ADVANTAGES OF A REPRESENTATION WITHOUT USE OF THE ISOSPIN FORMALISM, AND PRECISE STUDY OF FEW-NUCLEON SYSTEMS

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The analysis of the isospin formalism is carried out in order to study a few-nucleon system with high and controlled precision. The commonly used isospin formalism is shown to give the unreasonable complication of the total wave function and to increase the number of equations for spatial components. Using the example of three nucleons with the general central exchange NN-interaction potential, the complete equivalence is grounded for two approaches with and without the isospin formalism. New sets of equations are obtained for the systems of three nucleons with total spin $S = 1/2$ and four nucleons with total spin $S = 0$. Optimal variational schemes are developed with the use of the Gaussian basis for a precise studying of the main properties of a few-nucleon system. For the three-nucleon systems, the binding energies, r.m.s. radii, density distributions, and formfactors are calculated with high and controlled precision within the proposed approach. The qualitative detailed analysis is carried out for the structure peculiarities of the three-nucleon systems. Advantages of the proposed approach without use of the isospin representation are demonstrated. The obtained results are of superior accuracy in comparison with the known ones and give new possibilities for constructing the realistic variants of nuclear potentials for a complete description of all the main low-energy parameters of few-nucleon systems.

The latest achievements in precise variational approaches [1-3] for few-particle systems of various physical nature put forward the problem of elaborating the optimal variational procedures to achieve high-precision results for the main characteristics of the bound states with a minimal number of the variational basis functions involved [4]. The possibilities to achieve a high precision in calculations of the nuclear system parameters are connected both with the finite number of internal spatial coordinates of nucleons and the limited number of independent components of the total wave function [2-6]. In the standard scheme commonly used for light nuclei in the framework of the isospin formalism treating nucleons as identical particles, one has to deal with a complicated system of equations for a number of components of the wave function in order to take into account a coupling of the components with different total isospin in the case of central exchange nuclear potentials and the Coulomb interaction involved. In particular, it is necessary to solve a system of six equations for spatial components of the wave function in the doublet state of ^{3}He, and one has twelve equations for ^{4}He. At the beginning of the theoretical semi-qualitative study of nuclei, some rough approximations were assumed for the structure of wave functions (see [7-9]), and the total number of the wave function components was not too important. But precise and reliable calculations, with all the spatial components of the wave function taken into account, are already confronted by nontrivial difficulties, which requires the optimization of calculational schemes even for the above-mentioned light nuclei. Precise calculations in the framework of the standard isospin formalism are by far more complicated for heavier nuclei.

In the present work, we put forward an approach [10,11] free from the isospin formalism and show the general and complete equivalence of the proposed approach to the prevalent standard isospin one by using the example of a three-nucleon system. The obtained system of equations for the spatial components of the wave function appears to be of rather simple structure and thus has significant advantages in comparison with the standard isospin scheme in order to carry on precise calculations for few-nucleon systems.

1. Consider the ^{3}He system of three nucleons in the doublet state (with spin $S = 1/2$) in the framework of the standard isospin approach with conventional assumptions about the central exchange NN-potentials

$$\hat{V}(i,j) = V_{11}(r_{ij})\,P_s(\sigma)\,P_s(\tau) + V_{13}(r_{ij})\,P_s(\sigma)\,P_t(\tau) + \\ + V_{31}(r_{ij})\,P_t(\sigma)\,P_s(\tau) + V_{33}(r_{ij})\,P_t(\sigma)\,P_t(\tau) + \\ + \frac{e^2}{2}\frac{T_3(i,j)\,(T_3(i,j)+1)}{r_{ij}}, \qquad (1)$$

containing the corresponding spin and isospin projection operators onto the triplet and singlet states, as well as

the two-nucleon isospin projection operator $T_3(i,j) = \frac{1}{2}(\tau_3(i) + \tau_3(j))$. The $(2p,n)$ system in the doublet state (with spin $S = 1/2$ and the spin Young scheme [2,1]) is in a mixed isospin state (with both $T = 1/2$ and $T = 3/2$) and has the projection of isospin $T_3 = 1/2$. In the isospin approach, the total antisymmetric (in nucleons) wave function of the $(2p,n)$ system is represented in terms of spin ζ and isospin χ (as well as spin-isospin ξ) wave functions and has six spatial components corresponding to four different nonreducible representations of the permutation group for coordinates:

$$\Psi^a = \psi^s\xi^a + (\psi'\xi'' - \psi''\xi') + \psi^a\xi^s + (\varphi'\zeta'' - \varphi''\zeta')\,\chi_{3/2}. \quad (2)$$

