ELECTROWEAK PHASE TRANSITION IN A HYPERMAGNETIC FIELD

V. V. SKALOZUB, V. I. DEMCHIK

Dnipropetrovsk National University
(13, Naukovy Prov., Dnipropetrovsk 43625, Ukraine)

UDC 539.12
‰ 2001

The electroweak phase transition in a strong hypermagnetic field H_Y is investigated in the Standard Model. We use the effective potential of scalar and hypermagnetic fields at finite temperature, which takes into consideration the contributions of one-loop and ring diagrams of all fermions and bosons of the model. The only free parameter is the Higgs boson mass chosen to be in the energy interval $75 \leq m_H \leq 115$ GeV. It is found that, for the field strengths $H_Y \sim (10^{22} \div 10^{23})$ G, the electroweak (EW) phase transition is of first order but the baryogenesis condition is not fulfilled. For stronger fields, it turns to crossover. The stability of the vacuum in a field at high temperatures is studied. A comparison with the results of other approaches is done.

Introduction

The presence of strong magnetic fields in the early Universe as well as possible effects of them is widely discussed nowadays (see survey [1]). As is believed, the primordial magnetic fields could serve as the seed ones necessary for producing the large-scale magnetic fields observed at present time. An interesting idea was put in forth that a strong hypermagnetic field is able to increase considerably the strength of the first-order phase transition (PT), that is needed for baryogenesis [2]. Interest in this problem appeared recently when it has been realized that the standard scenario of baryogenesis does not hold without external fields in the minimal standard model (SM) for the Higgs boson masses permitted by experiment (see [3] and references therein).

The assumption that, by analogy to superconductivity, the strong external hypermagnetic field can increase the strength of the 1st order PT making it sufficient to preserve baryogenesis in the SM was accepted in [2, 4 – 6] where the influence of the constant hypermagnetic field H_Y on the EW PT has been investigated. In [2, 4], by using perturbative methods of calculations, it was concluded that the H_Y makes the weak-1st-order PT stronger. Moreover, standard baryogenesis could survive in the SM for $H_Y \sim (0.3 \div 0.5)\, T^2$, and the Higgs particle mass $m_H \leq 100$ GeV. In [5], by applying the approach combining a dimension reduction at high temperature and lattice simulations, it was found that, for the field strengths mentioned, the 1st order phase indeed becomes stronger. However, for $m_H \geq (80 \div 90)$ GeV, the end point of the 1st order PT line has nevertheless been observed, as at zero field. In addition, a mixed PT was found. No observations for the lattice structure similar to that in the type II superconductors in magnetic fields (that was expected for these values of m_H and field strengths) have been determined. In a certain sense, these nonperturbative calculations have indicated either the lack of the standard baryogenesis scenario in the SM or the necessity to substitute the model by other one. As a possible candidate, the minimal supersymmetric standard model is widely discussed (see the recent paper [7] and references therein). At the same time, the absence of the lattice vacuum structure is the unexpected observation requiring further investigations. It is also interesting to find out the origin of discrepancies in the perturbative and nonperturbative results noted above.

In [4], the role of fermions in the phase transition dynamics was not investigated in detail. However, due to some peculiarities of fermion interactions with the field, all fermions (heavy and light) are essential. This point will be discussed in detail below. Here, we only notice that the term $\sim H_Y^2 \log(T/m_f)$ (m_f – fermion mass, T – temperature) of the high temperature asymptotics of the one-loop effective potential (EP) is dominating for light particles. So, a more detailed consideration of the fermion effects is of interest.

In the present paper, the EW PT in the hypermagnetic field is investigated within the consistent EP, which includes one-loop and ring diagram contributions. All bosons and fermions are taken into account with their actual masses (the *t*-quark mass is chosen to be 175 GeV). The only free parameter is m_H, and we assume $75 \leq m_H \leq 115$ GeV, in order to take account of a nowadays experimental low limit $m_H \geq 90$ GeV. We calculate also the contributions of ring diagrams in the external field. In both the restored and broken phases, these diagrams cancel the imaginary terms of the one-loop EP making the total contribution real at sufficiently high temperatures [10]. Our main objective is to clarify the influence of fermions in the PT dynamics in strong fields and the realization of the necessary conditions for the creation of a lattice vacuum structure.

As in [4, 5], we investigate the case of constant external hypermagnetic field. This is a good approximation for strong fields and the 2nd order PT and/or the 1st order PTs, when the bubbles are not too large.

1. Boson Field Contributions to $V^{(1)}(T, H_Y, \varphi_c)$

To compute the EP $V^{(1)}$, we use the proper time (s-representation) formalism. Detail of calculations based on the s-representation can be found, for example, in [13, 14].

