WEAKLY BOUND PARTICLE IN THE ELECTRIC FIELD OF A NUCLEUS

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UDC 539.171

The closed expression for the internal wave function of a weakly bound particle in the external electric field has been obtained. The approximate analytical representation of the function is found. It is shown that the internal stationary state of a weakly bound particle in the electrical field of a heavy nucleus is transformed in a quasi-stationary one (the particle may polarize or break up). It leads to a significant distortion of internal wave functions. It has been shown that the spherical symmetry of this function fails as the particle comes to the force source. The wave function calculations for d, ^{6}He, and ^{11}Li ions in the electric field of ^{208}Pb nucleus are fulfilled.

The problem on motion of weakly bound particles in the external field rises in many areas of physics. Weakly bound negative ions in electric or magnetic fields and radioactive neutron-excess exotic particles in the heavy nucleus field can be the examples of such systems [1–6]. As a rule, they are two-component (two-cluster) systems. Low binding energy is the reason why an external field reacting upon system particles in different ways could significantly affect the internal state of such a particle. Under these circumstances, the stationary state of the particle is transformed into a quasi-stationary state with a certain width and a different binding energy.

Here, we solve the problem on finding the wave function of a weakly bound deuteron-like particle ("deuteron" with weight m_d and charge Z_p) consisting of charged ("proton" with mass and charge m_p, Z_p, correspondingly) and neutral ("neutron" with weight m_n) clusters in the external electric field of a charge Z_T [4, 6–8]. The particle is modelled by a potential well with the level close to zero ($\varepsilon_0/V_0 \ll 1$, $\varepsilon_0 = \hbar^2\alpha_0^2/2\mu$ — binding energy, V_0 — potential well depth, μ — reduced particle mass, $1/\alpha_0$ — effective "deuteron" radius [9]).

If the external field is absent, the state of "deuteron" is described as [10]:

$$[-\varepsilon_0 + (\hbar^2/2\mu)\Delta_{\mathbf{r}} - V_{np}(r)]\varphi_0(r) = 0, \tag{1}$$

where $(\hbar^2/2\mu)\Delta_{\mathbf{r}}$ — kinetic energy operator with respect to the coordinate $\mathbf{r}$, $\mathbf{r}$ — the distance between "proton" and "neutron" in "deuteron", $\mu = m_n m_p/m_d$, $m_d = m_n + m_p$— "deuteron's" mass, $V_{np}(r)$ — cluster interaction potential in "deuteron", $\varphi_0(r)$ — free "deuteron's" wave function. For a zero-range radius $V_{np}(r)$ [9],

$$\varphi_0(r) = \sqrt{\frac{\alpha_0}{2\pi}}\frac{u_0(r)}{r}, u_0(r) = \exp(-\alpha_0 r). \tag{2}$$

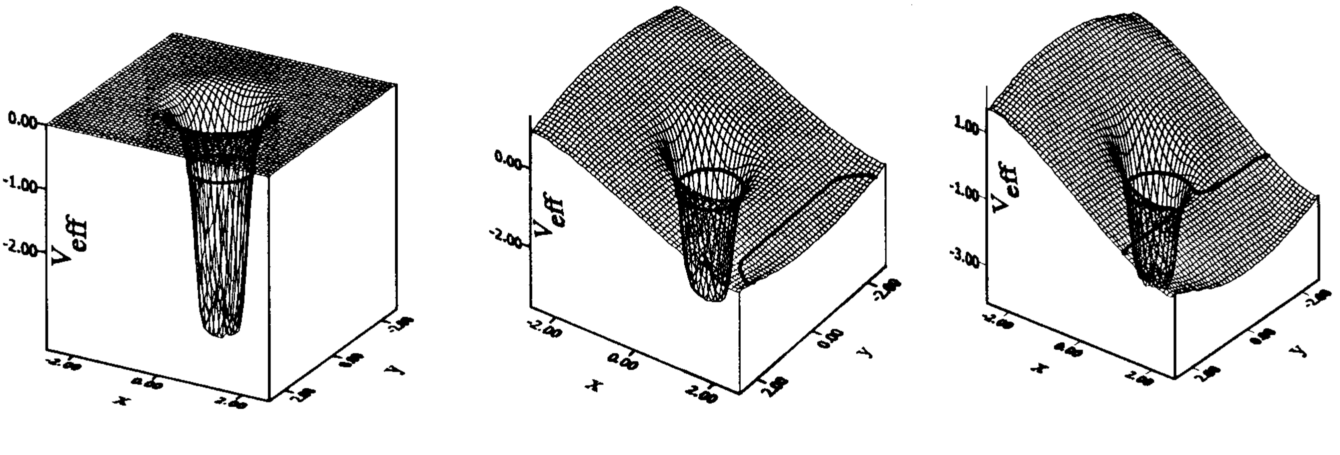
In the external field ($Z_T > 0$), the "deuteron's" wave function of the internal state is a solution of the equation [8]:

$$[-\varepsilon_{\mathbf{R}} + (\hbar^2/2\mu)\Delta_{\mathbf{r}} + \Delta V - V_{np}(r)]\varphi_{\mathbf{R}}(\mathbf{r}) = 0, \tag{3}$$

where $\varepsilon_{\mathbf{R}} = \varepsilon_0 - \delta V$ — the energy of "deuteron's" state in the field, δV — the unknown complex additive stipulated by the field's influence upon the relative motion of "neutron" and "proton" in "deuteron", $\Delta V = Z_p Z_T e^2/R - Z_p Z_T e^2/r_p$, $\mathbf{R}$ — coordinates of the "deuteron's" center of mass, $\mathbf{r}_p = \mathbf{R} - (m_n/m_d)\mathbf{r}$ — coordinates of "proton". As $\varepsilon_{\mathbf{R}}$ is unknown, we come to a typical problem on finding the eigenvalue and eigenfunctions of Eq.(3). The assumption is made that the wave function $\varphi_{\mathbf{R}}(\mathbf{r})$ of this system in the external field parametrically depends on $\mathbf{R}$ (adiabatic approximation).

In the zero-range approximation, Eq.(3) takes the following form:

$$[-\varepsilon_{\mathbf{R}} + (\hbar^2/2\mu)\Delta_{\mathbf{r}} + \Delta V]\varphi_{\mathbf{R}}(\mathbf{r}) = D_0\delta(\mathbf{r}),$$



$\lambda = 0$ $\lambda = 0.5$ $\lambda = 0.9$

Fig. 1. Polarization of ^{6}He in the electrical field of target nucleus ^{208}Pb in the dipole approximation

$$D_0 = -(2\pi\hbar^2/\mu)(\alpha_0/2\pi)^{1/2}, \quad (4)$$

with the boundary condition [11]

$$\lim_{r\to\infty}\{(\partial/\partial r)[r\varphi_{\mathbf{R}}(\mathbf{r})]\} = -\alpha_0. \quad (5)$$

Solution (4) is proportional to the Coulomb Green's function $G_C(E_p, \mathbf{R}, \mathbf{r}_p)$ [8] and can be presented as

$$\varphi_{\mathbf{R}}(\mathbf{r}) = \sqrt{\frac{\alpha_0}{2\pi}}\frac{u_{\mathbf{R}}(\mathbf{r})}{r},$$

$$u_{\mathbf{R}}(\mathbf{r}) = [\partial/\partial\rho_1 - \partial/\partial\rho_2]\{H_0^+(\rho_1)F_0(\rho_2)\}, \quad (6)$$

where $H_0^+ = G_0 + iF_0$, F_0 and G_0 — are regular and irregular, respectively, in the null Coulomb functions of complex arguments; $\rho_{1,2} = (q/2)(R + r_p \pm |\mathbf{r}_p - \mathbf{R}|)$, $q^2 = (2m_p/\hbar^2)E_p$, $E_p = (m_d/m_n)\times \times(Z_pZ_Te^2/R - \varepsilon_{\mathbf{R}})$. At great distances, $(R \to \infty)$ $u_{\mathbf{R}}(\mathbf{r}) \to u_0(r)$. A complex shift of the internal state energy δV can be obtained from (6) [6] and the boundary condition (5).

