MAGNETIZATION PROCESSES IN QUANTUM SPIN CHAINS WITH REGULARLY ALTERNATING INTERSITE INTERACTIONS

O. DERZHKO

Institute for Condensed Matter Physics
(1, Svientsitskii Str., Lviv 79011, Ukraine)

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We consider the dependence of magnetization on field at zero temperature for spin-$\frac{1}{2}$ chains in which intersite interactions regularly vary from site to site with period p. In the limiting case, where the smallest value of the intersite interactions tends to zero, the chain splits into noninteracting identical fragments of p sites and the dependence of magnetization on field can be examined rigorously. We demonstrate explicitly the appearance of plateaus in such a dependence and discuss the presence of the magnetization values m predicted by the condition $p\left(\frac{1}{2}-m\right)=$ integer [1]. We comment on the influence of an anisotropy in the interspin interaction on the magnetization profiles. Finally, we show how the case of a nonzero smallest value of the intersite interactions can be considered.

The theoretical study of (quantum) spin chains attracts much attention during last years. On the one hand, a number of quasi-one-dimensional magnetic compounds, the properties of which can be reasonably described by one-dimensional quantum spin models, becomes available. On the other hand, quantum spin chains should exhibit various interesting properties, whose examining is of great importance from the academic point of view. Thus, the analysis of magnetization at low temperatures may yield a step-like dependence 'magnetization vs field“. The latter problem has received a lot of interest in a numerous theoretical, numerical, and experimental papers concerning a variety of spin chains and ladders [1 – 3].

In what follows, we discuss one mechanism which generates a step-like dependence 'magnetization vs field“, that is a regular alternation of intersite interactions. Namely, we consider a chain of $N\to\infty$ spins $\frac{1}{2}$ governed by the Hamiltonian

$$H=-h\sum_{n=1}^{N}s_n^z+\sum_{n=1}^{N}(J_n^x s_n^x s_{n+1}^x+J_n^y s_n^y s_{n+1}^y+$$

$$+J_n^z s_n^z s_{n+1}^z), \qquad (1)$$

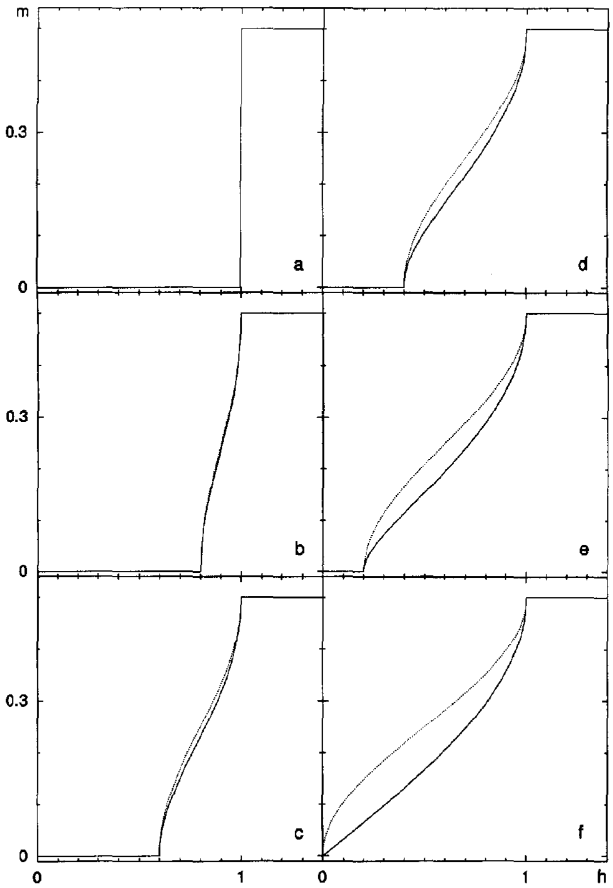
assuming that the intersite (antiferromagnetic) interactions $J_n^\alpha(\geq 0)$ vary regularly from site to site with period p, i.e., a sequence of parameters is $\{J_1^x, J_1^y, J_1^z, \ldots, J_p^x, J_p^y, J_p^z, J_1^x, J_1^y, J_1^z, \ldots\}$. If $J_n^x=$ $=J_n^y=J_n^z=J_n$, Eq. (1) is the Hamiltonian of an isotropic Heisenberg (XXX) chain. But if J_n^x, $J_n^y\neq 0$, and $J_n^z=0$, Eq. (1) corresponds to an anisotropic XY chain. For the latter chain, one can distinguish two limiting cases, namely, i) $J_n^x=J_n^y=J_n$ – an isotropic XY (XX) chain and ii) $J_n^x=J_n$, $J_n^y=0$ – an Ising chain. In addition, Hamiltonian (1) contains a uniform external field h directed along the z axis (which is called a transverse field for XY chains).