To incorporate the interaction with an external hypermagnetic field, we add the term $\frac{1}{2}\vec{H}\vec{H}_Y$ to the Lagrangian. A value of the macroscopic magnetic field generated in the system will be determined from the equations of motion.

A one-loop contribution into the EP is given by the expression

$$V^{(1)} = \frac{1}{2}\,\mathrm{Tr}\log G^{ab}, \tag{1}$$

where G^{ab} stands for the propagators of all the quantum fields $W^{\pm}, \varphi^{\pm}, ...$ in the background magnetic field H. Details of calculations can be found, for instance, in [8, 11], where the case of usual magnetic field has been investigated. The difference between these cases lays in the boundary conditions and the description of the restored phase. It is convenient to introduce the dimensionless quatities: $h = H/H_0$, $(H_0 = M_W^2/e)$, $\varphi = \frac{\varphi_c}{\delta(0)}$, $K = \frac{m_H^2}{M_W^2}$, $B = \beta M_W$, $\tau = \frac{1}{B} = \frac{T}{M_W}$, $\upsilon = \frac{V}{H_0^2}$ and $M_W = \frac{g}{\sqrt{2}}\delta(0)$, where φ_c − vacuum expectation value of the Higgs field, $\delta(0)$ is that at $H = 0$.

We make use of the gauge-fixing conditions [15]

$$\partial_\mu W^{\pm\mu} \pm ie\bar{A}_\mu W^{\pm\mu} \mp i\frac{g^2\varphi^2}{4\xi}\varphi^{\pm} = C^{\pm}(x),$$

$$\partial_\mu Z^{\mu} - \frac{i}{\xi'}(g^2 + g'^2)^{1/2}\varphi_z = C_z, \tag{2}$$

where $e = g\sin\theta_W$, $\mathrm{tg}\,\theta_W = g'/g$, $\varphi^{\pm}$, φ_Z are the Goldstone fields, ξ, ξ' are the gauge fixing parameters, $C^{\pm}, C_Z$ are arbitrary functions and φ_c is a scalar condensate value. In what follows, all calculations will be done in the general relativistic renormalizable gauge (2) and then we set $\xi, \xi' = 0$ by choosing the unitary gauge.

After reparametrization, the scalar field potential is directly expressed in terms of the ratio K:

$$\upsilon^{(0)} = \frac{h^2}{2} + K\left(-\frac{\varphi^2}{4} + \frac{\varphi^4}{8}\right). \tag{3}$$

Recall that h is the electromagnetic component of the hyperfield h_Y, which is unscreened in the broken phase. In the restored phase, it is convenient to make use of initial fields, and we shall carry out the corresponding calculations later.

The renormalized one-loop EP is given by the sum of functions:

$$\upsilon_1 = \upsilon^{(0)} + \upsilon^{(1)}(\varphi, h, K) + \omega^{(1)}(\varphi, h, K, \tau), \tag{4}$$

where $\upsilon^{(1)}$ is the one-loop EP at $T = 0$, which was studied already in [15]. It has the form:

$$\upsilon^{(1)} = \upsilon^{(1)}_{W,Z} + \upsilon^{(1)}_{\varphi}, \tag{5}$$

$$\upsilon^{(1)}_{W,Z} = \frac{3\alpha}{\pi}\Big[h^2\log\Gamma_1\left(\frac{1}{2} + \frac{\varphi^2}{2h}\right) + h^2\zeta(-1) + $$
$$+ \frac{1}{16}\varphi^4 - \frac{1}{8}\varphi^4\log\frac{\varphi^2}{2h} + \frac{1}{24}h^2 - \frac{1}{24}h^2\log(2h)\Big] + $$
$$+ \frac{\alpha}{2\pi}[-2h^2 + (h^2 + h\varphi^2)\log(h + \varphi^2) + $$
$$+ (h^2 - h\varphi^2)\log|h - \varphi^2|] + i\frac{1}{2}\alpha h(\varphi^2 - h)\,\theta(h - \varphi^2), \tag{6}$$

$$\upsilon^{(1)}_{\varphi} = K\sin^2\theta_W\left(-\varphi^2 + \frac{1}{2}\varphi^4\right) + $$
$$+ \frac{3\alpha}{4\pi}\left(1 + \frac{1}{2\cos^2\theta_W}\right)\left(\frac{1}{2}\varphi^4\log\varphi^2 - \frac{3}{4}\varphi^4 + \varphi^2\right) + $$
$$+ \frac{\alpha K^2}{32\pi}\left[\left(\frac{9}{2}\varphi^4 - \frac{3}{4}\varphi^2 + \frac{1}{2}\right)\times\right.$$
$$\left.\times\log\left|\frac{3\varphi^2 - 1}{2}\right| - \frac{27}{4}\varphi^4 + \frac{21}{2}\varphi^2\right] \tag{7}$$

and $\omega^{(1)}$ is the T-dependent contribution to the EP,

$$\mathrm{Re}\,\omega^{(1)} = \omega^{(1)}_W + \omega^{(1)}_Z + \omega^{(1)}_{\varphi}. \tag{8}$$