Therefore, $\varphi_{\mathbf{R}}(\mathbf{r})$ and δV are complex, i.e., the particle's state becomes quasi-stationary in the external field. The real part δV is stipulated by polarization (the shift of the quasi-stationary state energy relative to that of a stationary state) [1–2], and the imaginary part depends on the possibility of the break up of "deuteron" [4, 6], tunneling through the arising barrier (the width of the quasi-stationary state). Obviously, the condition of quasi-stationarity [4]

$$\left|\frac{\delta V}{\varepsilon_0}\right| \ll 1 \quad (7)$$

should be fulfilled.

In the literature, there is a wide discussion on the explanation of the nature of specific properties of weakly bound nuclei far from the valley of β-stability: neutron halo [12], creation of neutron associations (di- and tetra-neutrons in the field of a charged core [12, 13]), observation of two modes of dipole resonance in such nuclei [13], and others. All these properties of the internal structure can be revealed in the interaction of such ions with nuclei, whose characteristics are well known. In order to illustrate the influence of the external field upon the internal state of a weakly bound particle, we have taken, as an example, the interaction of an exotic radioactive nucleus ^{6}He ($\varepsilon_0 = 0.975$ MeV), which can be presented [13] as dineutron and α-particle with a great probability, and a heavy target nucleus ^{208}Pb at the energy $E_{^6\mathrm{He}} = 16$ MeV.

The change of particle's internal state can be viewed in Fig. 1, where the schematic ^{6}He nucleus is modelled by the effective potential well $V_{\text{eff}} = (V_{np} + \Delta V)/\varepsilon_0$ (V_{np}— is a Hulten-type potential, ΔV is taken in the dipole approximation); $\lambda = F_R r_p^{\max}/\varepsilon_0$, where $F_R = Z_pZ_Te^2/R^2$, and the maximum shift of "proton" within "deuteron" $r_p^{\max} = (m_n/m_d)\alpha_0^{-1}$; $x = r\sin\theta$, $y = r\cos\theta$, θ — is the angle between the vectors $\mathbf{R}$ and $\mathbf{r}$; bold solid line ($V_{\text{eff}} = -1$) — energetic level relevant to the basic state of the free "deuteron" ($\lambda = 0$). Starting from some distances ($\lambda = 0.5$) to the center of force, the external field distorts the potential well and a potential barrier of finite width appears (between two solid lines), through which the tunneling is possible (break up). At $\lambda = 0.9$, the well is significantly deformed, the energy state shift is great, the

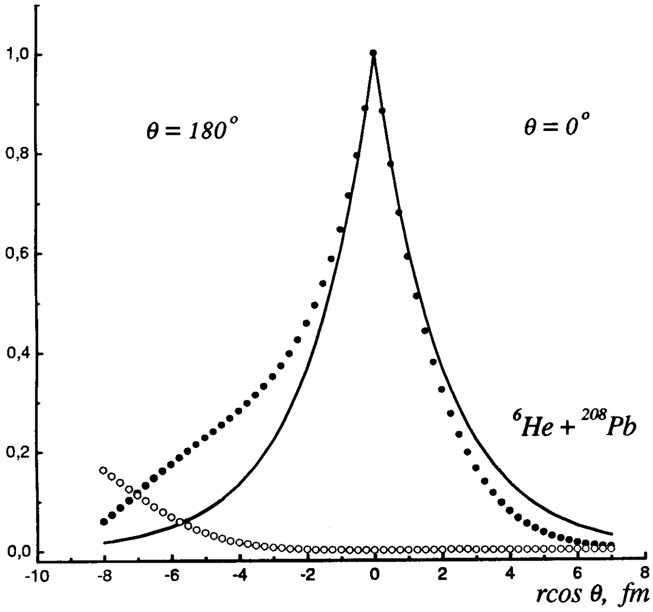


Fig. 2. Relative nuclear density of distribution of ^{6}He nucleus

condition of quasi-stationarity fails, and, as a result, the problem condition becomes incorrect.

An approximate analytical expression for the wave function of a weakly bound particle in the external electric field can be obtained by decomposition of $u_{\mathbf{R}}(\mathbf{r})$ in the Taylor series at a small ratio r/R, limiting oneself by terms of the second order, and using the WKB-approximation [17] of the first order for Coulomb functions. Then, after easy transformations, we obtain

$$u_{\mathbf{R}}(\mathbf{r}) \approx \exp(-\alpha r)[1-\beta(\mathbf{n_R n_r})(\alpha_0 r)^2], \quad (8)$$

where $\alpha=\sqrt{(2\mu\varepsilon_0/\hbar^2)(1+\beta^2)}$, $\beta=(F_R r_p^{\max}/4\varepsilon_0)(1-\delta V/\varepsilon_0)^{-3/2}$, $\beta\approx F_R r_p^{\max}/4\varepsilon_0$.

