QED CORRECTIONS TO ELECTRON-POSITRON ANNIHILATION CROSS-SECTION WITH TAGGED COLLINEAR PHOTONS AND $e^+ e^-$-PAIR

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The possibility of a precise scanning of the hadron cross-section in the process of electron-positron annihilation is considered. It can be done by selection and analysis of the events with collinear photons and an $e^+ e^-$-pair radiated from the initial state. The energy distribution for all particles, hitting a detector placed along the electron beam direction, is derived taking into account radiative corrections. The corresponding contribution due to photon emission is calculated with next-next-to-leading order (NNLO) accuracy, and the contribution caused by the pair production with next-to-leading order (NLO) one. The results obtained are illustrated with numerical estimations.

Introduction

The precise measurement of the standard model (SM) parameters and the search for a possible deviation from it are the priority aims of electron-positron collider experiments. The high-energy experiments likely allow one to reveal and investigate the SM Higgs region in detail [1] and make definite conclusions about its supersymmetric extension and the existence of additional heavy neutral vector bosons [2]. At the same time, experiments at low and intermediate energies have to improve the accuracy in the determination of such SM parameters as the QED running coupling constant evaluated at the mass of a Z-boson, $\alpha_{\rm QED}(M_Z^2)$, and the anomalous magnetic moment of a muon, a_μ [3].

The precision of these parameters directly depends on that of the measurement of the total hadronic cross-section $\sigma_t(s)$ in the single-photon annihilation process

$$e^-(p_1) + e^+(p_2) \to \gamma^* \to \text{hadrons}, \qquad (1)$$

at the energy region $\sqrt{s}$ up to several GeV, $s = (p_1 + p_2)^2$.

The hadronic cross-section $\sigma_t(s)$ was measured in many experiments over a large energy region from the threshold up to the Z-boson mass and over. At high energies, $\sqrt{s} > 5$ GeV, the experimental values for $\sigma_t(s)$ agree with theoretical predictions well. These predictions are based on the perturbative quantum chromodynamics (QCD) and give the impressing confirmation for the colour quark hypothesis. Unfortunately, at low energies, the perturbative QCD has no prediction force, and experimental values have an uncertainty of the order of 15% in this region [4]. This uncertainty restricts the precision of a_μ and $\alpha_{\rm QED}(M_Z^2)$ that, in turn, prevents from the accurate determination of the Higgs boson mass which can be evaluated in terms of $\alpha_{\rm QED}(M_Z^2)$ by radiative corrections (RC) within SM [5]. Therefore, there is an important physical reason for calculation of $\sigma_t(s)$ from the threshold up to $\sqrt{s} = 5$ GeV with the most high precision. One part of these experiments was already carried out at the B-fabric in Beijing, where it is planned to cover the range from 2.3 up to 5 GeV [6].

The problem consists in measuring $\sigma_t(s)$ not only at several points but rather in the scanning it at the continuously changing energy of electron-positron collision. The point is that $\sigma_t(s)$ determines a hadronic contribution into the vacuum polarization by a dispersion integral, and the vacuum polarization comes in directly in the determination of the $(g-2)_\mu$ and $\alpha_{\rm QED}(M_Z^2)$ values [7].

For the scanning of the hadronic cross-section, one can use an analysis of radiative events in the process of electron-positron annihilation into hadrons

$$e^-(p_1) + e^+(p_2) - \gamma(k) \to \gamma^*(q) \to \text{hadrons}(q), \qquad (2)$$

if the statistics of such events is high enough [8 – 10]. We write $-\gamma(k)$ on the left side of Eq. (2) to emphasize that a real photon with 4-momentum k is radiated from the initial state (ISR). In this case, the total hadronic cross-section depends on the virtuality of an intermediate heavy photon q^2 only:

$$q^2 = sx, \quad s = 4E^2, \quad 1 - x = \frac{\omega}{E}, \qquad (3)$$

where E (ω) is the beam (photon) energy. Therefore, we can obtain information about $\sigma_t(q^2 = sx)$ by measuring the photon spectrum in process (2). If the photon is emitted in the final state (FSR), then q^2 remains to have a fixed value. Such events do not scan $\sigma_t(s)$ and they must be eliminated from the analysis. The energy of the emitted photon in process (2) can be measured by a photon detector (PD). We suggest that it is placed along the electron beam direction and has the opening angle $2\theta_0$.