Here, $\omega_W^{(1)}$, $\omega_Z^{(1)}$, and $\omega_\varphi^{(1)}$ are the temperature contributions of W-, Z-bosons and the scalar field, correspondingly,

$$\mathrm{Re}\,\omega_W^{(1)} = -4\frac{\alpha}{\pi}\frac{h}{B}\left(3\omega_0^W + \omega_1^W - \omega_2^W\right),$$

$$\mathrm{Re}\,\omega_Z^{(1)} = -6\frac{\alpha}{\pi}\sum_{n=1}^{\infty}\frac{\varphi^2}{\cos^2\theta_W n^2 B^2}K^2\left(\frac{nB\varphi}{\cos\theta_W}\right),$$

$$\mathrm{Re}\,\omega_\varphi^{(1)} = \begin{cases} -2\dfrac{\alpha}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{t^2}{B^2n^2}K_2(nBt), \\ \quad t = \left[K\left(\dfrac{3\varphi^2-1}{2}\right)\right]^{1/2}, \quad 3\varphi^2 > 1, \\ \alpha\displaystyle\sum_{n=1}^{\infty}\frac{|t|^2}{n^2B^2}Y_2(nB|t|), \\ \quad t = \left[K\left(\dfrac{1-3\varphi^2}{2}\right)\right]^{1/2}, \quad 3\varphi^2 < 1, \end{cases} \tag{9}$$

where $K_n(x)$ and $Y_n(x)$ are the Macdonald and Bessel functions, respectively,

$$\omega_0^W = \sum_{p=0}^{\infty}\sum_{n=1}^{\infty}\frac{x_p}{n}K_1(nBx_p), \quad x_p = (\varphi^2 + h + 2ph)^{1/2},$$

$$\omega_1^W = $$

$$= \begin{cases} \displaystyle\sum_{n=1}^{\infty}\frac{y}{n}K_1(nBy), & y = (\varphi^2 - h)^{1/2}, \quad \varphi^2 > h, \\ -\dfrac{\pi}{2}\displaystyle\sum_{n=1}^{\infty}\frac{|y|}{n}Y_1(nB|y|), & y = (h - \varphi^2)^{1/2}, \quad \varphi^2 < h, \end{cases}$$

$$\omega_2^W = \sum_{n=1}^{\infty}\frac{z}{n}K_1(nBz), \quad z = (\varphi^2 + h)^{1/2}. \tag{10}$$

There is an imaginary part in (8) for $h > \varphi^2$ appeared due to the tachyonic mode $\varepsilon^2 = p_3^2 + M_W^2 - eH$ in the W-boson spectrum [15]. The imaginary part of EP is signalling an instability of the system. In what follows, we consider the symmetry behaviour described by the real part of the EP. As the imaginary part is concerned, it will be cancelled in consistent calculation including the one-loop and ring diagram contributions to the EP.

Expressions (5) and (8) – (10) will be used in the numerical studying of symmetry behaviour at various H, T. There is the cancellation of a number of terms from the zero-temperature contributions given by (5) and T-depended ones. This fact has a general nature. It was used in checking the correctness of calculations.

2. Fermion Contributions to $V^{(1)}(H, T, \varphi_c)$

The fermion part of the SM in magnetic fields is well studied [17]. The zero-temperature fermion contribution to the dimensionless EP reads

$$\upsilon_f(h,\varphi) = \frac{\alpha}{4\pi}\sum_f K_f^2\left(-2\varphi^2 + \frac{3}{2}\varphi^4 - \varphi^4\log\varphi^2\right) -$$

$$-\frac{\alpha}{\pi}\sum_f\left(q_f^2\frac{h^2}{6}\log\frac{2|q_f|\,h}{K_f}\right) - \frac{\alpha}{\pi}\sum_f\left[2q_f^2h^2\times\right.$$

$$\times\log\Gamma_1\left(\frac{K_f\varphi^2}{2|q_f|\,h}\right) + \left(2\zeta'(-1) - \frac{1}{6}\right)q_f^2h^2 +$$