Substituting (8) into (6), we obtain the analytical expression of the internal state wave function of a weakly bound particle in the external field for small values of r:

$$\varphi_{\mathbf{R}}(\mathbf{r})=\sqrt{\alpha_0/2\pi}r^{-1}\exp(-\alpha r)[1-\beta(\mathbf{n_R n_r})(\alpha_0 r)^2]. \quad (9)$$

It is clear that this wave function is different from the wave function of a free "deuteron" having the spherically asymmetric term proportional to r^2. When $\beta\to 0$, $\varphi_{\mathbf{R}}(\mathbf{r})\to\varphi_0(r)$.

Applicability of all approximations to (9) requires small values of the parameter β, i.e., it is necessary that the external force work at the maximum shift of "proton" from the "deuteron" center of mass be less than the

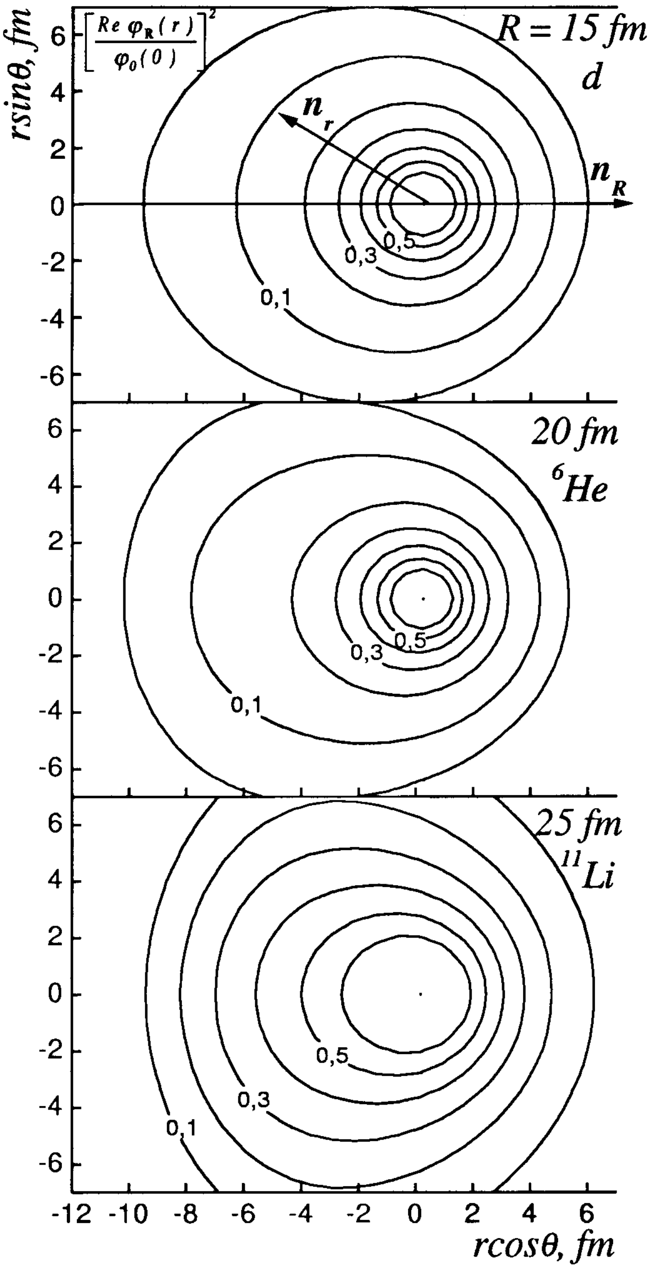


Fig. 3. Relative nuclear matter density distributions of d, ^{6}He, and ^{11}Li ions in the electric field of ^{208}Pb nuclei

"deuteron" binding energy. That is,

$$|F_R r_p^{max}/\varepsilon_0|<1. \quad (10)$$

This limitation is practically the same as the criteria of the WKB-approximation [18] and therefore, can be viewed as a new adiabatic and quasi-stationarity criterion of weakly bound system's state.