In order to eliminate the FSR background and have no loss in the events statistics, it is suitable to choose the parameter θ_0 such that $\theta_0 \ll 1$, $E^2\theta_0^2 \gg m^2$ (m is the electron mass). In this case, the cross-section of process (2) has a logarithmic amplification $\sim L_0 = \ln\frac{\theta_0^2 E^2}{m^2}$ [11] and the background cross-section, which is caused by FSR, is of the order of θ_0^2. That is why we can neglect this contribution for $\theta_0^2 \sim 10^{-3}$ without sufficient loss in the events statistics.

The method of hadron cross-section scanning, by means of the radiative events in process (2), can be competitive if the accuracy of the measured cross-section will be better than 1% [10]. Such a high accuracy requires adequate theoretical calculations. Thus, RC have to be taken into account.

In [8], the photonic RC to the Born cross-section of process (2) were calculated with the leading and NLO accuracy. In order to control the accuracy of these contributions, we consider here also the NNLO contribution to the photonic RC. In addition, we consider RC for a calorimeter type detector (CD) which does not distinguish a photon and charged pairs (it is an electron-positron pair in our case) and measures the total energy of all particles hitting it. In this case, the process

$$e^-(p_1) + e^+(p_2) - (e^-(p_-) + $$
$$+ (e^+((p_+)) \to \gamma^*(q) \to \text{hadrons}(q), \tag{4}$$

with a collinear e^+e^--pair contributes into the scanned cross-section on the level of RC to the Born cross-section.

1. The Born approximation and Photonic RC

The differential, with respect to the tagged photon energy fraction ($\omega = (1-x)E$), cross-section of process (2) in the Born approximation, for an arbitrary hadronic final state, can be expressed in terms of the ratio R as [8]

$$\frac{d\sigma^B}{dx} = \frac{4\pi\alpha^2(q^2)}{3}\frac{\alpha}{2\pi}P_1(x, L_0)\frac{R(q^2)K(q^2)}{q^2},$$

$$K(q^2) = \left(1 + \frac{2m_\mu^2}{q^2}\right)\sqrt{1 - \frac{4m_\mu^2}{q^2}}, \tag{5}$$

$$P_1(x, L_0) = \frac{1+x^2}{1-x}L_0 - \frac{2x}{1-x}, \tag{6}$$

m_μ is the muon mass and $R(q^2) = \frac{\sigma_t(q^2)}{\sigma_{e^+e^-\to\mu^+\mu^-}}$. The Born cross-section (5) is the same for PD and CD types of a detector.

Let us consider RC in the Born approximation due to additional (real and virtual) photon emission. The corresponding contribution with the NNLO accuracy can be written as follows:

$$\frac{d\sigma_{\text{ph}}}{dx} = \frac{4\pi}{3}\left(\frac{\alpha}{2\pi}\right)^2\left(\alpha^2(q^2)\left(\frac{1}{2}P_{2\theta}(x)L_0^2 + \right.\right.$$
$$\left. + G(x)L_0 + \tilde{K}(x)\right)\frac{R(q^2)K(q^2)}{q^2} +$$
$$+ P_1(x, L_0)\left(\int_0^{1-\delta/x} C_1\frac{R(\bar{q}^2)K(\bar{q}^2)}{\bar{q}^2}dz + \right.$$
$$\left.\left. + \ln\frac{4}{\theta_0^2}\int_0^{x-\delta} C_2\frac{R(\tilde{q}^2)K(\tilde{q}^2)}{\tilde{q}^2}dz\right)\right) + \frac{d\sigma_{sc}^H}{dx}. \tag{7}$$

