
ELEMENTARY PARTICLES IN A NEW QUANTUM SCHEME. 1

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Proceeding from the principles of the non-unitary quantum theory of relativistic bi-Hamiltonian systems [1], a system of Lagrangian fields characterized by a certain dispersion law (particle mass spectrum), interactions between fields, and their coupling constants are derived. The mass spectrum formula for "bare" fundamental hadrons is obtained and the *a priori* normalization of particle fields is found in the realistic model of a bi-Hamiltonian system. Numerical values of some dimensionless parameters of the present theory are determined.

1. Introduction

To construct an adequate theory of elementary particles, it is necessary to refuse geometric concepts of space-time and a differential topology on it [1] (in this respect, changes in particle theory should be not less radical than that in atomic theory). In particular, this remark concerns the field concept, which, of course, loses force at supersmall distances.

If the classical mechanics of a particle is connected with space and trajectories in it, so the quantum one is connected with a fiber bundle, being built over the space-time (base) and bundle sections – wave functions $\psi(X)$. Based on this observation, we see that a progress in fundamental physics is connected with the development and application of new and more perfect mathematical tools¹.

Now we have to switch to another paradigm: to a new concept of space-time structure at supersmall distances, where, as the research shows [1], space-time should be regarded as a discontinuum instead of a continuum and has to consider the dynamical structure of a fiber. This structure is a non-standard (bi-Hamiltonian) dynamical system. It is essential that this statement is not an axiom, but a theorem.

¹Let us add to this that before the creation of quantum theory, attempts to geometrize matter [2] (Clifford, Einstein) did not seem to be doomed. At present, many people consider this approach to be outdated or even erroneous. Most probably, apposite attempts should be made to derive space from matter (Leibniz).

Before, the action quantum h had a fundamental significance for the transition from trajectories to fields. But now a third constant k with dimensionality cm^{-1} has the fundamental significance for a transition from continuum to discontinuum and from the field system (elementary particles) to a relativistic bi-Hamiltonian system.

In the present article, it will be shown that one of the main assignments of this system is to create fundamental particle fields (see also [1]). Therefore, we reject composite models of particles, in particular, the quark-gluon one, because their objects – quarks, gluons, preons, etc. – are non-observable in space. Whereas composite models tried to answer the question: *what do fundamental particles consist of?*, the present theory answers another question: *how do fundamental particles arise?*

The purpose of this paper is to formulate the main laws responsible for creation of particles. Heisenberg already formulated the main task of an adequate particle theory [3]. It should explain: 1) the real observable mass spectrum of particles; 2) their symmetry properties; 3) the kinds and constants of interactions (particle charges).

The realization of this program was begun in [1] in the case of a nonrealistic model of the bi-Hamiltonian system based on the Heisenberg algebra $h_8^{(*)}$. Here, we consider a realistic model based on the algebra $h_{16}^{(*)}$.

1) In our approach, the mathematical apparatus of a new quantum theory, based on the extended Fock representation of the Heisenberg algebra $h_{16}^{(*)}$ advanced in [4], plays an important role. First of all, we consider the states of the relativistic bi-Hamiltonian system – fundor fields $f(x), \dot{f}(x')$ (or waves of space) called quanta f and \dot{f} . Following von Neumann's terminology [5], we call this part of the theory as non-unitary quantum theory I.

Then we investigate the amplitudes of the transition $f \rightarrow \dot{f}$ (quantum theory II), derive the equations which they satisfy, and establish the connection of bilocal fundamental particle fields $\psi^\Sigma(X, Y)$ with transition amplitudes $O^\Sigma(X, Y)$.

Considering ensembles of the states f and \dot{f} , we take into account the Gibbs distribution function in the relativistic Juttner form [6] (quantum theory III). Here, temperature parameters T_f and $T_{\dot{f}}$ (calculable in the theory) as well as their product $\mu^2 = T_f T_{\dot{f}}$ appear.

The existence of two branches of fundamental particles, baryons and mesons, and their mass formulae are essentially defined by the statistics of quanta f . To create leptons, the statistical properties of quanta f are not important. As a consequence, the lepton spectrum is much poor than the hadron one (see [1]). The existence of two families of particles, hadrons and leptons, is connected with two topological different transitions in the bi-Hamiltonian dynamical system [1]. Fields of fundamental hadrons appear with their *a priori* normalization (calculable in the theory) playing important role.

2) In the realistic model, the isotopic symmetry is represented by the group $U(2)$, see [4]. The Lorentz symmetry in both models ($h_8^{(*)}$ and $h_{16}^{(*)}$) is identical.

With this symmetry, the symmetry of the manifold of coordinates X is connected. It is very important to emphasize that the space-time continuum in the form of the Poincare space $\mathbf{A}_{3,1}$ is formed after the appearance of particle fields $\psi(X, Y)$ and switching on interactions. So, in spite of the usual point of view, we come to the idea of a space-time after a certain set of functions $\psi(X, Y)$ is constructed.

3) Particles arising from the quantum transition $f \rightarrow \dot{f}$ are called fundamental. When considering a continuous degeneration of states f , which is described in the model $h_{16}^{(*)}$ by the degeneration group $U_i(2) \otimes U_e(1) \otimes \dot{T}_{3,1}$, interactions and their fields are appeared as the parameters of the degeneration group (compare with the Yang - Mills theory [7]). In accordance with three kinds of degeneration, there are three kinds of interactions and three kinds of quanta of degenerate fields: strong ($U_i(2)$, first of all, these are η - and π -mesons), electromagnetic ($U_e(1)$, photons) and gravitational ($\dot{T}_{3,1}$, gravitons) ones (it is interesting to note that $U_e(1)$ is a special subgroup of $U_i(2)$; therefore, we can speak about unificated electro-strong interaction). In the present theory, weak interactions are distinguished by their nature from the above-mentioned ones (so the standard electro-weak model is invalid in the frame of the suggested theory) and, being given by correlations between various fibers

of the fiber bundle, of a completely different non-gauge kind [8].