$$\left. + \frac{1}{8}K_f^2\varphi^4 + \left(\frac{1}{4}K_f^2\varphi^4 - \frac{1}{2}K_f|q_f|\,h\varphi^2\right)\log\frac{2|q_f|\,h}{K_f\varphi^2}\right], \tag{11}$$

where q_f is the fermion electric charge, the sum $\sum_f = \sum_{f=1}^{3}(\text{leptons}) + 3\sum_{f=1}^{6}(\text{quarks})$ counts the contributions of leptons and quarks with their electric charges. The Γ_1 function is defined as follows:

$$\log\Gamma_1(x) = \int_0^x dy\log\Gamma(y) + \frac{1}{2}x(x-1) - \frac{1}{2}x\log(2x). \tag{12}$$

The finite temperature part looks as follows:

$$\omega_f = -4\frac{\alpha}{\pi}\sum_f\left\{\sum_{p=0}^{\infty}\sum_{n=1}^{\infty}(-1)^n\left[\frac{(2ph + K_f\varphi^2)^{1/2}h}{Bn}\times\right.\right.$$

$$\times K_1((2ph + K_f\varphi^2)^{1/2}Bn) + \frac{((2p+2)h + K_f\varphi^2)^{1/2}h}{Bn}\times$$

$$\left.\left.\times K_1(((2p+2)h + K_f\varphi^2)^{1/2}Bn)\right]\right\}. \tag{13}$$

Expressions (11) and (13) will be used in numerical investigations.

3. Contribution of Daisy Diagrams

The sum of daisy diagrams describes a dominant contribution of long distances [10]. It differs from zero only in the case where massless states appear in a system. So, this type of diagrams has to be calculated when a symmetry restoration is investigated. To find the correction $V_{\rm ring}(H, T)$ at high temperatures and a magnetic field, the polarization operators of neutral bosons at considered background have to be calculated.

In order to calculate the contribution of ring diagrams, not the complete polarization operators $\Pi_{\mu\nu}(k, T, H)$ but only their limiting expressions at zero momenta, $\Pi_{00}(k = 0, T, H)$, are sufficient. This limit, named the Debye mass, can be calculated from the EP of a special type. This fact considerably simplifies our task.

Now, let us turn to the calculations of $V_{\rm ring}(H, T)$. It is given by the standard expression [10, 16, 11]:

$$V_{\rm ring} = -\frac{1}{12\pi\beta}\,{\rm Tr}\,\{[M^2(\varphi) + \Pi_{00}(0)]^{3/2} - M^3(\varphi)\}, \tag{14}$$

where the trace means the summation over all the contributing states, $M(\varphi)$ is the tree mass of the corresponding state for the Higgs particle and $\Pi_{00}(0) = \Pi_{00}(k = 0, T, H)$ are the zero-zero components of the polarization operators in the magnetic field taken at zero momenta. The above contribution has the order $\sim g^3(\lambda^{3/2})$ in coupling constant whereas the two-loop terms are to be of order $\sim g^4(\lambda^4)$. As $\Pi_{00}(0)$, the high temperature limits of polarization functions have to be substituted which have the order $\sim T^2$ for leading terms and $\sim g\varphi_c T$, $\sqrt{gH}\,T$ $(\varphi_c/T, \sqrt{gH}/T << 1)$ for subleading ones.

Detailed calculations of the polarization functions $\Pi_{00}(0)$ were carried out in [8] in case of a usual magnetic field. We refer readers to this paper for computational details and here adduce the necessary expressions. As $\Pi_\varphi(0)$, we have

$$\Pi_\varphi(0) = \left.\frac{\partial^2 V^{(1)}(\varphi, H, T)}{\partial\varphi^2}\right|_{\varphi=0} =$$

$$= \frac{1}{24\beta^2}\left(6\lambda + \frac{6e^2}{\sin^2 2\theta_W} + \frac{3e^2}{\sin^2\theta_W}\right) +$$

$$+ \sum_f \frac{2G_f^2}{\beta^2} + \frac{(eH)^{1/2}}{8\pi\sin^2\theta_W\,\beta}\,e^2\left(3\sqrt{2}\,\zeta\left(-\frac{1}{2}, \frac{1}{2}\right) - 1\right). \tag{15}$$

Substituting (15) into (14), we obtain (in dimensionless variables)

$$\upsilon^\varphi_{\rm ring} = -\frac{1}{12B}\left\{\frac{3\varphi^2 - 1}{2}K + \Pi_\varphi(0)\right\}^{3/2} +$$

$$+ \frac{\alpha}{3B}K\left(\frac{3\varphi^2 - 1}{2}\right)^{3/2}. \tag{16}$$

The last term of this expression cancels the term in Eq.(9), which becomes imaginary at $3\varphi^2 < 1$. This is the important cancellation preventing the infrared instability at high temperature.