The first two lins on the right side of Eq. (7) describe the events with one hard photon and with two ones, provided both hit PD. The second two lins correspond to the events with two hard photons where only one from them hits PD. All terms on the right side of the last equation, except $\tilde{K}(x)$, were derived in [8]. We do not repeat all calculations and represent the final results for them as

$$P_{2\theta}(x) = 2\frac{1+x^2}{1-x}\left(\frac{3}{2} + \ln\frac{(1-x)^2}{x}\right) +$$
$$+ (1+x)\ln x - 2(1-x), \tag{8}$$

$$G(x) = \frac{3+4x+3x^2}{1-x}\ln x + \frac{(1+x)}{2}\ln^2 x -$$
$$- \frac{2(1+x)^2}{1-x}\ln(1-x) + \frac{2\pi^2}{3}\frac{1+x^2}{1-x} + \frac{1-16x-x^2}{2(1-x)},$$

$$C_1 = \alpha(\bar{q}^2)\left(P_1\left(1-z, \ln\frac{4E^2}{m^2}\right)\theta(z-\Delta) + \right.$$

$$+ \left(-1 + \ln \frac{4E^2}{m^2} \right) \left(2 \ln \Delta + \frac{3}{2} \right) \delta(z) \Bigg),$$

$$C_2 = \alpha(\tilde{q}^2) \left(\frac{x^2 + (x - z^2)}{x^2 z} \theta(z - \Delta) + \right.$$

$$\left. + \left(2 \ln \Delta + \frac{3}{2} - 2 \ln x \right) \delta(z) \right),$$

$$\frac{d\sigma^H_{sc}}{dx} = \frac{8\pi^2}{s} \left(\frac{\alpha}{2\pi} \right)^2 P_1(x, L_0) \times$$

$$\times \int\limits_{-1}^{1} dc \int\limits_{0}^{2\pi} \frac{d\phi}{2\pi} \int\limits_{0}^{x_0} \frac{dz}{z} \left(\frac{2F(c)}{1 - c^2} - \frac{F(1)}{1 - c} \times \right.$$

$$\left. \times\, \theta(x - z - \delta_0) - \frac{F(-1)}{1 + c} \theta(x - xz - \delta_0) \right),$$

$$F(c) = \frac{\alpha^2(\hat{q}^2)}{x^2} \frac{R(\hat{q}^2) K(\hat{q}^2)}{\hat{q}^2},$$

$$F(1) = \frac{\alpha^2(\tilde{q}^2)}{x^2} (x^2 + (x - z)^2) \frac{R(\tilde{q}^2) K(\tilde{q}^2)}{\tilde{q}^2},$$

$$F(-1) = \alpha^2(\bar{q}^2)(1 + (1 - z)^2) \frac{R(\bar{q}^2) K(\bar{q}^2)}{\bar{q}^2},$$

$$\tilde{q} = (x - z) p_1 + p_2, \ \hat{q} = x p_1 + p_2 - k_1,$$

$$\bar{q} = x p_1 + (1 - z) p_2,$$

$$\delta_0 = \frac{M^2_{\min}}{s}, \ x_0 = \frac{2x}{1 + x + c(1 - x)},$$

$$\alpha(q^2) = \frac{\alpha}{1 - \Pi(q^2)}, \ \Pi(q^2) = \frac{1}{3} \ln \frac{q^2}{m^2} - \frac{5}{9},$$

where k_1 is the 4-momentum of a photon hitted PD and $M^2_{\min}$ is the minimum invariant mass of the final state. The main result of our calculations is just the $\tilde{K}(x)$ term. It includes the contribution caused by the emission of additional virtual and soft photons and the radiation of an additional hard photon inside and outside PD, and reads

$$\tilde{K}(x) = \frac{20}{3(1 - x)} + 2x \ln^2(1 - x) + (1 - 7x/3) \times$$

$$\times \operatorname{Li}_2(1 - x) + \left(4x - 6 + \frac{5}{1 - x} \right) \operatorname{Li}_2(x) +$$

$$+ \frac{4x}{1 - x} \ln(1 - x) + \left(\frac{5}{2} - \frac{7}{6} x - \frac{6}{1 - x} + \right.$$

$$\left. + \frac{16}{3(1 - x)^2} - \frac{4}{3(1 - x)^2} \right) \ln^2 x +$$

$$+ \left(\frac{25}{3} - \frac{14}{3(1 - x)} - \frac{8}{3(1 - x)^2} \right) \ln x +$$

$$+ \frac{\pi^2}{6} \left(\frac{3 - 34x - 8x^2}{3(1 - x)} \right) - \frac{23}{3} +$$

$$+ \int\limits_{0}^{1-x} \left(\frac{x^2 + (1 - z)^4}{z(1 - z - x)(1 - z)^2} J + \right.$$

$$\left. + \frac{z(1 - z - x) - 3x}{2(1 - z)^2} L_2 + \frac{x + z}{2(1 - z)} L_1 \right) dz,$$

$$\operatorname{Li}_2(x) = -\int\limits_{0}^{x} \frac{\ln(1 - z)}{z} dz. \qquad (9)$$