Other kinds of particles (composite ones) are connected with correlations. The junction of $U_i(2)$ -multiplets from various fibers gives composite particles of higher symmetries $SU(3)$, $SU(4)$ (etc.)-multiplets (without quarks).

4) We would find the wrong (namely, Planck's) constant of electromagnetic interactions, if we would determine the Heisenberg algebra $h_{16}^{(*)}$ by the commutation relations

$$[\phi_{\alpha k}, \bar{\phi}_{\beta m}] = \delta_{\alpha\beta} \delta_{km}, [\phi_{\alpha k}, \phi_{\beta m}] = [\bar{\phi}_{\alpha k}, \bar{\phi}_{\beta m}] = 0. \tag{1}$$

More general relations are written in the form

$$[\phi_{\alpha k}, \bar{\phi}_{\beta m}] = \Lambda \delta_{\alpha\beta} \delta_{km} \tag{2}$$

(others are zero as well as in (1)), where Λ is a dimensionless constant (analogous to the Planck constant) which can impossibly be an arbitrary number and is connected with dimensionality of the Lie algebra of dynamical variables of our system, equal to 136, by the formula $\Lambda = \sqrt{136}$ (see [9]). So, in the theory, the "bare" electromagnetic charge of a particle is equal

$$\text{to } e = \sqrt{\frac{hc}{136}}.$$

2. Realistic model of Relativistic bi-Hamiltonian System

Generators of the Heisenberg algebra $h_{16}^{(*)}$, being canonical variables of our dynamical system, denoted by $\phi_{\alpha k}, \bar{\phi}_{\beta m}$ (here, $\alpha, \beta = 1, 2, 3, 4$ are Lorentz or Dirac indices, and $k, m = 1, 2$ are isotopic), obey the commutation relations (2).

1) At first, we describe briefly the structure of this system given by relations (1). Dynamical variables of the system represent every possible bilinear form of canonical variables: $\bar{\phi}_{\alpha k} \phi_{\beta m}, \phi_{\alpha k} \bar{\phi}_{\beta m}, \bar{\phi}_{\alpha k} \bar{\phi}_{\beta m}$. They form a Lie algebra of dynamical variables, denoted by d , that is isomorphic to the Cartan algebra $sp^{(*)}(8, \mathbf{C})$ of dimensionality 136. Real variables $\bar{\phi}_{\alpha k} \phi_{\beta m}$ which are conveniently written down using the Dirac matrices γ_N and isotopic Pauli matrices τ_k in the form $\bar{\phi} \gamma_N \tau_k \phi$ or

$$P_\mu = i \bar{\phi} \gamma_\mu P_+ \phi, \dot{P}_\mu = -i \bar{\phi} \gamma_\mu P_- \phi, I_{\mu\nu} = \bar{\phi} \Sigma_{\mu\nu} \phi, \tag{3}$$

$$A = -\bar{\phi} \phi - 4, B = -\bar{\phi} \gamma_5 \phi - 4,$$

$$\vec{i} = \frac{1}{2} \bar{\phi} \vec{\tau} \phi, \vec{k} = \frac{1}{2} \bar{\phi} \gamma_5 \vec{\tau} \phi, Q = \frac{1}{2} \bar{\phi} (1 + \tau_3) \phi$$

as well as $\bar{p}_\mu^\pm = \pm i\bar{\phi} \gamma_\mu P_\pm \bar{\tau} \bar{\phi}$ and $\bar{I}_{\mu\nu}^\pm = \bar{\phi} \Sigma_{\mu\nu} \bar{\tau} \bar{\phi}$ play the most important role. The physical sense of these variables follows from the commutation relations which they satisfy and is analogous to that of the corresponding variables in the model $h_8^{(*)}$, see [1]; in (3), Q is the operator of electric charge.

The extended Fock representation is given by the operators [1]

$$\phi = \begin{pmatrix} \partial/\partial\bar{\phi}_{\alpha k} \\ \phi_{\alpha k} \end{pmatrix}, \quad \bar{\phi} = \begin{pmatrix} \bar{\phi}_{\alpha k}, & -\frac{\partial}{\partial\phi_{\alpha k}} \end{pmatrix}, \quad \alpha, k = 1, 2, \quad (4)$$

and is constructed in the dual pair of topological vector spaces $(\hat{\mathbf{F}}, \mathbf{F})$, where \mathbf{F} (and $\hat{\mathbf{F}}$) has the known (see [1]) structure $\mathbf{F} = \bar{F}_F \otimes F_0$. Here, \bar{F}_F is a space of functions depending on the variables $\phi_{\alpha k}, \bar{\phi}_{\alpha k}$ (coordinates on the Lagrangian plane $L \subset h_{16}^{(*)}$), and F_0 is a space of functions depending on the additional variables $\phi_k \doteq \phi_{2k}, \bar{\phi}_k \doteq \bar{\phi}_{2k}$ (Lorentzian scalars, i.e., scalars of the group $GL_l(2, \mathbf{C})$ with generators $I_{\mu\nu}, A_l, B_l$), see [1], representing isospinors (spinors of the group $GL_l(2, \mathbf{C})$ with the generators $\vec{i}, \vec{k}, A_i, B_i$; concerning a splitting of operators A and B into A_l, A_i and B_l, B_i , see [1]).