As for the H-dependent Debye mass of photons and Z-bosons, the following expressions have been obtained:

$$m^2_{D_A} = m^2_{D_A}(\text{fermions}) + m^2_{D_A}(\text{bosons}), \tag{17}$$

$$m^2_{D_A}(\text{fermions}) = \sum_{\rm ferm} g_f^2\sin^2\theta_W \times$$

$$\times\left(\frac{T^2}{3} - \frac{1}{2\pi^2}m_f^2 + O((m_f\beta)^2, (eH\beta))\right), \tag{18}$$

$$m^2_{D_A}(\text{bosons}) = 3g^2\sin^2\theta_W\left[\frac{1}{3}T^2 - \frac{1}{2\pi}T(m^2 +\right.$$

$$+ g\sin\theta_W H)^{1/2} - \frac{1}{8\pi^2}(g\sin\theta_W H) +$$

$$\left.+ O\left(\frac{m^2}{T^2}, \left(\frac{g\sin\theta_W H}{T^2}\right)^2\right)\right]. \tag{19}$$

This is the W-boson contribution to the Debye mass of the electromagnetic field. As is seen, spin does not contribute in the leading order. Other interesting point is that the next-to-leading terms are negative.

The contribution of the W-boson sector to the Z-boson mass m_D^2 is given by expression (19) with the replacement $g^2\sin^2\theta_W \to g^2\cos^2\theta_W$.

Summing up (18) and (19) and substituting them in (14), we obtain the photon part $V^Z_{\rm ring}$, where it is necessary to express masses in terms of the vacuum value of the scalar condensate φ_c. In the same way, the contribution of Z-bosons $V^Z_{\rm ring}$ can be calculated. The only difference is the additional mass term of the Z-field and the additional term in the Debye mass due to the neutral current $\sim\bar\nu\gamma_\mu\nu Z_\mu$. These three fields φ, γ, Z, which becomes massless in the restored phase, contribute into $V_{\rm ring}(H, T)$ in the presence of a magnetic field. At zero field, there is also a term due to W-boson loops. But when $H \neq 0$, charged particles acquire $\sim eH$ masses. The corresponding fields remain short-range ones in the restored phase of the vacuum and therefore do not contribute.

A separate consideration should be spared to the tachyonic (unstable) mode in the W-boson spectrum: $p_0^2 = p_3^2 + M_W^2 - eH$. It should be included in $V_{\rm ring}(H, T)$ side by side with other considered neutral fields. See [8] for details. The result looks as follows:

$$V_{\rm ring}^{\rm unstable} = \frac{eH}{2\pi\beta}\{(M_W^2 - eH + \Pi(H,T))^{1/2} -$$

$$- (M_W^2 - eH)^{1/2}\}. \qquad (20)$$

By summing up the one-loop EP and all the terms $V_{\rm ring}$, we arrive at the total EP consistent in leading order.

Notice the most important features of the above expression. The last term in Eq. (20) exactly cancels the 'dangerous" term in the one-loop EP. So, no instabilities appear at sufficiently high temperatures when $\Pi(H, T) > M_W^2 - eH$ and the EP is real. To make a quantitative estimate of the range of validity of the total EP, it is necessary to calculate the W-boson mass operator in a magnetic field at finite temperature and hence to find $\Pi(H, T)$. This is a separate problem, which has been investigated in [18]. Below, we adduce the result of $\Pi(H, T)$ calculations:

$$\Pi_{\rm unstable}(H, T) = \langle n = 0, \sigma = 1 \,|\, \Pi_{\mu\nu}^{\rm charged} |\, n = 0,$$

$$\sigma = 1\rangle = \alpha\,[12.33\,(eH)^{1/2}\,T + 4i\,(eH)^{1/2}\,T], \qquad (21)$$

where the average value of the mass operator in the ground state of the W-boson spectrum $|\, n = 0, \sigma = +1\rangle$ was computed. (21) has been obtained in the limit $eH/T^2 \ll 1$, $B = M_W / T \ll 1$, which is a good approximation since, as we show below, typical inverse temperatures for the symmetry restoration are $B \sim 0.1$. Along with the real part responsible for the radiation mass squared, (21) contains the imaginary one describing the decay of the state. Its value is small as the real part is compared and of the order of usual damping constants at high temperature. So, $\mathrm{Im}\,\Pi(H, T)$ can be ignored in our problem. The radiation mass squared is positive and acts to stabilize the spectrum. With this result obtained, we conclude that our EP is real for temperatures corresponding to the PT epoch[1].