Some formulae necessary for the calculation of RC are quoted in Appendix A (for the emission of virtual and additional soft photons) and Appendix B (for the radiation of two hard photons inside PD), the quantities J, L_1 and L_2 are defined in the Appendix B too.

2. Contribution due to a Tagged Collinear e^+e^--Pair

If an experimental setup includes CD, that does not distinguish between photons, electrons, and positrons, then the contribution caused by process (4) effectively changes RC. In this case, the improvement of RC, defined by Eq. (7), due to the events with an e^+e^--pair hitting CD must be taken into consideration.

The numerical estimations (see Section 3) show that the corresponding contribution is considerably less as compared with the photonic RC. Therefore, we can restrict ourselves with the NLO accuracy only.

If the e^+e^--pair in process (4) hits CD and deposits its energy, that is equal to $(1 - x)E$, we can use the approximation collinear kinematics. The corresponding calculations were performed in [12]. The result reads

$$\frac{d\sigma_{\text{pair}}}{dx} = \frac{4\pi}{3} \left(\frac{\alpha}{2\pi} \right)^2 \alpha^2(q^2)(A(x) L_0^2 +$$

$$+ B(x) L_0) \frac{R(q^2) K(q^2)}{q^2}. \qquad (10)$$

The functions A and B can be obtained from the result in [12], where the effect of collinear pair production

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was considered in the cases of deep inelastic scattering and small-angle Bhabha scattering, and they can be written in the following form:

$$A(x) = \frac{1+x^2}{3(1-x)} + (1+x)\ln x - $$

$$- \frac{2}{3}\left(x^2 - \frac{1}{x}\right) + \frac{1}{2}(1-x), \qquad (11)$$

$$B(x) = \frac{136}{9}x - \frac{2}{3}x^2 - \frac{4}{3x} +$$

$$+ \left(-\frac{8}{3}x^2 - \frac{10}{3}x + \frac{2}{3} + \frac{8}{3x} + \frac{8}{3(1-x)}\right)\ln(1-x) +$$

$$+ \left(\frac{8}{3}x^2 + \frac{5}{3}x - \frac{7}{3} - \frac{13}{3(1-x)}\right)\ln x - \frac{2}{1-x}\ln^2 x +$$

$$+ 2(1+x)\left(\frac{\pi^2}{6} - \mathrm{Li}_2(x) + \ln x \ln(1-x)\right) -$$

$$- \frac{4x^2}{1-x}\mathrm{Li}_2(x) - \frac{107}{9} - \frac{20}{9(1-x)}. \qquad (12)$$

The function B can be affected also by the contribution of semicollinear kinematics when $\mathbf{p}_- || \mathbf{p}_1$. For the corresponding calculation, we can use, in principle, the result in [13], where the leptonic tensor due to pair production was derived. But in our estimations we restrict ourselves the case of collinear kinematics.

3. Numerical Estimations

For the illustration of the obtained results, we consider two different 'hadronic" final states. One of them is a structureless fermion pair (for simplicity, we choose a muon pair), other is a pion pair. For an arbitrary two-particle final state in the radiative process (2), we can write the quantity R in the following form [14]:

$$R = 3\sqrt{1 - 4m_h^2/q^2}\left[D_1 - \frac{D_2 q^2}{6} \times\right.$$

$$\left. \times \left(1 - \frac{4m_h^2}{q^2}\right)\right] / [q^2 K(q^2)], \qquad (13)$$

where D_1 and D_2 depend on q^2 and can be expressed in the terms of the corresponding form factors, m_h is the mass of a particle. For the muon channel $D_1 = q^2/2$, $D_2 = 1$, whereas $D_1 = 0$, $D_2 = -|F_\pi(q^2)|^2/2$ for the pion-pair production, where $F_\pi(q^2)$ is the pion electromagnetic formfactor.

