$$= \frac{\sin 2\sqrt{A}}{2\sqrt{A}} = \frac{\sin a^2 P^2}{a^2 P^2}. \tag{27}$$

The asymptotics of the formfactor ρ at large $B >> A$ $(C \approx -B)$ may be easily obtained from (21) if we write it in the form

$$\rho(P,k) = \frac{-i}{16\pi B} \sum_{n=0}^{\infty} \frac{1}{B^n} \left(\int_{-\infty}^{\infty} x^{n-1/2} e^{ix} dx\right)^2.$$

As $\int_{-\infty}^{\infty} e^{ix} x^{n-1/2} dx = 2i^{n+1/2}\,\Gamma(n+\frac{1}{2})$, so $\rho(P,k) =$ $= \frac{1}{4\pi B} \sum_{n=0}^{\infty} \frac{\Gamma^2(n+\frac{1}{2})}{(-B)^n}$. The main part of this asymptotic series $(n = 0)$ gives

$$\rho(P,k) = \frac{1}{4B} = \frac{1}{P^2 k^2}.$$

The asymptotics of the formfactor ρ at large $A >> B$ $(C \approx A)$ can be obtained from (20), making there the substitution $x = u + \frac{A}{y}$. In result, we have

$$\rho(P,k) = \frac{i}{16\pi} \int \frac{e^{i(u+y+A/y)}}{\sqrt{uy\,(uy+C)}} du\,dy.$$

Decomposing $(uy + C)^{-1}$ in the series in C^{-1}, we get

$$\rho(P,k) = \frac{i}{16\pi C} \sum_{n=0}^{\infty} \left(-\frac{1}{C}\right)^n \times$$

$$\times \int e^{i(u+y+A/y)} u^{n-1/2} y^{n-1/2} du\,dy.$$

The integral over u is already known, and the integral over y is (see [7])

$$\int_{-\infty}^{\infty} e^{i(y+A/y)} y^{n-1/2} dy =$$

$$= 2\pi i^{-n+1/2} (\sqrt{A})^{n+1/2} J_{-n-\frac{1}{2}}(2\sqrt{A}).$$

As $J_{-n-\frac{1}{2}}(z) = (-1)^{n+1} N_{n+\frac{1}{2}}(z)$ (N_a are the Neumann functions),

$$\rho(P,k) = \frac{1}{4A^{3/4}} \sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{A}}\right)^n \Gamma\left(n+\frac{1}{2}\right) N_{n+\frac{1}{2}}(2\sqrt{A}).$$

The main part $(n = 0)$ of this asymptotic series gives

$$\rho(P,k) = \frac{\Gamma(\frac{1}{2})}{4A^{3/2}} N_{\frac{1}{2}}(2\sqrt{A}) = -\frac{\cos 2\sqrt{A}}{4A} = -\frac{\cos Pk}{(Pk)^2}. \tag{28}$$

It is interesting to note that the asymptotic values of formfactor (28), (19) may be represented in the form of integrals

$$\rho(P,k) = -\int_\Omega \frac{d\beta}{2} \int_I \frac{d\alpha}{2} e^{i\alpha\beta Pk} =$$

$$= -\int_1^{\infty} d\beta \int_0^1 d\alpha \cos(\alpha\beta P k);$$

$$\rho(P,k) = \int_I \frac{d\alpha}{2} e^{i\alpha Pk} = \int_0^1 d\alpha \cos(\alpha Pk), \tag{29}$$

where Ω is a complementary subset to the interval $I = [-1, 1]$ in the **R**. It may be said that there are three kinds of formfactors in the suggested theory. If both particles are massless, the formfactor is e^{iPk} (or $\cos Pk$). Note that this expression does not coincide with the asymptotic value of formfactor (19) at $P^2 = k^2 = 0$ equaled $\sin Pk/Pk$. The latter is the result of integrating the expression $e^{i\alpha Pk}$ with respect to α (the second formula in (29)). (28) is obtained as a result of integrating the expression $\frac{\sin \beta P k}{\beta P k}$ with respect to β (the first formula in (29)). Thus, the following picture of the echolonized structure of a particle 'dress“ arises: the next (inner) fiber of particle 'clothes“ is obtained from the previous (external) one by means of a stochastic process – stochastic integration. (In local theory, the method of renormgroup is used for this purpose.)

We emphasize that, in specific calculations, exact expressions for formfactors will be used rather than their asymptotic expression.

7. Comparison with Axiomatic Approach

In axiomatic field theory, the most restricting axiom is a local one. In the suggested theory, the local field operators appear but only in the form of asymptotic bilocal fields $\psi(X, Y)$ when $|X| >> |Y|$. This circumstance changes the situation concerning the Wightman theorem (about non-existence of local operators) and the Haag theorem (about triviality of S-matrix in local theory) radically.