We are interested in the magnetization per site $m=\frac{1}{N}\sum_{n=1}^{N}\langle s_n^z\rangle$ (the angle brackets denote the thermodynamical canonical average) or more precisely in the dependence of m on h at zero temperature. Oshikawa et al. [1], by using the Lieb – Schultz – Mattis theorem and bosonization techniques for a general quantum spin s chain with axial symmetry, argued that the magnetization obeys the relation $p(s-m)=$ integer, i.e., in the case $s=\frac{1}{2}$, the posssible values of magnetization are $m=\frac{p-2k}{2p}$, $k=0, 1, 2, \ldots, p$. A lot of work was done to check the above-mentioned criteria using various approximate analytic approaches and numerical techniques. On the other hand, for the XX chain in a transverse field, the magnetization m can be calculated rigorously. The result at zero temperature reads

$$m=\frac{1}{2}\int_{-\infty}^{\infty}dE\,\rho(E)\,(2\,\theta(E)-1), \qquad (2)$$

where $\rho(E)$ is the density of the states of Jordan – Wigner fermions which is known explicitly for any finite period p [4] and $\theta(E)$ is the Heaviside step function. Regular nonuniformity leads to a splitting of the fermion band of a uniform chain into several subbands that, in turn, immediately leads to plateaus in the dependence 'm vs h" as follows from (2). The values of the characteristic fields at which plateaus start and end up are the solutions of two algebraic equations of the pth order, whereas the possible values of m are connected with the possible differences in the numbers of subbands at $E < 0$ and at $E > 0$ (see (2)). One more related work concerning a regularly alternating transverse Ising chain was reported in [5]. Contrary to a transverse XX chain, the regular alternation of exchange interactions for a transverse Ising chain does not lead to plateaus in the dependence ' m vs h".

In what follows, we consider in some detail a particular case of the regularly alternating spin-$\frac{1}{2}$ chain (1) when the smallest value of the intersite interactions equals zero. Without a loss of generality, we may put $J_p^{\alpha} = 0$. In such a limiting case, a simple picture for explanation of the zero temperature magnetization profiles emerges. Really, in this limit, the chain consists of noninteracting clusters, every one of which contains p sites. The magnetization of the chain per site m follows from the magnetization of the cluster M_p after dividing by p, and $M_p = \langle \mathbf{GS} \mid S_p \mid \mathbf{GS} \rangle$, where $S_p = s_1^z + \ldots + s_p^z$ and $\mid \mathbf{GS} \rangle$ is the ground state eigenvector of the cluster Hamiltonian. The appearance of plateaus is due to a change of the ground state with varying the field. Such a viewpoint is known as the strong-coupling approach. It was exploited in a number of papers devoted to the spin chains with a periodic modulation of the intersite interactions and the spin ladders [3]. We discuss the strong-coupling limit to get a better understanding of the obtained earlier rigorous results for the transverse XX and transverse Ising chains by means of the continued fraction approach [4, 5] as well as to discuss the influence of an anisotropy in the spin interaction on the zero temperature magnetization profiles. We explain the difference between magnetization curves for the transverse XX chain and the transverse Ising chain (Eqs. (4), (5), and Eq. (11)) and show a possibility of disappearance of the plateau at $m = 0$ for the XX chain of period 4 (discussion after Eq. (10)). We obtain the approximate Hamiltonian, which can describe the magnetization processes in the regularly alternating XX chain of period 2 (Eq. (21), $h \geq 0$) if the smaller interaction has a nonzero value, and compare the approximate results with the exact ones which are available for the chain under consideration (Figure). Demonstrating how does the strong-coupling approach work in the exactly solvable case, we can reveal a region of validity of this approximate method.