After calculating the spin-isospin matrix elements of the total Hamiltonian, one obtains the system of six equations for the spatial components of the wave function (or the set of four equations [8,9] in the case of the $(2p,n)$ system without Coulomb interaction):

$$\left[\hat{K} - E + \frac{1}{2}(U^s_{31} + U^s_{13}) + \frac{1}{3}U^s_c\right]\psi^s - \frac{1}{2}\left[(U'_{31} - U'_{13})\,\psi' + (U''_{31} - U''_{13})\,\psi''\right] + \frac{1}{3}(U'_c\psi' + U''_c\psi'') +$$

$$+\frac{1}{3}(U'_c\varphi' + U''_c\varphi'') = 0,$$

$$\left[\hat{K} - E + \frac{1}{4}(U^s_{33} + U^s_{31} + U^s_{13} + U^s_{11}) + \frac{1}{3}U^s_c\right]\psi' - \frac{1}{4}\left[(U'_{33} - U'_{31} - U'_{13} + U'_{11})\,\psi'' + (U''_{33} - U''_{31} - U''_{13} + U''_{11})\,\psi'\right] -$$

$$-\frac{1}{2}\left(U'_{31} - U'_{13} - \frac{2}{3}U'_c\right)\psi^s + \frac{1}{2}\left(U''_{33} - U''_{11} + \frac{2}{3}U''_c\right)\psi^a + \frac{1}{3}(U''_c\varphi' + U'_c\varphi'') = 0,$$

$$\left[\hat{K} - E + \frac{1}{4}(U^s_{33} + U^s_{31} + U^s_{13} + U^s_{11}) + \frac{1}{3}U^s_c\right]\psi'' - \frac{1}{4}\left[(U'_{33} - U'_{31} - U'_{13} + U'_{11})\,\psi' - (U''_{33} - U''_{31} - U''_{13} + U''_{11})\,\psi''\right] +$$

$$-\frac{1}{2}\left(U''_{31} - U''_{13} - \frac{2}{3}U''_c\right)\psi^s - \frac{1}{2}\left(U'_{33} - U'_{11} + \frac{2}{3}U'_c\right)\psi^a + \frac{1}{3}(U'_c\varphi' - U''_c\varphi'') = 0,$$

$$\left[\hat{K} - E + \frac{1}{2}(U^s_{33} + U^s_{11}) + \frac{1}{3}U^s_c\right]\psi^a + \frac{1}{2}\left[(U''_{33} - U''_{11})\,\psi' - (U'_{33} - U'_{11})\,\psi''\right] - \frac{1}{3}(U'_c\psi'' - U''_c\psi') +$$

$$+\frac{1}{3}(U'_c\varphi'' - U''_c\varphi') = 0,$$

$$\left[\hat{K} - E + \frac{1}{2}(U^s_{13} + U^s_{33}) + \frac{1}{3}U^s_c\right]\varphi' + \frac{1}{2}\left[(U'_{13} - U'_{33})\,\varphi'' + (U''_{13} - U''_{33})\,\varphi'\right] +$$

$$+\frac{1}{3}(U'_c\psi'' + U''_c\psi' + U'_c\psi^s - U''_c\psi^a) = 0,$$

$$\left[\hat{K} - E + \frac{1}{2}(U^s_{13} + U^s_{33}) + \frac{1}{3}U^s_c\right]\varphi'' + \frac{1}{2}\left[(U'_{13} - U'_{33})\,\varphi' - (U''_{13} - U''_{33})\,\varphi''\right] +$$

$$+\frac{1}{3}(U'_c\psi' - U''_c\psi'' + U''_c\psi^s + U'_c\psi^a) = 0, \quad (3)$$

where $\hat{K}$ is the kinetic energy operator for three nucleons of equal masses, and the complete set of the spatial Young schemes (symmetric [3] and mixed [21]) for the potentials (both nuclear and Coulomb) looks as follows:

$$U^s = V(r_{12}) + V(r_{13}) + V(r_{23}),$$

$$U' = \frac{\sqrt{3}}{2}\left(-V\left(r_{13}\right) + V\left(r_{23}\right)\right),$$

$$U'' = -V\left(r_{12}\right) + \frac{1}{2}\left(V\left(r_{13}\right) + V\left(r_{23}\right)\right). \quad (4)$$