2) The states of the system satisfy the equations [1]

$$-i \frac{\partial}{\partial x_\mu} f(x) = p_\mu f(x), \quad -i \frac{\partial}{\partial \dot{x}_\mu} \dot{f}(\dot{x}) = \dot{p}_\mu \dot{f}(\dot{x}). \quad (5)$$

As $p_\mu = \pi_\mu^{(1)} + \pi_\mu^{(2)}$ in this model, where $\pi_\mu^{(k)} = \bar{\phi}_k^+ \sigma_\mu \phi_k$ so we have $p_\mu^2 = 2\pi_\mu^{(1)}\pi_\mu^{(2)} = -4|\det \phi_{\alpha k}|^2 = -\kappa$ (as $\sum_{\mu=1}^4 (\pi_\mu^{(k)})^2 = 0$). From (5), we obtain $\left(p_\mu \frac{\partial}{\partial x_\mu} - 4i|\det \phi_{\alpha k}|^2\right)f = 0$. As the form $p_\mu \frac{\partial}{\partial x_\mu}$ is real, and $i|\det \phi_{\alpha k}|^2$ is strictly imaginary, the latter equation is equivalent to two equations:

$$p_\mu \frac{\partial}{\partial x_\mu} f = 0, \quad |\det \phi_{\alpha k}|^2 f = 0. \quad (6)$$

We cannot consider $\det \phi_{\alpha k} = 0$ since all $\phi_{\alpha k}$ are independent. Hence, in the model $h_{16}^{(*)}$ as opposed to $h_8^{(*)}$, there is no strong relation $p^2 = 0$. Using Dirac's terminology and notations [10], we say that equations (6) define a weak coupling $\det \phi_{\alpha k} \approx 0$ that only acts on states f or on the measures of initial states $d\mu_f$, see further (such states will be called quanta f).

The Lorentz-invariant measure $d\mu$ on the Lagrangian plane L has the form

$$d\mu = \prod_{\alpha, k=1,2} \frac{i}{4\pi} d\phi_{\alpha k} \wedge d\bar{\phi}_{\alpha k} \quad (7)$$

and can be expressed in terms of variables $\pi_\mu^{(k)}$ as follows:

$$d\mu = \frac{1}{16} \prod_{k=1,2} d\omega_k \delta(\pi^{(k)^2}) \theta(\pi_0^{(k)}) d^4\pi^{(k)} = \theta(\pi_0) \theta(-\pi^2) \delta(\pi^2 + \kappa) d^4\pi d^4\tilde{\nu} d\kappa, \quad (8)$$

where $\omega_k = \arg \phi_{2k}$ and

$$d^4\tilde{\nu} = \frac{1}{16} d\omega_1 d\omega_2 \delta(\Pi^2) \delta(\pi^2 - 2\pi\Pi) \times d^4\Pi \frac{\theta(\Pi_0)\theta(\pi_0 - \Pi_0)}{\theta(\pi_0)} \quad (9)$$

(here, we denote $p_\mu = \pi_\mu, \pi_\mu^{(2)} = \Pi_\mu$). According to (6), the measure $d\mu_f$ represents a contraction of the measure $d\mu$ on the "light" cone $\kappa = 0$ and is explicitly written in the form taking into account the weak coupling $\det \phi_{\alpha k} \approx 0$:

$$d\mu_f = \frac{1}{(2\pi)^{3/2}} \theta(\pi_0) \delta(\pi^2) d^4\pi d^4\nu, \quad (10)$$

where $d^4\nu = \left(\frac{2}{\pi}\right)^3 d^4\tilde{\nu}$. The measure $d\mu_f$ is normalized so that formula (13) (see below) is valid for coherent states. Thus, the measure $d^4\nu$ is normalized by the condition $\int d^4\nu = 1$ (see Appendix in Part 2).

A further plan of the theory building is the same as in the case of the $h_8^{(*)}$ -model (therefore, we give here only main formulae of the theory in the realistic model).

3) Quantum theory I. Solutions of equations (5) for $f(x)$ may be written in the form $f(x) = e^{ipx} f_0$, where $f_0 = f_0^\Sigma$ satisfies the equation

$$\hat{M}^2 f_0^\Sigma = F_\Sigma^0 f_0^\Sigma, \quad (11)$$

and are the polynomials $f_0^\Sigma = O^\Sigma(\phi_{\alpha k}, \bar{\phi}_{\beta m}; \phi_n, \bar{\phi}_r)$ of variables $\phi_{\alpha k}, \bar{\phi}_{\alpha k}, \phi_k, \bar{\phi}_k$ (compare with [1]). In (11), the operator $\hat{M}^2 = 2\hat{p}_\mu p_\mu$ and

$$F_\Sigma^0 = -(N+5)^2 - 7 + 4i(i+1) \quad (12)$$

are its eigenvalues on polynomials $O^\Sigma(\varphi)$ (see Appendix), which are named skeletons of fundamental particles characterized by quantum numbers Σ , see [1].

4) From the solutions of $f_0^\Sigma(x) = e^{i\pi x} O^\Sigma(\varphi)$, it is possible to construct coherent states - fields

$$O^\Sigma(x) = \int f_0^\Sigma(x) d\mu_f = \frac{1}{(2\pi)^{3/2}} \int e^{i\pi x} \theta(\pi_0) \delta(\pi^2) d^4\pi O^\Sigma(\pi), \tag{13}$$

where

$$O^\Sigma(\pi) = \int d^4v O^\Sigma(\varphi). \tag{14}$$

These (massless) fields exist on the group $T_{3,1}$, and should not be mixed up with Lagrangian fields given on the space-time $A_{3,1}$.

The differences between coherent fields and their secondary quantized Lagrangian analogs are: 1) the former are not quantized ($f_0^\Sigma(x)$ are c -number entities because they are characterized by only positive energies π_0 , see (10)), 2) under compression (collapse), they desintegrate into separate Fourier-components - non-Lagrangian fields $f^\Sigma(x)$, i.e., quanta f representing the solutions of Eqs. (5). In [1], this process is referred to as the phase transition 'particles \leftrightarrow quanta f '.