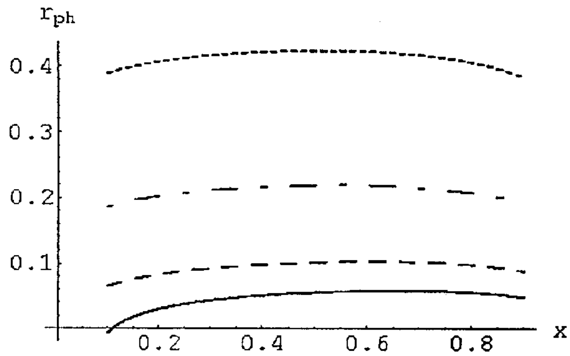

Fig. 1. Ratio $r_{ph} = \frac{d\sigma_{ph}/dx}{d\sigma^B/dx}$ as the function of the variable x at energies 0.5 GeV (the dotted line), 1 GeV (the dot-dashed), 5 GeV (the dashed) and 50 GeV (the solid)

For the case of structureless fermion production, the corresponding cross-section is a uniformly decreasing function of the energy. The numerical estimations have been performed for E(GeV) = 0.5, 1, 5, and 50 and $\theta_0 = 0.05$ rad. The photonic RC for the muon final state is shown in Fig. 1, and the corresponding RC due to pair production in Fig. 2. It follows from our calculations that the next-next-to-leading contribution is of the order of 1% of the total photonic correction. It is in agreement with the rough estimation $\sim \frac{1}{\ln^2(E^2/m^2)}$. For low energies, this contribution is approximately of 0.05% of the Born cross-section, and, for the highest energies, it reaches 1%. Thus, we conclude that the calculations obtained in [8] are sufficient for the scanning of $\sigma_t(s)$ to an accuracy of 1% (see Section 1) at the low energies but they have to be improved by the NNLO contribution at the high energy. The RC caused by the e^+e^--pair production is essentially less than the photon correction especially at low energies.

Looking at Fig. 2, one can see that the right side of Eq. (10) at $x > 0.5$ has became negative for energies below 1 Gev. This result is wrong because it describes the cross-section of process (4) that cannot be negative. We must admit here that the used approximation is not enough in this region and the contribution of semicollinear kinematics have to be taken into account. This contribution cancels the negative terms in the cross-section which are proportional to $\ln(E^2/m^2)\ln\theta_0^2$, as was shown in [13]. But we sure any way that the contribution of the e^+e^--pair production process into RC is negligible.

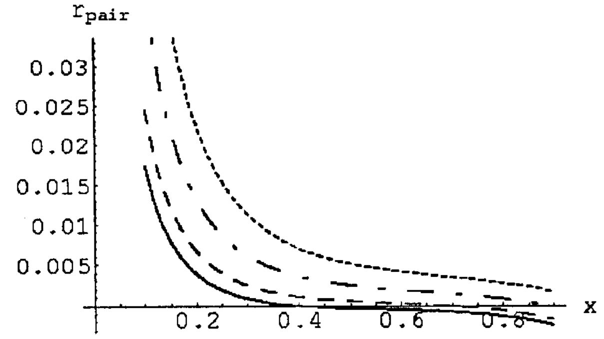

Fig. 2. The same as in Fig. 1 but for the ratio $r_{\text{pair}} = \dfrac{d\,\sigma_{\text{pair}}/\,dx}{d\,\sigma^B/\,dx}$

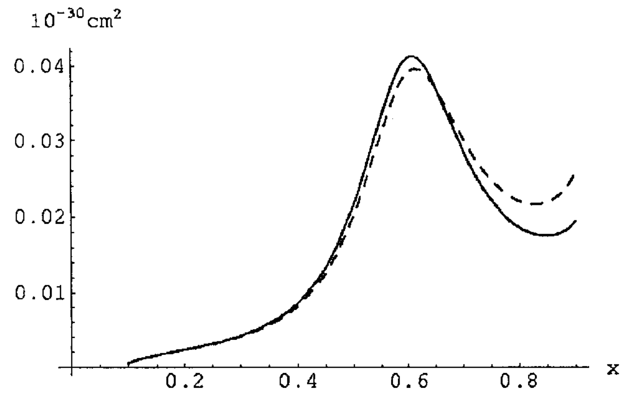

Fig. 3. Cross-section of the $e^+ e^- \to \gamma \pi^+ \pi^-$ reaction in the Born approximation (the solid line) and with the first-order RC (the dashed line)