The last two equations in (3) appear due to the Coulomb interaction potential mixing for the states with different total isospins (the total projection of the isospin $T_3 = 1/2$ being fixed). Neglecting the components φ' and φ'' as well as the Coulomb interaction matrix elements in (2), (3), one obtains the standard system of equations in the isospin representation for a ^{3}H nucleus (in the doublet state both in spin ($S = 1/2$) and isospin ($T = 1/2$)). The system of equations (3) for ^{3}He is the basic one for precise calculations with regard for all the details of the total antisymmetric wave function. We show the system of equations (3) for ^{3}He to be equivalent to a more simple one. We introduce, instead of six spatial components,

$$\psi^s,\ \psi',\ \psi'',\ \psi^a,\ \varphi',\ \varphi'' \quad (5)$$

the following new ones:

$$\phi_1',\ \phi_2',\ \phi_3',\ \phi_1'',\ \phi_2'',\ \phi_3'', \quad (6)$$

in accordance with

$$\begin{pmatrix} \phi_1' \\ \phi_2' \\ \phi_3' \\ \phi_1'' \\ \phi_2'' \\ \phi_3'' \end{pmatrix} = \begin{pmatrix} -\frac{1}{2\sqrt{3}} & -\frac{1}{2} & \frac{1}{2\sqrt{3}} & -\frac{1}{2} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{2\sqrt{3}} & \frac{1}{2} & \frac{1}{2\sqrt{3}} & \frac{1}{2} & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & 0 & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{2} & \frac{1}{2\sqrt{3}} & \frac{1}{2} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{2} & \frac{1}{2\sqrt{3}} & -\frac{1}{2} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \end{pmatrix} \cdot \begin{pmatrix} \psi^s \\ \psi' \\ \psi'' \\ \psi^a \\ \varphi' \\ \varphi'' \end{pmatrix}. \quad (7)$$

As the obvious result of this transformation, one has only two (from six) independent components, while the rest components are expressed in terms of these two ones using also the coordinate permutations. In particular, we express all the components from the new system (6) in terms of $\phi_3'(123)$ and $\phi_3''(123)$ as follows:

$$\phi_1'(123) = -\frac{1}{2}\phi_3'(321) + \frac{\sqrt{3}}{2}\phi_3''(321),$$

$$\phi_1''(123) = \frac{\sqrt{3}}{2}\phi_3'(321) + \frac{1}{2}\phi_3''(321),$$

$$\phi_2'(123) = -\frac{1}{2}\phi_3'(132) - \frac{\sqrt{3}}{2}\phi_3''(132),$$

$$\phi_2''(123) = -\frac{\sqrt{3}}{2}\phi_3'(132) + \frac{1}{2}\phi_3''(132). \quad (8)$$

Thus, the complete solution of the three-nucleon problem and finding the total antisymmetric function (2) using six spatial components (5) need only two equations for ϕ_3' and ϕ_3'' (instead of six ones). Further, we use the denotations $\Phi_1 \equiv \phi_3'$ and $\Phi_2 \equiv \phi_3''$.

The same two components Φ_1 and Φ_2 may serve as basic ones in the equivalent approach without any use of the isospin representation, where a proton and a neutron are considered to be different particles, the total wave function of ^{3}He (in the doublet state) being expressed as

$$\Psi\left(p_1, p_2, n_3\right) = \zeta'\Phi_1\left(p_1, p_2, n_3\right) + \zeta''\Phi_2\left(p_1, p_2, n_3\right), \quad (9)$$

where ζ' and ζ'' are two components of the spin wave function with $S = 1/2$, $\Phi_1\left(p_1, p_2, n_3\right)$ is a symmetric function, and $\Phi_2\left(p_1, p_2, n_3\right)$ is an antisymmetric one in identical protons. It is suitable to express the central exchange NN-potential (1) in the following equivalent form without explicit introduction of isospin:

$$\hat{V}_{\text{nucl}}(i,j) =$$

$$= \left[V_s^+\left(r_{ij}\right)P_s\left(\sigma\right) + V_t^+\left(r_{ij}\right)P_t\left(\sigma\right)\right]\left(1 + P_r\right)/2 +$$

$$+ \left[V_s^-\left(r_{ij}\right)P_s\left(\sigma\right) + V_t^-\left(r_{ij}\right)P_t\left(\sigma\right)\right]\left(1 - P_r\right)/2, \quad (10)$$

where P_r is the Majorana exchange operator of a permutation of spatial coordinates. Note the well-known relations to be held for any charge-independent nuclear potential for any pair of nucleons:

$$V_{13} = V_s^+, \quad V_{31} = V_t^+, \quad V_{11} = V_s^-, \quad V_{33} = V_t^-. \tag{11}$$