5) Quantum theory III. $f_0^\Sigma(x)$ and coherent fields (13) connected with them are the states of an isolated quantum f . In the compressed form when the irreversible quantum transitions $f \rightarrow \dot{f}$ take place (see further), they form an ensemble described by the Gibbs distribution function $w_f = \exp\left(-\frac{\overline{\Phi}_f}{T_f}\right)$, where T_f is the temperature of the ensemble of quanta f , which may be written in the relativistic Juttner form

$$w_f = \exp\left(-\frac{\pi p}{2\mu^2}\right) = \exp\left(-\frac{P^2}{4\mu^2}\right) \tag{15}$$

(see [1]). In the ensemble, the state of quanta f is written as

$$f^\Sigma = w_f f_0^\Sigma = e^{-\frac{P^2}{4\mu^2}} O^\Sigma(\varphi). \tag{16}$$

6) In the model $h_{16}^{(*)}$, the solutions of equations for \dot{f} are written in the form $\dot{f}(x) = e^{i\dot{p}x} \dot{f}_0$, where states \dot{f}_0 are determined as solutions of the stationary equations

$$\dot{p}_\mu \dot{f}_0 = \rho_\mu^{(0)} \dot{f}_0 \tag{17}$$

belonging to the space of additional variables F_0 . The latter (when they form an ensemble) are written as

$$\dot{f}_z = \frac{C}{Z} \exp(\bar{z}_k \Phi_k - \bar{\Phi}_k z_k) \tag{18}$$

(compare with [1]), where $z_k = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ are complex parameters representing an isospinor. It follows from (18) that $\rho_\mu^0 = -\bar{z}z(0, 0, 1, i)$ so that there is a weak coupling $p^2 \approx 0$ (in the model $h_8^{(*)}$, there was a strong coupling). Thus, as well as in the model $h_8^{(*)}$, ρ_μ , connected with ρ_μ^0 by an arbitrary Lorentz transformation (details see in [1]), takes values from the lower half of the light cone N_- which has the invariant measure $\frac{2}{\pi} \theta(-\rho_0) \delta(\rho^2) d^4\rho = d\mu_f$ (the measure of final states or quanta \dot{f}).

The measure connected with the vertex of the light cone is equal to $d\mu_0 = \delta^4(\rho) d^4\rho$, see [1]. In this case, parameters $z_k = 0$.

7) In (18), the Gibbs distribution function for quanta \dot{f}

$$w_{\dot{f}} = \exp\left(-\frac{\rho_0 + \mu_f}{T_f}\right) \approx \exp\left(-\frac{\mu_f}{T_f}\right) = \frac{1}{Z}, \tag{19}$$

in which μ_f , the chemical potential of quanta \dot{f} (see [1]), is taken into account.

8) Quantum theory II (amplitudes of the transition $f \rightarrow \dot{f}$). We definite now the transition matrix element

$$\langle \dot{f}(\dot{x}), f^\Sigma(x) \rangle = \int \overline{\dot{f}(\dot{x})} f^\Sigma(x) d\mu_f \tag{20}$$

and transition amplitude

$$O^\Sigma(X; Y) = \int d\mu_{f,0} \langle \dot{f}(X - Y), f^\Sigma(X + Y) \rangle = \langle \langle \dot{f}(X - Y), f^\Sigma(X + Y) \rangle \rangle, \tag{21}$$

where we denoted $X = \frac{1}{2}(x + \dot{x})$, $Y = \frac{1}{2}(x - \dot{x})$, see [1]. Thus, there are two topological different transitions: the transition from the upper half of the light cone (the measure $d\mu_f$) into the lower one (the measure $d\mu_{\dot{f}}$), that will be named hadronic, and the transition from the upper half into the vertex of the cone (the measure $d\mu_0$), that will be named leptonic. By the first transition, heavy particles (hadrons) are created, by the second - light particles (leptons).

9) It is not difficult to see that the amplitudes of lepton transitions differ only from the coherent fields

(13) by the factor $1/Z$, and are only non-zero if skeletons $O^\Sigma(\varphi) = P(\pi)$ or $O^\Sigma(\varphi) = \varphi_\alpha \bar{\varphi} P(\pi)$ where

$P(\pi)$ are polynomials of $\pi_\mu = \bar{\varphi} \bar{\sigma}_\mu \varphi$. The amplitudes appropriated for such skeletons are written as ($x = X + Y$)

$$\Psi^\Sigma(x) = P\left(-i \frac{\partial}{\partial x}\right) X^0(x), \quad \Psi^\Sigma(x) = P\left(-i \frac{\partial}{\partial x}\right) v(x), \quad (22)$$

where

$$X^0(x) = \frac{1}{(2\pi)^{3/2}} \frac{1}{Z} \int e^{i\pi x} \theta(\pi_0) \delta(\pi^2) d^4\pi, \\ v_\alpha(x) = \frac{1}{(2\pi)^{3/2}} \frac{1}{Z} \int e^{i\pi x} \theta(\pi_0) \delta(\pi^2) v_\alpha(\pi) d^4\pi, \quad (23)$$

and $v_\alpha(\pi) = \frac{1}{2} \begin{pmatrix} \pi_1 - i\pi_2 \\ \pi_0 - \pi_3 \end{pmatrix} = \bar{\pi} a$. Here, $\bar{\pi} = \bar{\sigma}_\mu \pi_\mu = \pi_0 + \bar{\sigma} \vec{\pi}$, and $a = \frac{1}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is a constant spinor. (22) are a special kind of bilocal fields representing gradient waves from fields $X^0(x)$, $v_\alpha(x)$. The latter result from a translation by the 4-vector Y_μ of a local field $\Psi^\Sigma(X)$ (and $\frac{\partial}{\partial X}$ herein they are specific): $\Psi^\Sigma(X+Y) = e^{Y \frac{\partial}{\partial X}} \Psi^\Sigma(X)$.