m vs h for a spin-$\frac{1}{2}$ transverse XX chain of period 2 with $J_1 = 1 + \delta$, $J_2 = 1 - \delta$, $\delta = 1$ (a), $\delta = 0.8$ (b), $\delta = 0.6$ (c), $\delta = 0.4$ (d), $\delta = 0.2$ (e), $\delta = 0$ (f) at zero temperature. Solid curves correspond to the exact results following from (2) [4], and dotted curves are obtained with the help of the approximate Hamiltonian (21)

To close the introductory part, let us remark that the strong-coupling approach, besides regularly alternating spin chains, appears to be an appropriate tool for a study of molecular magnets. Molecular magnets consist of macroscopic molecules each of which is described by a cluster spin Hamiltonian (of 6, 8, 10, 12 sites, however, usually with $s = \frac{5}{2}$) similar to the ones considered below (for details see, for example, [6]).

We start with the regularly alternating XX chain with $p = 2$. Asuming $J_2 = 0$, one splits the chain into noninteracting clusters containing two sites. The Hamiltonian of a cluster reads

$$H_2 = -h\,(s_1^z + s_2^z) + J_1\,(s_1^x s_2^x + s_1^y s_2^y). \tag{3}$$

The eigenvalues of (3) are

$$E_1 = -\frac{1}{2}J_1, \quad E_2 = -h, \quad E_3 = h, \quad E_4 = \frac{1}{2}J_1; \qquad (4)$$

the corresponding eigenvectors are

$$|1\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle), \quad |2\rangle = |\uparrow_1\uparrow_2\rangle,$$

$$|3\rangle = |\downarrow_1\downarrow_2\rangle, \quad |4\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle + |\downarrow_1\uparrow_2\rangle). \qquad (5)$$

Moreover, as follows from (5), $\langle 1|S_2|1\rangle = 0$, $\langle 2|S_2|2\rangle = 1$, $\langle 3|S_2|3\rangle = -1$, $\langle 4|S_2|4\rangle = 0$. For $0 \le h < \frac{1}{2}J_1$, one concludes from (4) that $|\mathbf{GS}\rangle = |1\rangle$ and therefore $M_2 = 0$. For $\frac{1}{2}J_1 < h$, one finds that $|\mathbf{GS}\rangle = |2\rangle$ and therefore $M_2 = 1$. Similarly, the case $h \le 0$ can be considered. As a result, one deduces that the magnetization curve m vs h should exhibit a plateau at $m = 0$ (if $-\frac{1}{2}J_1 < h < \frac{1}{2}J_1$) and at $m = \pm\frac{1}{2}$ (if $h > \frac{1}{2}J_1$ and $h < -\frac{1}{2}J_1$).

Let us consider further the case $p = 3$. The cluster Hamiltonian is as follows:

$$H_3 = -h(s_1^z + s_2^z + s_3^z) + J_1(s_1^x s_2^x + s_1^y s_2^y) +$$

$$+ J_2(s_2^x s_3^x + s_2^y s_3^y). \qquad (6)$$

The eigenvalues of Hamiltonian (6) are as follows:

$$E_1 = -\frac{1}{2}h - \frac{1}{2}\sqrt{J_1^2 + J_2^2}, \quad E_2 = \frac{1}{2}h - \frac{1}{2}\sqrt{J_1^2 + J_2^2},$$