Ultimately, the set of two equations for the three-nucleon system $(2p, n)$ with spin $S = 1/2$ looks as follows:

$$\left[\hat{K} + \frac{e^2}{r_{12}} + V^+_{s(pp)}(r_{12}) - E\right]\Phi_1(123) + \frac{1}{8}\sum_{ij=13,23}\sum_{+,-}\left[3V^{\pm}_{t(np)}(r_{ij}) + V^{\pm}_{s(np)}(r_{ij})\right][1 \pm P(ij)]\,\Phi_1(123) +$$

$$+\frac{\sqrt{3}}{8}\sum_{ij=13,23}\sum_{+,-}(-1)^{i+j}\left[V^{\pm}_{s(np)}(r_{ij}) - V^{\pm}_{t(np)}(r_{ij})\right][1 \pm P(ij)]\,\Phi_2(123) = 0,$$

$$\left[\hat{K} + \frac{e^2}{r_{12}} + V^-_{t(pp)}(r_{12}) - E\right]\Phi_2(123) + \frac{1}{8}\sum_{ij=13,23}\sum_{+,-}\left[V^{\pm}_{t(np)}(r_{ij}) + 3V^{\pm}_{s(np)}(r_{ij})\right][1 \pm P(ij)]\,\Phi_2(123) +$$

$$+\frac{\sqrt{3}}{8}\sum_{ij=13,23}\sum_{+,-}(-1)^{i+j}\left[V^{\pm}_{s(np)}(r_{ij}) - V^{\pm}_{t(np)}(r_{ij})\right][1 \pm P(ij)]\,\Phi_1(123) = 0, \tag{12}$$

where we take into account the difference in masses between a proton and a neutron in the kinetic energy operator $\hat{K} = \frac{1}{2M_p}\left(p_1^2 + p_2^2\right) + \frac{1}{2M_n}p_3^2$. We generalize the obtained system of equations to the case of different neutrons and protons with charge-dependent nuclear interaction potentials as well as to the case of different interactions in odd and even orbital partial states. Thus, Eqs.(12) for the $(2p, n)$ system contain generally six different nuclear potentials. In the case of a ^{3}H nucleus (with total spin $S = 1/2$), we have the system of two equations similar to (12), but without Coulomb interaction and with the exchanged indices $n \leftrightarrow p$ in all the potentials. We stress once more that the system of two equations (12) is complete for solving the three-nucleon problem for the total spin $S = 1/2$. In the general case, there are eight independent components of the nuclear interaction potential to determine the three-nucleon system properties in all the spin states. The following relations are to be held in the case of a charge-independent potential:

$$\begin{aligned} V^+_{s(nn)} &= V^+_{s(pp)} = V^+_{s(np)} = V_{13}, \quad V^+_{t(np)} = V_{31}, \\ V^-_{s(np)} &= V_{11}, \quad V^-_{t(nn)} = V^-_{t(pp)} = V^-_{t(np)} = V_{33}. \end{aligned} \tag{13}$$

Within the proposed approach, we have one equation for the spatial part of the wave function in the quartet state ($S = 3/2$) of the three-nucleon system. For a four-nucleon system within the same approach without use of isospin, we have one equation for the spatial wave function in the case of spin $S = 2$, a system of three equations for $S = 1$, and a system of two equations for $S = 0$, correspondingly to dimensions of the respective Young schemes for the spin functions of four nucleons. We present here, as an example, the system of two coupled equations for the spatial wave function of ^{4}He in the $S = 0$ spin state in the general case of central exchange charge-dependent NN-potential:

$$\left[\hat{K} + \frac{e^2}{r_{12}} + V^+_{s(pp)}(r_{12}) + V^+_{s(nn)}(r_{34}) - E\right]\Phi_1(1234) +$$

$$+\frac{1}{8}\sum_{ij=13,14,23,24}\sum_{+,-}\left[3V^{\pm}_{t(np)}(r_{ij}) + V^{\pm}_{s(np)}(r_{ij})\right][1 \pm P(ij)]\,\Phi_1(1234) +$$

$$+\frac{\sqrt{3}}{8}\sum_{ij=13,14,23,24}\sum_{+,-}(-1)^{i+j}\left[V^{\pm}_{s(np)}(r_{ij}) - V^{\pm}_{t(np)}(r_{ij})\right][1 \pm P(ij)]\,\Phi_2(1234) = 0,$$

$$\left[\hat{K}+\frac{e^2}{r_{12}}+V^{-}_{t(pp)}\left(r_{12}\right)+V^{-}_{t(nn)}\left(r_{34}\right)-E\right]\Phi_2\left(1234\right)+$$

$$+\frac{1}{8}\sum_{ij=13,14,23,24}\sum_{+,-}\left[V^{\pm}_{t(np)}\left(r_{ij}\right)+3V^{\pm}_{s(np)}\left(r_{ij}\right)\right]\left[1\pm P\left(ij\right)\right]\Phi_2\left(1234\right)+$$

$$+\frac{\sqrt{3}}{8}\sum_{ij=13,14,23,24}\sum_{+,-}\left(-1\right)^{i+j}\left[V^{\pm}_{s(np)}\left(r_{ij}\right)-V^{\pm}_{t(np)}\left(r_{ij}\right)\right]\left[1\pm P\left(ij\right)\right]\Phi_1\left(1234\right)=0. \quad (14)$$