It is essential to remark that, in the model $h_{16}^{(*)}$, all coherent states are electroneutral, since the integration (see (14)) of charged skeletons or skeletons with hypercharge $Y \neq 0$ such as $\bar{\varphi} \bar{\tau} \varphi_\alpha$ or φ_α gives zero. This conclusion is rather essential for cosmology: before the first Big Bang (total transition $f \rightarrow \hat{f}$, see [11]) the Universe consisted of a mix of two neutral gases - bosons X^0 (spin 0) and fermions v (spin 1/2), see (23). The fields of these particles satisfy the equations:

$$\square X^0 = 0, \quad \bar{\sigma}_\mu \frac{\partial}{\partial x_\mu} v = 0.$$

10) Further, we discuss hadronic transitions. If to denote $\mathbf{P} = \pi - \rho$, $\mathbf{Q} = \pi + \rho$, the amplitudes of hadron transitions can be represented in the form (see [1])

$$O^\Sigma(X; Y) = \frac{C}{(2\pi)^{3/2}} \times \\ \times \int e^{i\mathbf{P}X + i\mathbf{Q}Y} \theta(P_0 + Q_0) \theta(P_0 - Q_0) \times$$

$$\times \delta(Q^2 + P^2) \delta(PQ) O^\Sigma(P; Q) d^4P \frac{d^4Q}{2\pi}, \quad (24)$$

where

$$O^\Sigma(P; Q) = \frac{1}{Z} \int d^4v O^\Sigma(\varphi) e^{-\frac{P^2}{4\mu^2} - 2i \text{Im} \bar{z}_k \varphi_k}, \quad (25)$$

(these magnitudes are calculated in Appendix 1 of Part 2).

11) These amplitudes satisfy the equation (see [1])

$$\square_X O^\Sigma(X; Y) = \langle \langle \hat{f}(X - Y), \hat{M}^2 f^\Sigma(X + Y) \rangle \rangle, \quad (26)$$

where $\hat{M}^2 = 2\hat{p}_\mu p_\mu$ is the operator introduced above.

Denoting $[\hat{M}^2, e^{ipx}] e^{-ipx} = \hat{T}(x)$, $\hat{T}^\Sigma(X; Y) = \langle \langle \hat{f}(X - Y), \hat{T}(X + Y) f^\Sigma(X + Y) \rangle \rangle$ and taking into account that

$$\hat{M}^2 f^\Sigma = F_\Sigma(-\mathbf{P}^2) f^\Sigma, \quad (27)$$

where $f^\Sigma = w_f O^\Sigma(\varphi)$ and

$$F_\Sigma(-\mathbf{P}^2) = F_\Sigma^0 + (N + 6) \frac{-\mathbf{P}^2}{\mu^2} - \frac{(-\mathbf{P}^2)^2}{4\mu^4}, \quad (28)$$

(see Appendix) where F_Σ^0 is defined by (12), we obtain the inhomogeneous equation for transition amplitudes [1]

$$(\square_X - F_\Sigma(\square_X)) O^\Sigma(X; Y) = \hat{T}^\Sigma(X; Y). \quad (29)$$

The latter can be written in the form of the Bopp equation (the left-hand side in (29) is a second-degree polynomial in \square_X)

$$(\square_X - M_{\Sigma B}^2)(\square_X - M_{\Sigma M}^2) O^\Sigma(X; Y) = 4\mu^4 \hat{T}^\Sigma(X; Y) \quad (30)$$

in which $\hat{T}^\Sigma(X; Y)$ plays the role of a source for transition amplitudes $O^\Sigma(X; Y)$.

12) Fields of Lagrangian particles. A solution of the inhomogeneous equation (29) can be written in the form of $O^\Sigma = \Psi^\Sigma + G^\Sigma$, where G^Σ is a partial solution of the inhomogeneous equation (29) and Ψ^Σ is a general solution of the homogeneous equation

$$(\square_X - M_{\Sigma B}^2)(\square_X - M_{\Sigma M}^2) \Psi^\Sigma(X; Y) = 0. \quad (31)$$

Solution of (31) have a physical sence, since they may be written as

$$\psi^\Sigma(X; Y) = \frac{1}{(2\pi)^{3/2}} \int e^{iP_X} \psi^\Sigma(P; Y) \delta(P^2 + M_\Sigma^2) d^4P. \tag{32}$$

We call $\psi^\Sigma(X; Y)$ bilocal Lagrangian fields. In the asymptotics $|X| \gg |Y|$, they go over to the usual local fields observable in the space $A_{3,1}$

$$\begin{aligned} \psi^\Sigma(X; 0) = \psi^\Sigma(X) &= \frac{1}{(2\pi)^{3/2}} \int e^{iP_X} \theta(P_0) \psi^\Sigma(P) \times \\ &\times \delta(P^2 + M_\Sigma^2) d^4P, \end{aligned} \tag{33}$$

where $\theta(P_0) \psi^\Sigma(P) = \psi^\Sigma(P; 0)$, which are characterized by the certain dispersion law $P^2 = -M_\Sigma^2$ (see further).

We require further the orthogonality condition of the solution G^Σ to the field ψ^Σ in the sense of the Stueckelberg scalar product (details see in [1])

$$\int \psi^\Sigma(X; Y) \overline{G^\Sigma(X; Y')} d^4X = 0. \tag{34}$$

This condition allows us to express the function $\psi^\Sigma(P; Y)$ in the general solution $\psi^\Sigma(X; Y)$ as

$$\begin{aligned} \psi^\Sigma(P; Y) &= \int e^{iQ_Y} \theta(P_0 + Q_0) \theta(P_0 - Q_0) \times \\ &\times \delta(P^2 + Q^2) \delta(PQ) O^\Sigma(P; Q) \frac{d^4Q}{2\pi} \end{aligned} \tag{35}$$

and, for the normalized constant C in (18), to obtain $C = \delta(0)$, see [1].

13) Mass spectrum. (29), (30) imply the mass spectrum formula for "bare" hadrons (see also Appendix):

$$\begin{aligned} M_\Sigma^2 &= 2\mu^2 \left(\frac{kh}{c} \right)^2 \left\{ N + 6 - \mu^2 + \right. \\ &\left. + (-1)^{F+1} \sqrt{\mu^4 - 2\mu^2(N+6) + 4i(i+1) + 2N+4} \right\} \end{aligned} \tag{36}$$

where i and N are the spin and isotonic quantum number of a skeleton $O^\Sigma(\varphi)$, and $\mu^2 = 3T_f T_{f'}$, where $T_{f'} = \frac{1}{3} \bar{z}_k z_k$. In (36), $F = 1$ for the barionic branch (B) and $F = 0$ for the mesonic branch (M).