It is accepted to consider that the troubles of quantized field theory are connected with the usage of infinite-dimensional Heisenberg and Clifford algebras [8]. In this respect, the relativistic bi-Hamiltonian system essentially differs from field systems: the finite-dimensional Heisenberg algebra $h_{16}^{(*)}$ underlies this theory. Hereby the field variables $f^{\Sigma}(x)$ and $\dot{f}_z(\dot{x})$ of the system are not fields on the space-time continuum, but on the group $T_{3,1}$ and $\dot{T}_{3,1}$ respectively, i.e. at an isolated point of the discontinuum. They are pure c-number magnitudes, because there not exist the rules for quantization of fields with arbitrary spin. So the Wightman's theorem does not concern them.

Further in axiomatic theory, the interaction field $\Psi(X)$ is connected with a non-interacting $\psi(X)$ by means of a unitary transformation and therefore nothing differs from the latter (Haag's theorem). Moreover, $\psi(X)$ does not exist at all (Wightman's theorem). In the suggested theory, the interacting (bilocal) field $\psi(X, Y)$ is connected with a local one $\psi(X)$ by means of the smearing operator: $\psi(X, Y) = F\left(Y, -i\frac{\partial}{\partial X}\right)\psi(X)$. It is impossible to present such a field in the form of $U_Y\,\psi(X)\,U_Y^{-1}$, where U_Y is a unitary operator on H. Bilocal field is therefore quite another realization of the idea of interacting field and the Haag's theorem may not be applied to it.

It is essential that the local field $\psi(X)$, as asymptotic, is given not on the whole space-time but only in the region $|X| >> |Y|$ (on the whole space-time, the bilocal field $\psi(X, Y)$ is given). As $\psi^{\Sigma}(X, Y)$ and $\psi^{\Sigma}(X)$ describe one and the same particle (in the first case, it is smearing, in the second – point-like), both these fields transform under the same unitary representation with weight (s^{Σ}, M_{Σ}) of the Poincare group:

$$\psi^{\Sigma}_{\alpha}(X, Y) \to V^{\Sigma}_{\alpha\beta}(\upsilon)\,\psi^{\Sigma}_{\beta}(\Lambda_{\upsilon}^{-1}(X - a), \Lambda_{\upsilon}^{-1}Y) =$$

$$= U(\upsilon, a)\,\psi^{\Sigma}_{\alpha}(X, Y)\,U^{-1}(\upsilon, a),$$

$$\Lambda_{\upsilon} \in \mathrm{SO}(3,1), \quad \upsilon \in \mathrm{SL}(2, \mathbf{C}), \quad a \in T_{3,1} \qquad (30)$$

(the formula for the local field $\psi^{\Sigma}(X)$ is obtained at $Y = 0$). In particular, under the translations, we have $U(a)\,\psi(X, Y)\,U^{-1}(a) = \psi(X + a, Y)$. This formula is valid for all values of a. But the formula for the local field $U(a)\,\psi(X)\,U^{-1}(a) = \psi(X + a)$ is valid until $|X + a| >> |Y|$ and, of course, $|X| >> |Y|$, i.e. for not all a. The condition of the Wightman's theorem is not fulfiled here. Moreover, in the new theory, there are a lot of Lorentz invariant finite measures on $\mathbf{R}_{3,1}$. For example,

$$d\mu_{\Sigma}(P, Y) = \theta(P_0)\,\delta(P^2 + M_{\Sigma}^2)\,F_{\Sigma}(Y, P)\,d^4P,$$

where F_{Σ} is the smearing operator (in local theory, there is only one such a measure: $d\mu(P) = \delta^4(P)\,d^4P$ with which a trivial field $\psi(X) = \mathrm{const}$ is connected [2]). At $Y \neq 0$, we have ($Y^2 = \mathbf{Y}^2 - Y_0^2$)

$$\int d\mu_{\Sigma}(P, Y) = 2\pi M_{\Sigma}\left[\frac{\theta(-Y^2)}{\sqrt{-Y^2}}K_1(M_{\Sigma}\sqrt{-Y^2}) + \right.$$

$$\left. + \frac{\pi}{2}\frac{\theta(Y^2)}{\sqrt{Y^2}}N_1(M_{\Sigma}\sqrt{Y^2})\right] = 4\pi^2 D^1(iY),$$

where D^1 is the so-called antipermutation function, N_1 and K_1 are the Neumann and MacDonald functions.

It is important to note that, from the invariance of the Lagrangian of bilocal fields under transformation (30), the Noether theorem follows, i.e. the conservation laws of energy-momentum and angular momentum for the field system and the explicit form of the corresponding magnitudes.

Conclusion

We see that the wave properties of space-time (called also ether waves) play the role of the smearing medium due to which particles become smeared objects. Interactions of such objects are described by any formfactor cutting the infinites in Feynman’s integrals. Neither quark models nor the standard model accepted at the present time do not claim to the role of a true theory of elementary particles; they are considered as some phenomenological approach to the interaction problem [9]. It is possible that the way of unification of interactions will not lead to the construction of an adequate theory of particles. Obviously, in the 60s when granules were identified with quarks, physicians had hurried. The quark interpretation of granules is ambiguous and not sole. There exists quite another mathematics for description of particle fundamental interactions. And only experiment may decide which from these theories is right — the quark model or the model suggested here.

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