$$E_3 = -\frac{3}{2}h, \quad E_4 = -\frac{1}{2}h, \quad E_5 = -E_4,$$

$$E_6 = -E_3, \quad E_7 = -E_2, \quad E_8 = -E_1. \qquad (7)$$

Moreover, $\langle 3|S_3|3\rangle = \frac{3}{2}$, $\langle 1|S_3|1\rangle = \langle 4|S_3|4\rangle = \langle 7|S_3|7\rangle = \frac{1}{2}$, $\langle 2|S_3|2\rangle = \langle 5|S_3|5\rangle = \langle 8|S_3|8\rangle = -\frac{1}{2}$, $\langle 6|S_3|6\rangle = -\frac{3}{2}$. Therefore, for $0 < h < \frac{1}{2}\sqrt{J_1^2 + J_2^2}$ (since $|\mathbf{GS}\rangle = |1\rangle$ (see (7))), one finds $M_3 = \frac{1}{2}$ and $m = \frac{1}{6}$, whereas one finds $M_3 = \frac{3}{2}$ and $m = \frac{1}{2}$ for $\frac{1}{2}\sqrt{J_1^2 + J_2^2} < h$ (since $|\mathbf{GS}\rangle = |3\rangle$). As a result, one concludes that the dependence m vs h exhibits plateaus at $m = \frac{1}{2}$ (if $\frac{1}{2}\sqrt{J_1^2 + J_2^2} < h$), at $m = \frac{1}{6}$ (if $0 < h < \frac{1}{2}\sqrt{J_1^2 + J_2^2}$), at $m = -\frac{1}{6}$ (if $-\frac{1}{2}\sqrt{J_1^2 + J_2^2} < h < 0$), and at $m = -\frac{1}{2}$ (if $h < -\frac{1}{2}\sqrt{J_1^2 + J_2^2}$).

It is interesting to note that the low-lying levels of Hamiltonians (3) and (6) relevant for zero temperature magnetization follow from the following Hamiltonians:

$$H_2 = -hS^z + \frac{1}{2}J_1((S^z)^2 - 1), \quad S^z = \{\pm 1, 0\} \qquad (8)$$

and

$$H_3 = -hS^z + \frac{1}{4}\sqrt{J_1^2 + J_2^2}\left((S^z)^2 - \frac{9}{4}\right),$$

$$S^z = \left\{\pm\frac{3}{2}, \pm\frac{1}{2}\right\}, \qquad (9)$$

respectively. The appearance of plateaus becomes evident from (8) (or (9)) since, e.g., at small $h > 0$, spins are fixed to $S^z = 0$ (or to $S^z = \frac{1}{2}$) and they should be in the $S^z = \frac{3}{2}$ (or $S^z = 1$) state for large $h > 0$.

We pass to the case $p = 4$. The eigenvalues of the cluster Hamiltonian are as follows:

$$E_1 = -\frac{1}{2}\sqrt{J_2^2 + (J_1 + J_3)^2} = -E_{16},$$

$$E_2 = -h - \frac{1}{2\sqrt{2}}(J_1^2 + J_2^2 + J_3^2 + (J_1^4 + J_2^4 + J_3^4 -$$

$$-2J_1^2J_3^3 + 2J_1^2J_2^2 + 2J_2^2J_3^2)^{1/2})^{1/2} = -E_{15},$$

$$E_3 = h - \frac{1}{2\sqrt{2}}(J_1^2 + J_2^2 + J_3^2 + (J_1^4 + J_2^4 + J_3^4 -$$

$$-2J_1^2J_3^3 + 2J_1^2J_2^2 + 2J_2^2J_3^2)^{1/2})^{1/2} = -E_{14},$$

$$E_4 = -\frac{1}{2}\sqrt{J_2^2 + (J_1 - J_3)^2} = -E_{13},$$

$$E_5 = h - \frac{1}{2\sqrt{2}}(J_1^2 + J_2^2 + J_3^2 - (J_1^4 + J_2^4 + J_3^4 -$$