4. The Restored Phase in the Hypermagnetic Field

To describe the restored phase, one has to calculate radiation corrections to the external field H_Y at high temperature. Before doing that, let us recall that, at $\varphi = 0$, this field is completely unscreened whereas the non-Abelian constituents of electromagnetic and Z-fields are screened on the scales $\geq (g^2 T)^{-1}$. This fact means that, in the covariant derivative describing the interaction with an external field, one should include the $U(1)_Y$ term only: $D_\mu = \partial_\mu + \frac{1}{2} g B_\mu^{\rm ext}$. We set the potential as before, $B_\mu^{\rm ext} = (0, 0, H_Y, 0)$. In the restored phase, W-bosons do not interact with H_Y. The H_Y-dependent part of the EP $V(\varphi = 0, H_Y, T)$ is produced due to contributions of fermions and scalars. However, the fermion contribution depends logarithmically on temperature $\sim \frac{g^2}{4\pi} H^2 \log \frac{T}{T_0}$ and can be neglected in comparing with the three level term $\frac{1}{2} H_Y^2$. This is not the case for the scalar field, whose contribution to the one-loop EP is

$$V_{\rm sc}^{(1)}(H_Y, T) = -\frac{g^2 H_Y^2}{24\pi^2} \ln\frac{T}{T_0} + \frac{(gH_Y)^{3/2}\,T}{12\pi} + O\left(\frac{1}{T}\right). \qquad (22)$$

The term logarithmically depending on T can again be neglected but one linear in T should be retained. Since 'hyperphotons" are massless in the restored phase, we also include the contribution of the corresponding ring diagrams:

$$V_{\rm ring}^{\rm rest}(H_Y, T) =$$

$$= -\frac{T}{12\pi}\left[\frac{1}{3} g^2 T^2 - \frac{(gH_Y)^{1/2}\,T}{2\pi} - \frac{1}{8\pi^2} gH_Y\right]^{3/2}, \qquad (23)$$

where the terms logarithmic in T are omitted. Both these expressions have been calculated in a way described in [8]. For convenience of numerical

[1] (21) disagrees with the corresponding expression of [19] where the average value of the gluon polarization operator in an Abelian chromomagnetic field was calculated in the weak field approximation and $\Pi(H, T)$ has been found to be zero. Most probably, the discrepancy is a consequence of the calculation procedure adopted by these authors when the gluon polarization operator was calculated at zero external field and then its average value has been calculated in the state $|\, n = 0, \sigma = +1\rangle$. Our expression is the high temperature limit of the mass operator, which takes into account the external field exactly.

 ISSN 0503-1265. Ukr. J. Phys. 2001. V. 46, N 8

investigations, let us express Eqs. (22) and (23) in terms of the dimensionless variables h, τ:

$$V^{\text{rest}}(H_Y, T) = (H_0)^2\, \upsilon^{\text{rest}}(h, \tau),$$

$$\upsilon^{\text{rest}}(h, \tau) = \frac{1}{2}\frac{h^2}{\cos^2\theta_W} + \frac{e^{1/2}}{12\pi\cos^3\theta_W}h^{3/2}\tau -$$

$$-\frac{1}{3}\alpha\tau\left[\frac{4\pi\alpha\tau^2}{3\cos^2\theta_W} - \frac{h^{1/2}\tau}{2\pi e^{1/2}\cos\theta_W} - \frac{h}{8\pi^2 e\cos^2\theta_W}\right]^{3/2}, \quad (24)$$

where $\alpha = e^2/4\pi$ and $h_Y = h/\cos\theta_W$.

$V^{\text{rest}}(H_Y, T)$ also includes the terms describing field-independent corrections. They can be found by setting $h = 0$, $\varphi = 0$ in the broken phase EP.

5. Symmetry Behaviour

Now, let us investigate the EW PT in a hypermagnetic field for various values of m_{H}. It can be done by considering the Gibbs free energy in the broken and unbroken phases [4]:

$$G_b = V(\varphi) - \frac{1}{2}\cos^2\theta_W (H_Y^{\text{ext}})^2,$$

$$G_u = V(0) - \frac{1}{2}(H_Y^{\text{ext}})^2. \quad (25)$$

The 1st order PT can be determined from two equations:

$$G_u(H_Y, T) = G_b(H, T, \varphi_c(H)), \quad (26)$$

describing the energetic advantage of creation of the broken phase, where $\varphi_c(H)$ is a scalar field vacuum expectation value at given H, T, and the total EP,