To estimate the cross-section of pion-pair production in the final state, we use the following form for F_π [15]:

$$| F_\pi (q^2)|^2 = \frac{m_\rho^4}{(m_\rho^2 - q^2)^2 + m_\rho^2 \Gamma_\rho^2}, \qquad (14)$$

where $m_\rho\,(\Gamma_\rho)$ is the ρ-meson mass (width) [16]. We choose $\theta_0 = 0.05$ and $E = 0.5$ GeV because, just in this energy region, the radiative return onto the ρ-resonance defines the main contribution into the cross-section of process (2) for the final $\pi^+ \pi^-$ hadronic state. The form of the corresponding distribution, respect to the tagged-particle energy fraction, is shown in Fig.3.

Our numerical estimations indicate that RC increase the effect of radiative return to the resonance that is well known for the ISR processes: the decrease of a resonance maximum and the appearance of a radiative tail.

The largest contribution to RC arises from the emission of an additional hard photon outside CD. In the case under consideration, the value of RC changes from 10% (for small x) up to 40% (at the end of the spectrum) respect to the cross-section in the Born approximation. Note also that RC in this case change their sign after the resonance maximum.

For the integrated cross-section (contributions (5), (7), (10) are integrated over the resonance region, changing the variable x from $(m_\rho - \Gamma_\rho)^2 / s$ up to $(m_\rho + \Gamma_\rho)^2 / s$, we obtained the following results: $\sigma^B = 11.58$ nb, $\sigma^B + \sigma_{\text{pair}} + \sigma_{\text{ph}} = 11.72$ nb.

We estimated also the background caused by the double-photon mechanism of pion pair production provided a hard photon hits PD. For this goal, we used the known expression for the contribution of the double-photon mechanism to the total cross-section in the equivalent-photon approximation [17]. To describe the radiation of a tagged photon with the energy fraction $1 - x$, we apply the quasireal electron method [11]. Obtained in that way, the differential cross-section respect to the variable x reads

$$\frac{d\sigma_{\gamma\gamma}}{dx} = \frac{\alpha}{2\pi} P_1 (x, L_0) \frac{2\alpha^2}{\pi^2} \ln \frac{E}{m_\pi} \times$$

$$\times \ln \frac{xE}{m_\pi} \int\limits_{4m_\pi^2}^{4xE^2} \frac{dM^2}{M^2} \sigma_{\gamma\gamma\to\pi^+\pi^-} (M^2) f\left(\frac{M}{2E}\right), \qquad (15)$$

where m_π is the pion mass, M is the invariant mass of the final $\pi^+ \pi^-$-system, f is an universal function of its argument defined as [17]

$$f(x) = -(2 + x^2)^2 \ln x - (1 - x^2)(3 + x^2). \qquad (16)$$

The quantity $\sigma_{\gamma\gamma\to\pi^+\pi^-}$ in Eq. (15) is the $\gamma + \gamma \to \pi^+ + \pi^-$ total cross-section and reads

$$\sigma_{\gamma\gamma\to\pi^+\pi^-} (M^2) = \frac{2\pi\alpha^2}{M^2} \Bigg((1 + y)\sqrt{1 - y} -$$

$$- 2y\left(1 - \frac{y}{2}\right)\left(\ln \frac{1}{\sqrt{y}} + \sqrt{-1 + \frac{1}{y}}\right)\Bigg), \qquad (17)$$

where $y = 4m_\pi^2 / M^2$. The results of numerical calculation according to formula (17) are shown in Fig. 4. We see that the spectrum of the tagged photon in this case is by about three orders smaller as compared with the Born one and can be neglected. The integration of $d\sigma_{\gamma\gamma}/\,dx$ over the x-variable from 0.1 to

0.9 leads to the result $\sigma_{\gamma\gamma} = 9.93$ pb. This contribution reduces up to 6.54 pb in the case where the x-variable interval corresponds to the ρ-resonance in the one-photon annihilation channel.