Formula (36) results in a number of verified predictions. According to this formula, the isovector Σ -hyperon mass M_Σ is more than the isoscalar Λ -

hyperon mass $M_\Lambda (M_\Sigma > M_\Lambda)$, and the isovector ρ -meson mass M_ρ is less than the isoscalar ω -meson mass ($M_\rho < M_\omega$) that is in overall agreement with experimental results (see [12]) (for these particles, $N = 2$). Moreover, the ratio $M_\Lambda M_\omega / M_\Sigma M_\rho$ depends neither on the parameter μ^2 nor on the fundamental constants c, h, k , and is equal to the number $\sqrt{\frac{7}{6}} \approx 1.08$. The experimental value of this ratio is $0,76 \div 1,16$ (the inaccuracy is caused by the large width of ρ -mesons).

14) In the model $h_{16}^{(*)}$, hadron fields are presented in the form (32), (35), (25), where $O^\Sigma(\varphi)$ are the skeletons of particles in the given model. Calculations (see Appendix of Part 2) give the following factorized expression for $O^\Sigma(P; Q)$:

$$\begin{aligned} O^\Sigma(P; Q) &= \frac{1}{Z} N_\Sigma(X) O_\Sigma^{(i)}(z) O_\Sigma^{(l)}(P; Q), \\ X &= \sqrt{-P^2}, \end{aligned} \tag{37}$$

where $O_\Sigma^{(l)}$ and $O_\Sigma^{(i)}$ are Lorentzian and isotopic wave functions. In particular, for the baryon octet, the skeletons are

$$\begin{aligned} O^N(\varphi) &= \varphi_{\alpha k}, \quad O^\Lambda(\varphi) = \varphi_{\alpha k} \bar{\varphi}_k, \quad O^\Sigma(\varphi) = \bar{\varphi} \tau \bar{\varphi}_\alpha, \\ O^\Xi(\varphi) &= \frac{2}{\sqrt{-P^2}} (\bar{\varphi} \varphi_\alpha) (\bar{\varphi}_k \bar{\rho} \varphi_m) \bar{\varphi}_m \end{aligned} \tag{38}$$

(the quantum numbers N for nucleons N -, Λ -, Σ - and Ξ -skeletons are equal to 1, 2, 2, 5, respectively), the isotopic factors are of the form

$$O_N^{(i)} = z_k, \quad O_\Lambda^{(i)} = 1, \quad O_\Sigma^{(i)} = \frac{\bar{z} \tau \bar{z}}{\bar{z} z}, \quad O_\Xi^{(i)} = \frac{\bar{z}_k}{\bar{z} z}. \tag{39}$$

Lorentzian factors for these particles (spin 1/2) are $O_\Sigma^{(l)} = \bar{\pi} a$ (a is a constant spinor, see above). In these cases, we have

$$\begin{aligned} N_N &= \left(\frac{2}{M_N} \right)^2 I_2(M_N) e^{-\frac{M_N^2}{4\mu^2}}, \quad N_\Lambda = \frac{2}{M_\Lambda} I_1(M_\Lambda) e^{-\frac{M_\Lambda^2}{4\mu^2}}, \\ N_\Sigma &= -\frac{2}{M_\Sigma} I_3(M_\Sigma) e^{-\frac{M_\Sigma^2}{4\mu^2}}, \quad N_\Xi = I_2(M_\Xi) e^{-\frac{M_\Xi^2}{4\mu^2}} \end{aligned} \tag{40}$$

where I_n is the Bessel function.

Similar expressions can be obtained for the vector octet of mesons as well as for other multiplets.

The factor $O_{\Sigma}^{(i)} N_{\Sigma}(M_{\Sigma})$ contains a very important information about the mechanism of particle creation: it defines the a priori probability of creation of a hadron Σ in the quantum transition $f^{\Sigma} \rightarrow f_z$ (in the nonunitary theory, the state f^{Σ} is sloped in relation to f_z , and the angle between these states defines the given probability). The mentioned probability W_{Σ} is defined as $|O_{\Sigma}^{(i)} N_{\Sigma}|^2$. Obviously, the total probability as the sum of partial probabilities W_{Σ} must be equal to one:

$$\sum_{\Sigma} W_{\Sigma} = 1. \tag{41}$$

3. A Fiber as a Sea of Additional Variables

1) Formula (41) results from the normalization condition of skeletons $O^{\Sigma}(\varphi)$. Due to the 100 per cent baryon-antibaryon asymmetry of the considered particle creation mechanism (see [4]), the skeletons are constructed (in the Lorentzian system connected with the space F_0) exclusively from additional variables φ_k . The basis of the irreducible finite-dimensional representation Σ of the group $G_{M^2}^{\wedge} = GL_r(2, \mathbf{C}) \otimes U_i(2) \otimes H_i(1)$ (see [1]; on the space F_0 , this group is reduced to the $U_i(2) \otimes H_i(1)$, since φ_k are Lorentzian scalars) is written in the form [13]

$$O^{\Sigma}(\varphi) = \frac{\varphi_1^m \varphi_2^{2i-m}}{\sqrt{m!(2i-m)!}} = f_m^{(i)}, \tag{42}$$

where i is the isospin, and m is its projection so that

$$\sum_{m=0}^{2i} |f_m^{(i)}|^2 = \frac{(|\varphi_1|^2 + |\varphi_2|^2)^{2i}}{(2i)!} = \frac{(\overline{\varphi\varphi})^{2i}}{(2i)!} = w_i.$$