$- 2J_1^2 J_3^3 + 2J_1^2 J_2^2 + 2J_2^2 J_3^2)^{1/2})^{1/2} = -E_{12},$

$E_6 = h - \frac{1}{2\sqrt{2}}(J_1^2 + J_2^2 + J_3^2 - (J_1^4 + J_2^4 + J_3^4 -$

$- 2J_1^2 J_3^3 + 2J_1^2 J_2^2 + 2J_2^2 J_3^2)^{1/2})^{1/2} = -E_{11},$

$E_7 = -2h = -E_{10},$

$$E_8 = 0 = E_9. \quad (10)$$

Usually, a chain with $p = 4$ (e.g., if $J_1 = J_2 = J_3 = J$) exhibits plateaus at $m = 0$ for $-h_1 < h < h_1$ when $|\mathbf{GS}\rangle = |1\rangle$, $\langle 1|S_4|1\rangle = 0$, at $m = \frac{1}{4}$ $(m = -\frac{1}{4})$ for $h_1 < h < h_2$ $(-h_2 < h < -h_1)$ when $|\mathbf{GS}\rangle = |2\rangle$, $\langle 2|S_4|2\rangle = 1$ $(|\mathbf{GS}\rangle = |3\rangle$, $\langle 3|S_4|3\rangle = -1)$, and at $m = \frac{1}{2}$ $(m = -\frac{1}{2})$ for $h_2 < h$ $(h < -h_2)$ when $|\mathbf{GS}\rangle = |7\rangle$, $\langle 7|S_4|7\rangle = 2$ $(|\mathbf{GS}\rangle = |10\rangle$, $\langle 10|S_4|10\rangle = -2)$. However, for special values of the parameters J_1, J_2, J_3, not all possible values of m, i.e. 0, $\pm\frac{1}{4}$, $\pm\frac{1}{2}$, are observed. For example, if $E_1 = E_2$ in (10) (this occurs when $J_1 = J_2 = J$, $J_3 = 0$), the plateau at $m = 0$ disappears. In terms of the Jordan – Wigner fermions, one has [4] $\rho(E) = \frac{1}{4}\delta(E + h + \frac{\sqrt{2}}{2}J) + \frac{1}{2}\delta(E + h) + \frac{1}{4}\delta(E + h - \frac{\sqrt{2}}{2}J)$ in such a case. Therefore, in accordance with (2), $m = 0$ occurs exactly when $h = 0$ since any small positive (negative) h immediately yields $m = \frac{1}{2}$ $\left(m = -\frac{1}{2}\right)$.

Let us turn to the transverse Ising chain. For $p = 2$, the cluster Hamiltonian eigenvalues are

$$E_1 = -\sqrt{h^2 + \frac{1}{16}J_1^2} = -E_4, \quad E_2 = -\frac{1}{4}J_1 = -E_3. \quad (11)$$

Note, that the ground state is $|1\rangle$ for any h. Moreover, $\langle 1|S_2|1\rangle = 0$ if $h = 0$ and $\langle 1|S_2|1\rangle \to 1$ if $h \to \infty$. Thus, the considered chain does not exhibit plateaus that is in agreement with the result for a general (without the restriction $J_2 = 0$) transverse Ising chain with $p = 2$ reported in [5]. In fact, the absence of plateaus in the magnetization curve is not surprising and is conditioned by the different symmetry of the transverse Ising chain. Thus, for this model with arbitrary J_2, $\Sigma_n s_n^z$ does not commute with the Hamiltonian (in contrast to the case of transverse XX chain) and hence $\langle \mathbf{GS}|S_p|\mathbf{GS}\rangle$ should vary continuously with changing h.

One can convinced himself that formulae (4), (7), (10), (11) are valid for the ferromagnetic sign of all or a part of interactions and, hence, the described picture is not restricted to the case $J_n \geq 0$. For an isotropic Heisenberg chain with $p = 2$ with the antiferromagnetic interaction $J_1 > 0$, the cluster Hamiltonian eigenvalues are $E_1 = -\frac{3}{4}J_1$, $E_2 = -h + \frac{1}{4}J_1$, $E_3 = \frac{1}{4}J_1$, $E_4 = h + \frac{1}{4}J_1$, and therefore one concludes that $m = 0$ for $-J_1 < h < J_1$ and $m = \frac{1}{2}$ $\left(m = -\frac{1}{2}\right)$ for $J_1 < h$ $(h < -J_1)$. However, for the ferromagnetic interaction $J_1 < 0$, one finds instead $E_1 = -h - \frac{1}{4}|J_1|$, $E_2 = -\frac{1}{4}|J_1|$, $E_3 = h - \frac{1}{4}|J_1|$, $E_4 = -\frac{3}{4}|J_1|$, and hence the plateau at $m = 0$ does not appear.

Evidently, in the considered limit $J_p^\alpha = 0$, an arbitrary type of intersite interaction (the anisotropic XY chain, Heisenberg – Ising (XXZ) chain, etc.) and the value of spin s can be examined easily. In addition to the total magnetization, the on-site magnetization can be calculated in a similar way. In addition, the treatment in that limit can be applied to the spin systems of higher dimensions.