We conclude therefore, that the contribution caused by the double-photon mechanism of pion pair production can be neglected on the one-percent level at energies below 1 GeV for the events with a tagged photon. But, for a CD, the corresponding background requires some special calculations because the scattered electron in this case can deposit its energy in the detector and such an event may imitate the ISR event.

Conclusion

We investigated QED corrections to the cross-section of process (1) accompanying with tagged collinear photons and an $e^+ e^-$-pair. The differential cross-section, respect to the energy deposited in the detector by tagged particles, is calculated. Such a kind of distribution can be effectively used to scan $\sigma_t(s)$ as a function of the hadron invariant mass.

In the Born approximation, this distribution describes the events with only one tagged photon. The QED RC to the Born cross-section arise due to emission of additional real and virtual photons. In addition, at the calorimeter-type detector, the events with a collinear $e^+ e^-$-pair contribute on the level of RC.

To be sure in high precision (better than one percent), we calculate both the NNLO contribution caused by the photonic RC and the effect due to collinear $e^+ e^-$-pair production with the NLO accuracy. The performed numerical estimations in the wide range of energy justify this choice.

The estimation of the main background, caused by the double-photon mechanism of pion pair production (with one tagged photon in PD), shows that its contribution can be neglected on the one percent level in the region of the ρ-resonance.

APPENDIX

In this Appendix, some formulae necessary for the calculation of photonic RC to the Born cross-section of process (2) with the NNLO accuracy are given. It is convenient to separate the contribution caused by the virtual and soft (with the energy less than ΔE, $\Delta \ll 1$) photon emissions (Appendix A) from the one due to the radiation of a hard (with the energy more than ΔE) photon that hits PD (Appendix B). The parameter Δ is auxiliary and is introduced to simplify our calculations. It disappears from the final result as one can see from Eq. (7). Let us recall that, in [8], such calculations were performed with the NLO accuracy.

APPENDIX A

Using the results of [8] and [18], we can write the contribution of the virtual and soft corrections to the cross-section of process (2) with a tagged ISR photon in the following form:

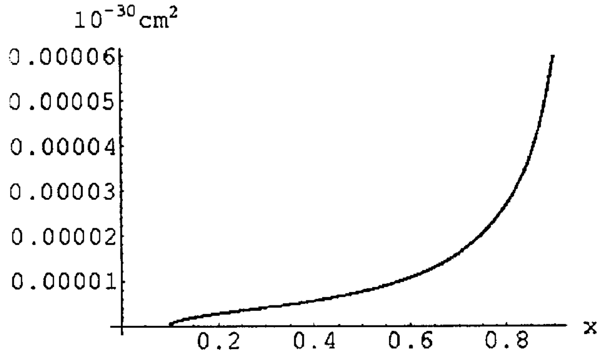

Fig. 4. Cross-section of the $e^+ e^- \to \pi^+ \pi^-$ reaction due to the double-photon mechanism

$$d\sigma^{s+v} = \frac{4\pi\alpha^2(q^2)}{3q^2}\frac{\alpha^2}{4\pi^3}\int\frac{d^3k}{\omega}\Bigg[\rho\left(-\frac{1+x^2}{(1-x)\,t_1} - \frac{2m^2x}{t_1^2}\right) - \frac{1}{t_1}\Bigg(\frac{1+x^2}{1-x}\left(-2l_s\ln x - \ln^2 x + 2\,\mathrm{Li}_2(1-x) + 2l_{t_1}\ln x\right) + \frac{1+2x-x^2}{2(1-x)} +$$

$$+\frac{4xm^2\ln x\, l_s}{t_1^2} + \frac{m^2}{t_1^2}\Bigg(\left(\frac{(4x-1)\,t_1}{t_1+m^2} + 2(1-x)\right)l_{t_1} - 2 + 2x\ln x\left(4\ln(1-x) - \ln x - l_{t_1}\right) +$$

$$+ 8x\int\limits_1^{1/x}\frac{dt}{t}\ln(1-t) + 2\left(x - \frac{m^2(1-x)}{t_1}F\right) - xn\Bigg)\Bigg)\Bigg],$$

$$F = \frac{\pi^2}{6} - \mathrm{Li}_2\left(1+\frac{t_1}{m^2}\right), \quad l_{t_1} = \ln\frac{-t_1}{m^2},$$