$$\frac{\partial V^{\text{total}}(H, T, \varphi)}{\partial\varphi} = 0, \quad (27)$$

h	K	T_c (GeV)	$\varphi_c(h,T_c)$	$\varphi_c^2(h,T_c)$	R	$M_W(h,T_c)$
0.01	0.85	106.47	0.301662	0.091	0.69699	0.2711
0.01	1.25	122.21	0.181659	0.033	0.36567	0.1664
0.01	2	145.56	0.094868	0.009	0.16033	0.1475
0.1	0.85	108.58	0.275681	0.076	0.62459	0.2452
0.1	1.25	123.54	0.130384	0.017	0.25963	0.1127
0.1	2	148.39	0.031623	0.001	0.05242	0.1263
0.5	0.85	108.89	0.248998	0.062	0.56253	0.2153
0.5	1.25			crossover		
0.5	2			crossover		

determining a minimum position of the total EP. Hence, the critical field strength can be calculated. In (27) (and below), we wrote H instead H^{ext}.

The results on the PT obtained by numerical investigations of the total EP are summarized in Table.

In the first collumn, we show the hypermagnetic field strength in the broken phase (in dimensionless units). In the second and third ones, the mass parameter $K = m_{\text{H}}^2/M_W^2$ and the critical temperature of the 1st order PT are adduced. Next two collumns give the local minimum position $\varphi_c(H, T)$ and their squared values at the transition temperatures. The last two columns fix the ratio $R = 246$ GeV $\varphi_c(h, T_c)/T_c$, which determines the advantage of baryogenesis, and the W-boson effective mass calculated at the local minimum of the EP at the corresponding h and T_c.

As is seen, an increase in h makes the PT weaker (not stronger), as was found in [2, 4, 5]. The ratio R is less than one for all the values of h, whereas the baryogenesis condition is $R \geq 1.2 \div 1.5$ [3]. Thus, we come to the conclusion that an external hypermagnetic field does not make the EW PT strong enough to produce baryogenesis. Moreover, for strong fields, the PT turns to crossover for all the values of K considered. This type of symmetry behaviour is expressed in appearing a plato without pronounced minima separated by a potential barrier. These are the main observations of our numerical study.

Let us continue the analysis of the data in Table. For the field strengths $H \geq (0.1 \div 0.5)\, H_0$ ($H_0 = M_W^2/e$), the PT is of weak-1st-order or cross-over. The W-boson effective mass squared $M_W^2(\varphi_c, h, B) = \varphi_c^2(h, B) - h + \Pi(h, B_c)$ is positive for $h = 0.01$, 0.1. Therefore, the local minimum is stable at the 1st order PT. Moreover, for these values of h, the effective mass squared is positive even at $\varphi = 0$. Therefore, there are no conditions for the vacuum instability, and, hence, no vortex-like structure expected in [5] can be generated. Recall the instability is resulted in the condensate of W- and Z-boson fields having a lattice structure.

We would like to note that our perturbative results are reliable for $K \sim 0.8 \div 0.9$. They are in agreement with nonperturbative analysis at zero field. The external field is taken into account exactly. For these masses m_{H}, we observed the change of the 1st order PT to crossover when the field strength is increased. The same behaviour takes place for $K \geq 1$ when perturbative analysis may be not trusty. But, as we have found out, the general picture of field influence is only quantitatively changed for heavy scalar particles. In any case, an increase in field strength makes the PT weaker, and baryogenesis does not survive due to the constant hypermagnetic field.

6. Discussion

In [2, 4], the influence of strong external hypermagnetic fields on the EW PT has been investigated in the tree approximation. A further study has to allow the radiation and correlation corrections. This is the problem that we have addressed to in the present paper. The main idea was to determine the form of the EP curve in the broken phase and find the range of the parameters H_Y, K when the EW PT is of 1st order. To elaborate that, the consistent EP including the one-loop and ring diagrams of all the fundamental particles has been calculated. As we have seen, the role of fermion and ring diagrams in the external field is crucial when the properties of the broken phase are investigated. The external field was taken into consideration exactly through Green's functions. For the field strengths considered, $H_Y \sim (0.01 \div 0.1)\, M_W^2$, the minima of the EP are found to be stable at sufficiently high temperatures when the PT happens. This important property is fulfilled even in case of crossover, and, hence, no conditions for *W*- and *Z*-boson condensates are realized at high temperatures. Our perturbative investigation shows that the baryogenesis condition does not hold in strong hypermagnetic fields. This conclusion disagrees with the results of [2, 4] where perturbative methods were used, and supports the conclusion of the nonperturbative computations in [5, 6].