As can be seen, the value m satisfies the Bernoulli distribution for a fixed i . The isospin i satisfies the Poisson distribution since

$$\sum_{i=0, 1/2, 1, \dots} w_i e^{-\overline{\varphi\varphi}} = 1. \tag{43}$$

As φ accepts small values (small oscillations, see below), it is possible to put

$$\sum_i w_i = 1. \tag{44}$$

So, if a bi-Hamiltonian fiber is imagined as a sea of coupled additional variables $\overline{\varphi\varphi}$, the configuration $(\overline{\varphi\varphi})^n$ arises in it with the probability given by the

Poisson distribution

$$P(n) = \frac{(\overline{\varphi\varphi})^n}{n!} e^{-\overline{\varphi\varphi}}.$$

It is natural to consider the functions $u_m^{(i)} = \exp\left(-\frac{\overline{\varphi\varphi}}{2}\right) f_m^{(i)}$ forming an orthonormal system of functions in the space F_0 with the scalar product $(f, g) = \int \frac{\overline{f(\varphi)g(\varphi)} d\mu(\varphi)}{c^2}$

and with the measure

$$d\mu(\varphi) = \prod_{k=1,2} \frac{i}{4\pi} d\varphi_k \wedge d\overline{\varphi}_k$$

(in fact, $(u_m^{(i)}, u_{m'}^{(i')}) = \delta_{ii'} \delta_{mm'}$), for which the condition

$$\sum_{i=0, 1/2, 1, \dots} \sum_{m=0}^{2i} |u_m^{(i)}|^2 = \sum_i e^{-\overline{\varphi\varphi}} w_i = 1$$

is satisfied.

Let us return to condition (44) which took place before the transition $f \rightarrow \dot{f}$. After the transition $f \rightarrow \dot{f}$ and the creation of fundamental hadron fields, this normalization condition for skeletons passes to condition (41).

We now show that the exponent in (43) can be put to equal 1, so $|\varphi|$ is a small value. Both the Bernoulli and Poisson distributions are realized in every fiber. Another distribution used by us, the Gibbs distribution $\exp\left(\frac{-\overline{\varphi\varphi}}{T_f}\right)$, describes the statistics of fibers. It follows from this that the values $\overline{\varphi\varphi} \sim T_f$ as well as $|\varphi| \sim \sqrt{T_f}$. We now see that $T_f \sim 10^{-6}$ so that the value $|\varphi| \sim 10^{-3}$ is really small.

2) Dimensionless parameters of the theory, such as T_f, z_k (or the sum $T_{\dot{f}} = \frac{1}{3}(|z_1|^2 + |z_2|^2)$ and ratio $\varepsilon = z_1/z_2$) and $1/Z$, are necessary to perform calculations in the framework of the present theory. The factor $1/Z$ was calculated in [8]. Here, the parameters $T_f, T_{\dot{f}}$, and ε will be found.

At first, we define the parameter $\mu^2 = 3T_f T_{\dot{f}}$. It is used in formula (36) defining the mass spectrum of "bare" (non-interacting) hadrons and is found from conditions (generally named the minimum principle; this principle is the mathematical formulation of the Leibnizian statement: our World is the best of worlds) put on factors of the quadratic form $P_{\Sigma}(X) = (X - M_{\Sigma B}^2)(X - M_{\Sigma M}^2)$. Denote by $P_{\Sigma_0}(X)$ the form for which a baryon root $M_{\Sigma B}$ has the least value.

This value corresponds to the point with $i_0 = -\frac{1}{2}$ and $N_0 = -1$ and is equal to the $M_{\Sigma_0 B}^2 = 2\mu^2\{5 - \mu^2 + \sqrt{\mu^4 - 10\mu^2 + 1}\}$. The form $P_{\Sigma_0}(X)$ accepts the least value at $X = \frac{1}{2}(M_{\Sigma_0 B}^2 + M_{\Sigma_0 M}^2)$. This value is equal to $-\frac{1}{4}(M_{\Sigma_0 B}^2 - M_{\Sigma_0 M}^2)^2 = -4\mu^4(\mu^4 - 10\mu^2 + 1)$. The latter expression as a function of the parameter μ^2 has a minimum at $\mu^2 = \frac{15}{4} \times \left(1 - \sqrt{1 - \frac{8}{225}}\right) \approx 0.067$ (determined by the equation $2\mu^4 - 15\mu^2 + 1 = 0$).

The other parameter $\bar{z}_k z_k$ enters condition (41). Since a proton and a neutron take on the greatest probability in sum (41) (herein, it is not difficult to be convinced looking at formulae (39), (40)), condition (41) with large accuracy is written in the form

$$\bar{z}z \left(\frac{2}{M_N}\right)^4 I_2^2(M_N) e^{-\frac{M_N^2}{2\mu^2}} \approx 1.$$

If M_N takes the 'bare' value $M_N = 1.14$ given by formula (36) at $\mu^2 = 0.067 \left(N = 1, i = \frac{1}{2}\right)$. For $\bar{z}z$, we obtain $\bar{z}z = 0.86 \cdot 10^5$. Thus, $T_f = 0,3 \cdot 10^5$. Then, from $T_f = \frac{\mu^2}{3T_f}$, we obtain $T_f = 0,78 \cdot 10^{-6}$. Another dimensionless parameter $\eta = \frac{3T_f}{T_f}$, playing an important role in cosmology, is equal to $\eta = 10^{11}$. The parameter ε will be determined in the Part 2 of this paper.