2. For studying the three- and four-nucleon systems ^{3}H, ^{3}He and ^{4}He on the basis of Eqs.(12), (14), we develop optimized variational procedures with the use of a Gaussian basis (see also [5,6]) and calculate all the main parameters of bound states with high precision. Here, we present only the results of calculations for the three-nucleon nuclei ^{3}H (Table I) and ^{3}He (Table II) with some NN-potentials (ATS3 and Minnesota see [3], K2 and EH see [5,6]). All the results are given with the accuracy of one unit in the last digit with great probability (the exact values of energy are lower, of course, but not more than by 1 KeV). It is important that a high precision of calculations can be achieved within the proposed approach on the basis of Eqs.(12), (14) with use of the optimization schemes of variational calculations at a comparatively small number of Gaussian basis functions involved. In particular, we have used not more than 60-100 Gaussian basis functions (for Φ_1 and Φ_2 together) in the ultimate calculations of ground state wave functions. Our results are of higher precision in comparison with the best known results [3] for the same potentials, being obtained with the significantly less number of variational Gaussian basis components. For Minnesota potential, we have 8.386(1) MeV as the ultimate result for the binding energy of ^{3}H (with a possible error less than 0.001 MeV) in comparison with 8.380 MeV in [3], and we have 8.765(1) MeV for the same nucleus with the ATS3 interaction potential in comparison with 8.753 MeV in [3]. It is mainly due to the advantage of system (12) to have only two coupled equations instead of generally six ones in (3). The hierarchy of the basis components considered is as follows: the number of the basis components in the symmetric channel (Φ_1) is generally about three times greater as compared to that in the antisymmetric one (Φ_2) for various considered potentials. Such a regularity helps us to build the optimal scheme of increasing the dimension of a Gaussian basis in order to achieve a prescribed accuracy with the least number of basis components.

In our calculations (using the values $e^2 = 1.44$ MeV·fm, and $\hbar^2/M = 41.47$ MeV·fm^2), we consider the following cases: 1) the spinless approximation (see [3,4]) with the interaction potential being a half of the sum of the triplet and singlet potentials; 2) an approximation with only the component $\Phi_1$taken into account ($\Phi_2 = 0$); 3) the complete calculation with a pair interaction in even orbital states (prevalent and commonly used); 4) the complete calculation with regard for different values of the proton and neutron masses ($M_p \neq M_n$). An approximate "rule of intervals" is found for the calculated values of energies (there are roughly the same intervals in energy between the first three approximations), which enables one to build efficient schemes of calculations with the use of some approximations. Note that the more the difference between the triplet and singlet potentials and the value of a short-range repulsion, the more is the difference between the energy in the spinless approximation and that from the complete calculation. Such regularities are also useful when solving the rather difficult problem of finding the absolute minimum of an energy functional in the multidimensional space of nonlinear parameters within the variational approach (the number of local minima with close values of energy is rather large). A rather efficient way of optimal increase in the number of basis components and determination of the absolute minimum appeares to be the use of a number of methods, in particular, the stochastic procedure, the regular method with different ranges for nonlinear parameters, removing the minor components, etc. The use of various methods gave us a possibility to achieve the results with high and controlled accuracy with the optimal number of basis components. For three- and four-nucleon systems with the Minnesota and ATS3 potentials, our results are of higher precision in comparison with the results from [3], and they are obtained with a less dimension of the basis. This is mainly the result of the use of a more simple system of equations (12) without isospin representation.

T a b l e I. Calculated energies and r.m.s. radii for ^{3}H nucleus. R_p, R_n, R_m are the proton, neutron, and mass density distribution r.m.s. radii, respectively