Numerical calculations of wave functions (6) for ^{6}He nucleus in the field of ^{208}Pb target are presented in Fig.2, where the relative nuclear matter density distribution of unperturbated ^{6}He ion (solid line), $|\mathrm{Re}\varphi_{\mathbf{R}}(\mathbf{r})/\varphi_0(0)|^2$ (filled circles) and $|\mathrm{Im}\varphi_{\mathbf{R}}(\mathbf{r})/\varphi_0(0)|^2$ (empty circles) are shown for two different values of the angle θ between the vectors $\mathbf{R}$ and $\mathbf{r}$. The value R is taken in the vicinity of the classic Coulomb turning point, where the influence of the external field should be maximum.

The projection isolines of the surface of the same density in the plane XY of the relative nuclear matter density distribution for d, ^{6}He, and ^{11}Li are shown in Fig. 3, where $\mathbf{n_R}$ and $\mathbf{n_r}$ – are unit orts directed along the vectors $\mathbf{R}$ and $\mathbf{r}$, respectively. In this case, the value of R was chosen in such a way that the external fields for various particles are the same. As seen from Fig. 2 and 3, the relative nuclear matter density distributions of nuclei of interest significantly depend upon the angle between the vectors $\mathbf{R}$ and $\mathbf{r}$, and they are stretched in the direction to the force center for such a configuration of the incident particle, when it moves forward by neutrons, and is squeezed for the opposite orientation. Such a dynamic deformation of the nuclear matter density distribution allows the neutron cluster to come to the target nucleus at a distance smaller than classic turning points in a Coulomb field even at under-barrier energies. At these distances, the interaction with the target nucleus field becomes possible. This can be the reason of the unsuccessful description of oscillations of the experimental elastic scattering cross-sections of ions ^{6}He by nuclei ^{208}Pb in [19], where the analysis was done without taking this fact into account.

Conclusions

We have obtained the exact closed and approximate analytical expressions for the wave functions of the "deuteron's" internal state in the external electric field.

Numerical calculations of those functions have been made and have shown that the relative density distribution of "nucleons" in "deuteron" significantly depends upon the angle between the vectors $\mathbf{R}$ and $\mathbf{r}$ and is stretched in the direction to the force center for such a configuration of the incident particle when it is moving with "neutrons" directed forward.

The adiabatic and quasi-stationarity criterion of a weakly bound particle's state in the external electrical field is obtained.

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Received 20.07.01

СЛАБКОЗВ'ЯЗАНА ЧАСТИНКА
В ЕЛЕКТРИЧНОМУ ПОЛІ ЯДРА

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Р е з ю м е

Знайдено замкнутий вираз для внутрішньої хвильової функції слабкозв'язаної частинки у зовнішньому електричному полі. Знайдено наближене аналітичне представлення таких функцій. Показано, що в електричному полі важких ядер стаціонарний внутрішній стан слабкозв'язаних частинок стає квазістаціонарним (частинка може поляризуватися або розвалитися). Це приводить до значного спотворення їхніх внутрішніх хвильових функцій. Встановлено, що при наближенні до силового центра функції втрачають сферичну симетрію. Розраховано хвильові функції іонів d, ^{6}He, ^{11}Li у електричному полі ^{208}Pb.

СЛАБОСВЯЗАННАЯ ЧАСТИЦА
В ЭЛЕКТРИЧЕСКОМ ПОЛЕ ЯДРА

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Р е з ю м е

Найдено замкнутое выражение для внутренней волновой функции слабосвязанной частицы во внешнем электрическом поле. Найдено приближенное аналитическое представление таких функций. Показано, что в электрическом поле тяжелых ядер стационарное внутреннее состояние слабосвязанных частиц становится квазистационарным (частиц а может поляризов аться или разв алиться). Это приводит к значительному искажению их внутренних волновых функций. Установлено, что при приближении к силовому центру функции теряют сферическую симметрию. Рассчитаны волновые функции ионов d, ^{6}He, ^{11}Li в электрическом поле ^{208}Pb.