Finally, let us discuss briefly how the obtained results can be used for the analysis beyond the limit $J_p = 0$. The transverse XX chain or transverse Ising chain can be studied rigorously [4, 5] in contrast to the Heisenberg or more complicated chains. Different ways to take a nonzero value of the smallest interaction into account perturbatively at a certain value of m have been elaborated [3]. Let us demonstrate how it can be done considering to be specific a regularly alternating XX chain of period 2 in which now $J_1 >> J_2 \neq 0$. The Hamiltonian of that system naturally splits into two parts

$H = H_0 + V,$

$H_0 = \ldots - h(s_{101}^z + s_{102}^z) + \frac{1}{2}J_1(s_{101}^+ s_{102}^- +$

$+ s_{101}^- s_{102}^+) - \ldots,$

$$V = \ldots + \frac{1}{2}J_2(s_{102}^+ s_{103}^- + s_{102}^- s_{103}^+) + \ldots. \quad (12)$$

Only two lowest levels of the two-site cluster discussed above will be taken into account. We assume $h \geq 0$; the relevant states are $|1\rangle_{51} =$

$= \frac{1}{\sqrt{2}}(|\uparrow_{101}\downarrow_{102}\rangle - |\downarrow_{101}\uparrow_{102}\rangle)$ and $|2\rangle_{51} = |\uparrow_{101}\uparrow_{102}\rangle$. Let us introduce new spin $\frac{1}{2}$ operators σ_l^α which act in the following way:

$$\sigma_{51}^z|1\rangle_{51} = -\frac{1}{2}|1\rangle_{51}, \quad \sigma_{51}^+|1\rangle_{51} = |2\rangle_{51},$$

$$\sigma_{51}^-|1\rangle_{51} = 0, \quad \sigma_{51}^z|2\rangle_{51} = \frac{1}{2}|2\rangle_{51}, \quad \sigma_{51}^+|2\rangle_{51} = 0,$$

$$\sigma_{51}^-|2\rangle_{51} = |1\rangle_{51}. \tag{13}$$

Then H_0 can be written approximately (only the lowest levels are reproduced) as

$$H_0 = \sum_{l=1}^{L}\left(-\frac{1}{2}J_1\left(\frac{1}{2} - \sigma_l^z\right) - h\left(\frac{1}{2} + \sigma_l^z\right)\right) =$$

$$= -\frac{1}{2}\left(h + \frac{1}{2}J_1\right)L - \left(h - \frac{1}{2}J_1\right)\sum_{l=1}^{L}\sigma_l^z \tag{14}$$

with $L = \frac{1}{2}N$. Really,

$$\left(-\frac{1}{2}\left(h + \frac{1}{2}J_1\right) - \left(h - \frac{1}{2}J_1\right)\sigma_l^z\right)|1\rangle_l = -\frac{1}{2}J_1|1\rangle_l,$$

$$\left(-\frac{1}{2}\left(h + \frac{1}{2}J_1\right) - \left(h - \frac{1}{2}J_1\right)\sigma_l^z\right)|2\rangle_l = -h|2\rangle_l, \tag{15}$$

that are just two lowest levels of the lth cluster. To write the intercluster interaction, e.g., $V_{51,52} = \frac{1}{2}J_2(s_{102}^+ s_{103}^- + s_{102}^- s_{103}^+)$ in terms of the operators σ_l^α, let us consider the following equations:

$$V_{51,52}|1\rangle_{51} = \frac{1}{2}J_2 s_{103}^- \frac{1}{\sqrt{2}}|2\rangle_{51} =$$

$$= \frac{1}{2\sqrt{2}}J_2 s_{103}^- \sigma_{51}^+|1\rangle_{51},$$

$$V_{51,52}|2\rangle_{51} = \frac{1}{2}J_2 s_{103}^+|\uparrow_{101}\downarrow_{102}\rangle =$$

$$= \frac{1}{2}J_2 s_{103}^+\frac{1}{\sqrt{2}}|1\rangle_{51} = \frac{1}{2\sqrt{2}}J_2 s_{103}^+\sigma_{51}^-|2\rangle_{51} \tag{16}$$

(note, that although $|\uparrow_{101}\downarrow_{102}\rangle = \frac{1}{\sqrt{2}}(|1\rangle_{51} + |4\rangle_{51})$, we put $|\uparrow_{101}\downarrow_{102}\rangle = \frac{1}{\sqrt{2}}|1\rangle_{51}$ since we take into account only $|1\rangle_{51}$ and $|2\rangle_{51}$). Hence,

$$V_{51,52} = \frac{1}{2\sqrt{2}}J_2(s_{103}^-\sigma_{51}^+ + s_{103}^+\sigma_{51}^-). \tag{17}$$