Approximation	Potentials / Energies, radii	AT-S3$^+$	ATS3	Minnesota	K2	EH
spinless	$-E_0$, MeV	6.699	6.699	6.896	8.397	7.048
	R, fm	1.738	1.738	1.730	1.823	1.721
	$-E^*$, MeV	0.234	0.234	0.264	0.721	0.284
	R^*, fm	12.5	12.5	12.7	11.8	13.0
$\Phi_2 = 0$	$-E_0$, MeV	7.491	7.616	7.561	8.438	7.352
	R_p, fm	1.614	1.591	1.613	1.779	1.637
	R_n, fm	1.758	1.753	1.761	1.858	1.775
	R_m, fm	1.712	1.701	1.713	1.832	1.730
total	$-E_0$, MeV	8.491	8.765	8.386	8.484	7.718
	R_p, fm	1.576	1.546	1.586	1.784	1.637
	R_n, fm	1.749	1.733	1.763	1.872	1.800
	R_m, fm	1.693	1.673	1.706	1.843	1.748
total ($M_p \neq M_n$)	$-E_0$, MeV	8.495	8.769	8.389	8.488	7.722
	R_p, fm	1.576	1.546	1.586	1.784	1.637
	R_n, fm	1.748	1.733	1.762	1.871	1.800
	R_m, fm	1.693	1.673	1.705	1.842	1.747
[3]	$-E_0$, MeV	–	8.753	8.380	–	–
	R_m, fm	–	1.67	1.698	–	–
experiment	$-E_0\left(^3\mathrm{H}\right) = 8.481$ MeV,		$R_p\left(^3\mathrm{H}\right) = 1.57$ fm			

T a b l e II. Calculated energies and r.m.s. radii for ^{3}He nucleus. R_p, R_n, R_m are the proton, neutron, and mass density distribution r.m.s. radii, respectively

Approximation	Potentials / Energies, radii	AT-S3$^+$	ATS3	Minnesota	K2	EH
spinless	$-E_0$, MeV	5.998	5.998	6.165	7.714	6.327
	R_p, fm	1.772	1.772	1.766	1.854	1.755
	R_n, fm	1.756	1.756	1.750	1.839	1.740
	R_m, fm	1.766	1.766	1.760	1.849	1.750
$\Phi_2 = 0$	$-E_0$, MeV	6.833	6.963	6.882	7.771	6.673
	R_p, fm	1.790	1.784	1.796	1.892	1.812
	R_n, fm	1.632	1.608	1.632	1.796	1.657
	R_m, fm	1.739	1.727	1.743	1.860	1.762
total	$-E_0$, MeV	7.833	8.110	7.711	7.821	7.047
	R_p, fm	1.780	1.763	1.798	1.909	1.842
	R_n, fm	1.593	1.560	1.605	1.802	1.659
	R_m, fm	1.720	1.698	1.736	1.874	1.783
total ($M_p \neq M_n$)	$-E_0$, MeV	7.826	8.103	7.706	7.816	7.041
	R_p, fm	1.781	1.764	1.799	1.909	1.843
	R_n, fm	1.593	1.561	1.605	1.802	1.659
	R_m, fm	1.721	1.699	1.736	1.874	1.784
experiment	$-E_0\left(^3\mathrm{He}\right) = 7.716$ MeV,		$R_p\left(^3\mathrm{He}\right) = 1.70$ fm			

Some physical conclusions should be mentioned. The numerical results for r.m.s. radii (of the proton, neutron, and mass density distributions) show clearly the peculiarities of the ^{3}H and ^{3}He structures. In particular, the proton distribution r.m.s. radius is essentially less than that for the neutron one in the case of a ^{3}H nucleus (something like a neutron "halo"), while in the case of ^{3}He - vice versa (a proton "halo"). This results from the fact that the attraction of two nucleons in the singlet state is essentially weaker as compared

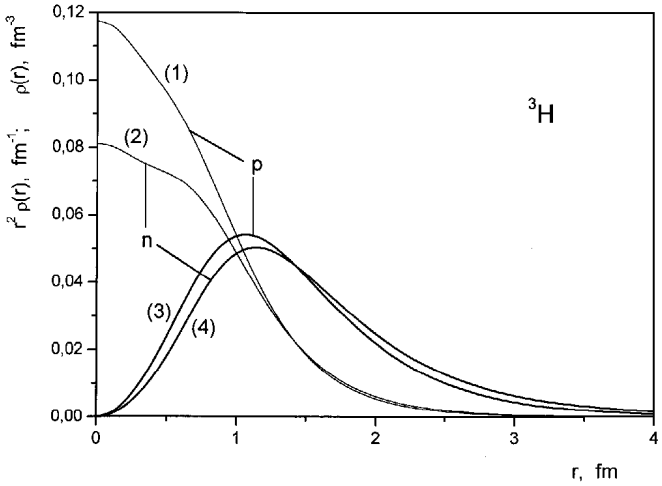

Fig. 1. Profiles of proton and neutron density distributions for ^{3}H nucleus (for the AT-S3$^+$ interaction potential and point-like nucleons). Curves (*1*), (*2*) depict $\rho(r)$, while curves (*3*), (*4*) show $r^2\rho(r)$