The influence of strong fields on the vacuum at high temperature is a complicate corporative effect described by the total EP. At some chosen values of *H, T, K,* the different terms of it are dominant. To better understand the role of fermions in symmetry behaviour, we adduce two terms of the asymptotic expansion of the EP in the limit $T \to \infty, H \to \infty$. The first one is $\sim H^2 \log(T/m_f)$. Due to this term, light fermions are important at high temperature. The second term comes from the zero temperature part of the EP and looks as follows (for details see [17]): $\sim - eHm_f^2 \times \times \log(eH/m_f^2)$. It acts to make the Higgs particle 'heavier" in the field, and weakening of the 1st order PT happens.

In [5, 6], a nonperturbative investigation of the EW PT was carried out. It has been observed that, although an increase in H_Y makes the 1st order PT stronger, the end point of the 1st order PT line was determined for the masses $m_{\rm H} \geq (80 \div 90)$ GeV. Our perturbative results are partially in agreement with this observation. We have seen a crossover for heavy masses $m_{\rm H}$. We were not able to see an end point in our approach. On the other hand, a number of fermion H-dependent terms were not taken into consideration in [5, 6]. Most probably, this may find reflection in some additional limit on field strengths H_Y when that consideration is trusty.

To complete our analysis, we note that, with the radiation mass of the W-boson included, the spectrum is stable for the field strengths investigated $H \sim (0.01 - 0.1)\, M_W^2/e$ even for zero scalar condensate. This makes impossible the generation of the W- and Z-boson condensate. We belive that one can explain in this way why the vortex-line phase has not been observed in [5]. It follows from our estimates that the unstable mode appears again in the spectrum for stronger fields. Hence, one may expect to observe the lattice vacuum structure for the case of stronger external fields applied.

The authors thank M.Bordag and A.Linde for interesting discussions and remarks and A.Batrachenko for checking the numerical calculations.

1. *Enqvist K.*//Intern. J. Mod. Phys.¯ 1998.¯ **D7**.¯ P.331.
2. *Goivannini M., Shaposhnikov M.*//Phys. Rev. D.¯ 1998.¯ **57**.¯ P.2186.
3. *Rubakov V., Shaposhnikov M.*//Uspehi Fizicheskih Nauk. ¯ 1996.¯ **166**.¯ P.493.
4. *Elmfors P., Enqvist K., Kainulainen K.*// Phys. Lett.¯ 1998.¯ **B440**.¯ P.269.
5. *Kajantie K., Laine M., Peisa J., Rummukainen K., Shaposhnikov M.*//Nucl. Phys. B.¯ 1999.¯ **544**.¯ P.345.
6. *Laine M.*//Magnetic Fields and the EW Phase Transition.¯ 1999.- hep-ph/9902282.
7. *Csikor F., Fodor Z., Hegedus P. et al.*¯hep-ph/0001087.
8. *Skalozub V., Bordag M.*//Intern. J. Mod. Phys.¯ 2000.¯ **A15**.¯ P.349.
9. *Kajantie K., Laine M., Rummukainen K., Shaposhnikov M.*//Nucl. Phys. B.¯ 1996.¯ **458.**¯ P.90.
10. *Takahashi K.*//Z. Phys.¯ 1985.¯ **C26**.¯ P.601.
11. *Vshivtsev A., Zhukovsky V., Starinets A.*//Ibid.¯ 1994.¯ **C61**.¯ P.285.
12. *Skalozub V.V.*//Sov. J. Nucl. Phys.¯ 1987.¯ **45**.¯ P.1058.
13. *Cabo A.*//Forschr. Phys.¯ 1981.¯ **29**.¯ P.495.
14. *Reznikov Yu.Yu., Skalozub V.V.*//Sov. J. Nucl. Phys.¯ 1987.¯ **46**.¯ P.1085.
15. *Skalozub V.V., Vanyashin V.S.*//Fortchr. Phys.¯ 1992.¯ **40**.¯ P.739.
16. *Carrington M.*//Phys. Rev. D.¯ 1992.¯ **45**.¯ P.2933.
17. *Dittrich W., Reuter M.* Effective Lagrangians in Quantum Electrodynamics: Lecture Notes in Physics.¯ Springer-Verlag, 1985.¯ Vol. 220.
18. *Skalozub V., Strelchenko A.*//Physics of Atomic Nuclei ¯ 2000.¯ **11**.¯ **63**.¯ P.2048.
19. *Elmfors P., Perrson D.*//Nucl. Phys. B.¯ 1999.¯ **539**.¯ P.309.

Received 12.12.00