Further,

$$V_{51,52}|1\rangle_{52} = \frac{1}{2\sqrt{2}}J_2\left(\sigma_{51}^+\frac{1}{\sqrt{2}}|3\rangle_{52} + \right.$$

$$\left. + \sigma_{51}^-\frac{1}{\sqrt{2}}(-|2\rangle_{52})\right) = -\frac{1}{4}J_2\sigma_{51}^-\sigma_{52}^+|1\rangle_{52},$$

$$V_{51,52}|2\rangle_{52} = \frac{1}{2\sqrt{2}}J_2\sigma_{51}^+|\downarrow_{103}\uparrow_{104}\rangle =$$

$$= -\frac{1}{4}J_2\sigma_{51}^+\sigma_{52}^-|2\rangle_{52}. \tag{18}$$

As a result, we conclude that

$$V_{51,52} = -\frac{1}{4}J_2(\sigma_{51}^+\sigma_{52}^- + \sigma_{51}^-\sigma_{52}^+) \tag{19}$$

and therefore

$$V = -\frac{1}{2}J_2\sum_{t=1}^{L}(\sigma_l^x\sigma_{l+1}^x + \sigma_l^y\sigma_{l+1}^y). \tag{20}$$

Hamiltonian (12), (14), (20) describes a uniform spin-$\frac{1}{2}$ transverse XX chain of L spins,

$$H = -\frac{1}{2}\left(h + \frac{1}{2}J_1\right)L - \left(h - \frac{1}{2}J_1\right)\sum_{l=1}^{L}\sigma_l^z -$$

$$-\frac{1}{2}J_2\sum_{l=1}^{L}(\sigma_l^x\sigma_{l+1}^x + \sigma_l^y\sigma_{l+1}^y). \tag{21}$$

Finally, by acting like while deriving (17), (19), we find the relation between the operators s_n^α and σ_l^α as follows:

$$s_{101}^+ = -\frac{1}{\sqrt{2}}\sigma_{51}^+, \quad s_{101}^- = -\frac{1}{\sqrt{2}}\sigma_{51}^-,$$

$$s_{102}^+ = \frac{1}{\sqrt{2}}\sigma_{51}^+, \quad s_{101}^z = \frac{1}{2}\left(\frac{1}{2} + \sigma_{51}^z\right),$$

$$s^{-}_{102} = \frac{1}{\sqrt{2}}\sigma^{-}_{51}, \qquad s^{z}_{102} = \frac{1}{2}\left(\frac{1}{2} + \sigma^{z}_{51}\right), \ldots \qquad (22)$$

Consider now the magnetization which owing to (22) can be written as $m = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{L}\sum_{l=1}^{L}\langle\sigma^{z}_{l}\rangle\right)$. Using the well-known results for a uniform spin-$\frac{1}{2}$ transverse XX chain (see, e.g., [4]), one concludes that $m = 0$ at $0 \le h \le \frac{1}{2}(J_1 - J_2)$ and $m = \frac{1}{2}$ at $\frac{1}{2}(J_1 + J_2) \le h$. If h increases from $\frac{1}{2}(J_1 - J_2)$ to $\frac{1}{2}(J_1 + J_2)$, the magnetization increases from 0 to $\frac{1}{2}$ as $\frac{1}{2} - \frac{1}{2\pi}\arcsin\sqrt{1 - \frac{(2h - J_1)^2}{J_2^2}}$.

In Figure, we plot the zero temperature magnetization profiles for the spin-$\frac{1}{2}$ transverse XX chain of period 2 with $J_1 = 1 + \delta$, $J_2 = 1 - \delta$ for $\delta = 1, 0.8, 0.6, 0.4, 0.2, 0$ as they follow from 1) the exact formula (2) [4] (solid curves) and 2) the approximate Hamiltonian (21) (dotted curves). These plots demonstrate how the strong-coupling approach works as δ deviates from 1.

To summarize, we have reconsidered the zero temperature magnetization processes in regularly alternating spin-$\frac{1}{2}$ XY chains within the frames of the strong-coupling approach. Besides discussing the magnetization plateaus in terms of spins rather than in terms of Jordan – Wigner fermions, we have demonstrated to what extent the strong-coupling approximation can reproduce the exact magnetization profiles.

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