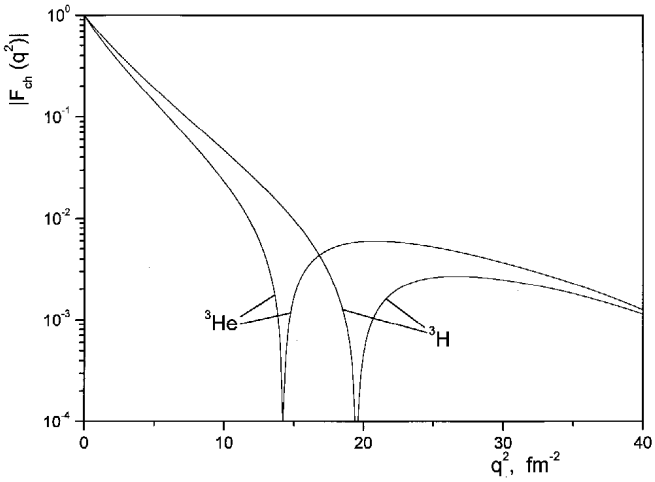

Fig. 2. Charge formfactors for ^{3}H and ^{3}He nuclei (for the AT-S3$^+$ interaction potential and point-like nucleons)

to the interaction in the triplet state. Among the potentials used in this work, the best coincidence with experimental data is attained with the AT-(S3)$^+$ potential with the only well-defined interaction in even orbital states. We give also the results for another version of the potential ATS3 (see [3]) in order to compare our ultimate precise calculations with the best ones available in the literature. Note that from the physical point of view, the latter version of the potential has no sense, because it supposes an unbelievable attraction in the odd singlet state. In the case of different masses of nucleons (see the rows $M_p \neq M_n$ in Tables I, II), the obtained results slightly differ from that with equal masses of nucleons (the upper Table rows) and confirm the general rule: the binding energy of ^{3}H slightly increases, while that of ^{3}He decreases with regard for the difference in masses. At the same time, the r.m.s. radii practically do not change. The ultimate complete calculation reveals the known discrepancy between experimental and theoretical Coulomb energy values. This means that it is necessary to take into account the charge dependence of the nuclear interaction potential. At last, we note that we have calculated the single excited state in the spinless approximation for all the potentials (see the part "spinless" in Tables I, II), which exists at about 50 KeV below the two-particle threshold obtained with the corresponding potential averaged over spin.

Fig.1 depicts the proton $\rho_p(r)$ and the neutron $\rho_n(r)$ density distributions in the case of ^{3}H with the potential AT-(S3)$^+$. Here, both the neutron peripheral "halo" effect and essential increase of the proton density distribution at the center of ^{3}H reveal themselves. We have almost the same dependences in the case of ^{3}He, but with exchange of the proton and neutron indices, since the main reason for the above dependences lies in the difference of the nuclear potentials in triplet and singlet states, while the role of the Coulomb interaction is not essential. Fig.2 depicts the charge formfactors of the nuclei ^{3}H and ^{3}He for the same potential. Note that the formfactor of ^{3}He decreases more rapidly, as compared to that of ^{3}H, already at small q^2 due to the larger proton radius, and it changes the sign at less values of q^2 due to the more important role of pair correlations between the protons at the center of ^{3}He resulting in a specific decrease of the proton density $\rho_p(r)$ at short distances r in ^{3}He.

3. To summarize, we clarify the main results. Using the example of a three-nucleon system, we show the complete physical and mathematical equivalence of both the isospin formalism (with the total wave function being antisymmetric in the space of spin, isospin, and coordinate variables) and the proposed representation without use of the notion of isospin (with the total wave function being antisymmetric in identical nucleons in the space of spin and coordinate variables). The approach proposed for studying the few-nucleon systems is much more simple and has essential advantages. A detailed description of interaction potentials in all the states is given with the three- and four-nucleon equations using no isospin representation. Optimal variational schemes are developed for studying the

bound states of few-nucleon systems with high and controlled accuracy.

Precise calculations are carried out for the bound states of three-nucleon systems, and the results surpassing the ones available in the literature in accuracy are obtained for the bound state energy and wave functions. An approximate rule of intervals is found for the binding energies calculated in different approximations, which gives a possibility to have an efficient scheme of calculations using some approximations. Note also that one can use a rather accurate approximation of Eqs.(12) without regard for the projection of the interaction onto odd and even pair states (compare the results for the AT-$(S3)^+$ and ATS3 versions of a potential). It is found that the account of a difference in the nucleon masses results in some increase of the binding energy of ^{3}H (decrease for ^{3}He), while r.m.s. radii are almost unchangeable. The neutron density distribution in a ^{3}H nucleus is revealed to be of essentially greater radius in comparison with the proton one (the neutron "halo" effect), while we have the inverse effect in a ^{3}He nucleus (the proton "halo"). These nuclei have also the charge density distributions distinctly different from the mass density ones. The charge density distribution in ^{3}H is very close to the neutron distribution in ^{3}He, and, vice versa, the neutrons in ^{3}H are distributed almost just as the protons in ^{3}He due to the role of nuclear forces, the Coulomb interaction being almost negligible.