APPENDIX

In the model $h_{16}^{(*)}$, the operator \hat{M}^2 is other than that in model $h_8^{(*)}$ (see [1]), as here $p_\mu = \bar{\varphi}_{\alpha k}^+ (\sigma_\mu^+)^{\alpha\beta} \varphi_{\beta k}$, $\dot{p}_\mu = -\partial_k^\beta (\bar{\sigma}_\mu)_{\beta\alpha} \bar{\partial}_k^\alpha \left(\partial_k^\alpha = \frac{\partial}{\partial \varphi_{\alpha k}}\right)$. Using the completeness condition for the matrices $\sigma_\mu^+ : \sum_{\mu=1}^4 \left(\sigma_\mu^+\right)^{\alpha\beta} (\bar{\sigma}_\mu)_{\gamma\delta} = 2\delta_\delta^\alpha \delta_\gamma^\beta$, we write

$$\hat{M}^2 = 2\dot{p}_\mu p_\mu = -4\partial_k^\alpha \bar{\partial}_k^\beta \bar{\varphi}_{\beta m} \varphi_{\alpha m}$$

and, using the completeness condition for the matrices $\tau_a^\pm = (\bar{\tau}^\pm \pm 1)$:

$\Sigma_a(\tau_a^+)_{km} (\bar{\tau}_a)_{np} = 2\delta_{kp} \delta_{nm}$, we have (here, τ^T is a transposed matrix)

$$\hat{M}^2 = -2\partial_a^+ \varphi \bar{\partial} \tau_a^T \bar{\varphi} = -2\partial_a^+ \varphi (\bar{\varphi} \tau_a \bar{\partial} + 4\delta_{a0}) = -2[\partial \bar{\tau} \bar{\varphi} \bar{\partial} + (\partial \varphi)(\bar{\varphi} \bar{\partial}) + 4\partial \varphi].$$

Using then the definitions $\hat{N} = \varphi \partial + \bar{\varphi} \bar{\partial}$, $\hat{Y} = \varphi \partial - \bar{\varphi} \bar{\partial}$, $\hat{i}^2 = -\frac{1}{2}(\varphi \bar{\tau}^T \partial - \bar{\varphi} \bar{\tau} \bar{\partial})$, $\hat{k}^2 = -\frac{1}{2}(\varphi \bar{\tau}^T \partial + \bar{\varphi} \bar{\tau} \bar{\partial})$, the expression for \hat{M}^2 may be written in the form

$$\hat{M}^2 = -4\left[\frac{1}{2}(\hat{k}^2 - \hat{i}^2) + \frac{1}{8}(\hat{N}^2 - \hat{Y}^2) + 2\hat{N} + 8\right].$$

Essentially, in this case, the operator \hat{M}^2 can only be expressed in terms of generators of the isotopic group $SL_\mu(2, \mathbf{C}) \otimes U_i(1) \otimes H_i(1)$; in so doing, generators of the Lorentz group $SL_1(2, \mathbf{C}) \otimes U_1(1) \otimes H_1(1)$ do not occur. As a consequence of this circumstance, the operator \hat{M}^2 has the same expression on both the whole space \mathbf{F} and subspace \mathbf{F}_0 . Therefore, we can consider a further operator \hat{M}^2 on the space \mathbf{F}_0 or (that is equivalent) to pass into a Lorentzian system such that $\varphi_{1k} = 0$.

The expression for \hat{M}^2 can be developed into

$$\hat{M}^2 = 4\hat{i}^2 - \hat{N}^2 - 10\hat{N} - 32 - 2\left(\Delta - \frac{1}{4}\hat{Y}^2 - \frac{1}{4}\hat{N}^2 - \hat{N}\right), \tag{45}$$

where $\Delta = \hat{i}^2 + \hat{k}^2$ is a Casimir operator of the algebra $sl_i(2, \mathbf{C})$. On \mathbf{F}_0 , we have $(\varphi_k = \varphi_{2k}$ are additional variables, $\partial_k = \frac{\partial}{\partial \varphi_k}$)

$$\Delta - \frac{1}{4}\hat{Y}^2 - \frac{1}{4}\hat{N}^2 = \frac{1}{2}[(\varphi \bar{\tau}^T \partial)^2 + (\bar{\varphi} \bar{\tau} \bar{\partial})^2] - \frac{1}{2}[(\varphi \partial)^2 + (\bar{\varphi} \bar{\partial})^2] = \frac{1}{2} \sum_{a=1}^4 [\varphi \tau_a^T \partial \varphi \tau_a^T \partial + \bar{\varphi} \tau_a \bar{\partial} \bar{\varphi} \tau_a \bar{\partial}].$$

Using the completeness condition for the matrices τ_a^\pm :

$$\sum_{a=1}^4 (\tau_a^+)_{km} (\tau_a^+)_{np} = 2\tau_{kn} \tau_{pm}$$

where $\tau = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, we obtain

$$\Delta - \frac{1}{4}\hat{Y}^2 - \frac{1}{4}\hat{N}^2 = (\varphi \tau)_k \partial_m \varphi_k (\partial \tau)_m + (\bar{\varphi} \tau)_k \bar{\partial}_m \bar{\varphi}_k (\bar{\partial} \tau)_m = \varphi \tau \partial \tau \bar{\partial} + \bar{\varphi} \tau \bar{\partial} \tau \bar{\partial} + \varphi \partial + \bar{\varphi} \bar{\partial}.$$

As $\varphi \tau \varphi = 0$, we have

$$\Delta - \frac{1}{4}\hat{Y}^2 - \frac{1}{4}\hat{N}^2 = \hat{N}.$$

Hence, the expression in the parentheses in (45) is equal to zero. Actually,

$$\hat{M}^2 = 4\hat{i}^2 - \hat{N}^2 - 10\hat{N} - 32.$$

Now it is not difficult to find the expression for $F_{\Sigma}(-\mathbf{P}^2)$ in the equation $\hat{M}^2 f^{\Sigma} = F_{\Sigma}(-\mathbf{P}^2) f^{\Sigma}$, where $f^{\Sigma} = w O^{\Sigma}$ (O^{Σ} is the skeleton, and w is the Gibbs distribution function). In the coordinate representation, we have

$$F_{\Sigma}(\square) = F_{\Sigma}^0 + \frac{N+6}{\mu^2} \square - \square^2/(4\mu^4), \quad (46)$$

where $F_{\Sigma}^0 = -(N+5)^2 - 7 + 4i(i+1)$, and N and i are the isotonic number and isospin of the skeleton O^{Σ} , respectively.

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