The representation without use of isospin enables us to carry on the comparatively simple precise calculations of four-nucleon bound states (the results obtained on the basis of Eqs.(14) will be published). The proposed approach gives one a possibility to carry on a complete study of the bound states of a five-particle system (five equations for spatial components in the case of $S = 1/2$) and, may be, of six-nucleon systems (also five equations both for ^{6}He and ^{6}Li). But it should be noticed that precise calculations without any simplifications for more complicated nuclear systems still remain to be a problem.

The advantages of the approach without use of the isospin representation enable one to achieve a superior accuracy in calculations of all the structure characteristics of nuclei as compared to the results available in the literature. The developed approach gives a real possibility to construct new realistic variants of NN-interaction potentials for a complete description of all the low-energy parameters of few-nucleon systems.

The authors dedicate this paper to the blessed memory of their teacher academician Oleksij Sitenko.

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ПЕРЕВАГИ ПРЕДСТАВЛЕННЯ БЕЗ ВИКОРИСТАННЯ ФОРМАЛІЗМУ ІЗОСПІНУ І ПРЕЦИЗІЙНІ ДОСЛІДЖЕННЯ МАЛОНУКЛОННИХ СИСТЕМ

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Р е з ю м е

З метою проведення досліджень властивостей малонуклонних систем з високою і контрольованою точністю проаналізовано доцільність використання формалізму ізоспіну для нуклонів. Показано, що використання, за традицією, формалізму ізотопічного спіну призводить до невиправданих ускладнень повної хвильової функції і збільшення кількості рівнянь для її просторових компонент. На прикладі системи трьох нуклонів з центральними обмінними потенціалами взаємодії в загальному вигляді встановлена повна еквівалентність двох підходів – представлення без використання формалізму ізоспіну і стандартного підходу з використанням формалізму ізоспіну. Сформульовані нові системи рівнянь для трьох нуклонів зі спіном $S = 1/2$ та чотирьох нуклонів зі спіном $S = 0$. Розроблені оптимальні варіаційні схеми з гаусоїдними базисами для прецизійного дослідження основних характеристик малонуклонних систем. Для трьох нуклонів на основі запропонованого підходу і сформульованих рівнянь отримано з високою і контрольованою точністю енергії зв'язку, радіуси, зарядові розподіли густини та форм-фактори. Виконано якісний аналіз тонких структурних

особливостей малонуклонних систем. Показана перевага підходу без використання представлення ізоспіну: отримані в даній роботі результати мають найкращу точність в порівнянні з відомими результатами, а також відкривають нові можливості побудови реалістичних варіантів ядерних потенціалів для одночасного і повного опису основних низькоенергетичних характеристик малонуклонних систем.

ПРЕИМУЩЕСТВА ПРЕДСТАВЛЕНИЯ БЕЗ ИСПОЛЬЗОВАНИЯ ФОРМАЛИЗМА ИЗОСПИНА И ПРЕЦИЗИОННЫЕ ИССЛЕДОВАНИЯ МАЛОНУКЛОННЫХ СИСТЕМ

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Р е з ю м е

Для проведения исследований свойств малонуклонных систем с высокой и контролируемой точностью выполнен анализ использования формализма изоспина для нуклонов. Показано, что использование, по традиции, формализма изотопического спина приводит к неоправданным усложнениям полной волновой функции и увеличению количества уравнений для ее пространственных компонент. На примере системы трех нуклонов с центральными обменными потенциалами взаимодействия в общем виде установлена полная эквивалентность двух подходов - представления без использования формализма изоспина и стандартного подхода с использованием формализма изоспина. Сформулированы новые системы уравнений для трех нуклонов со спином $S = 1/2$ и четырех нуклонов со спином $S = 0$. Разработаны оптимальные вариационные схемы с гауссоидальными базисами для прецизионного исследования основных характеристик малонуклонных систем. Для трех нуклонов на основе предложенного подхода и сформулированных уравнений получены с высокой и контролируемой точностью энергии связи, радиусы, зарядовые распределения плотности и формфакторы. Выполнен качественный анализ тонких структурных особенностей малонуклонных систем. Показаны преимущества подхода без использования представления изоспина: полученные в данной работе результаты имеют наилучшую точность по сравнению с известными результатами, а также открывают новые возможности построения реалистических вариантов ядерных потенциалов для одновременного и полного описания основных низкоэнергетических характеристик малонуклонных систем.