$$l_s = \ln\frac{s}{m^2}, \quad t_1 = -2(k_1 p_1), \qquad \text{(A.1)}$$

where

$$\rho = 4(l_s - 1)\ln\Delta + 3(l_s + \ln x) + \frac{2\pi^2}{3} - \frac{9}{2},$$

$$n = \frac{m^2}{t_1+m^2}\left(\frac{t_1}{t_1+m^2}\,l_{t_1} - 1\right),$$

and the integration over the PD angular acceptance must be carried out taking into account that

$$\frac{d^3k}{\omega} = \pi m^2(1-x)\,dx\,dz, \quad z = \frac{E^2\theta^2}{m^2},$$

$$\theta = \widehat{\mathbf{k}\mathbf{p}_1}, \quad z_0 = \frac{E^2\theta_0^2}{m^2} \gg 1. \qquad \text{(A.2)}$$

The terms proportional to m^2/t_1^2 (except the term $\frac{m^2}{t_1^2} l_s$) on the right side of Eq. (A.1) were not taken into account in [8]. Just they give a nontrivial NNLO contribution.

The list of necessary angular integrals is represented below.

$$\int \frac{d^3 k}{\omega} \frac{m^2}{t_1^2} = \pi \frac{dx}{1-x},$$

$$\int \frac{d^3 k}{\omega} \frac{m^4}{t_1^2 (t_1 + m^2)} = \pi \left(\frac{1}{1-x} + \ln \frac{x}{1-x} \right) dx,$$

$$\int \frac{d^3 k}{\omega} \frac{m^2}{t_1 (t_1 + m^2)} l_{t_1} = \pi \left(\frac{\pi^2}{6} + \mathrm{Li}_2 (x) + \frac{1}{2} \ln^2 (1-x) \right) dx,$$

$$\int \frac{d^3 k}{\omega} \frac{m^4}{t_1 (t_1 + m^2)^2} l_{t_1} = -\pi \left(\frac{\pi^2}{6} + \mathrm{Li}_2(x) + \frac{1}{2} \ln^2 (1-x) + \right.$$

$$\left. + \frac{\ln (1-x)}{x} + \ln \frac{x}{1-x} \right) dx,$$

$$\int \frac{d^3 k}{\omega} \frac{m^4}{t_1^3} \left(f \left(1 + \frac{t_1}{m^2} \right) - f(1) \right) =$$

$$= \frac{\pi}{2} \left(\frac{\pi^2}{6} + \mathrm{Li}_2 (x) + \frac{1}{2} \ln^2 (1-x) - \frac{1 + \ln (1-x)}{1-x} + \right.$$

$$\left. + \frac{1}{(1-x)^2} \left(\frac{\pi^2}{6} - \mathrm{Li}_2 (x) \right) \right) dx,$$

$$\int \frac{d^3 k}{\omega} \frac{m^2}{t_1^2} l_{t_1} = \pi (1 + \ln (1-x)) \frac{dx}{1-x}, \tag{A.3}$$

where $f(x) = \int\limits_0^x \frac{dz}{z} \ln (1-z)$. Using (A.3) as well as the definition of ρ, we can write the contribution of virtual and soft photons into the quantity $\tilde{K}$ (see (10), (11)) as follows:

$$\tilde{K}^{s+v} = \mathrm{Li}_2 (x) \left(4x - 6 + \frac{5}{1-x} \right) + \frac{8x}{1-x} \ln \Delta +$$

$$+ \frac{\pi^2}{3} \left(2x + \frac{1-7x}{1-x} \right) + \frac{8}{1-x} -$$

$$- 9 + 2x \ln^2 (1-x) + \frac{2x}{1-x} \ln^2 x - \frac{14}{1-x} \ln x. \tag{A.4}$$

APPENDIX B

The correction to the Born cross-section, caused by the radiation of two photons with 4-momentum k_1 and k_2 hitting PD, can be written as follows:

$$d\sigma^h = 4 \frac{\pi \alpha^2 (q^2)}{3q^2} \left(\frac{\alpha}{2\pi} \right)^2 \int \frac{d^3 k_1}{2\omega_1} \frac{d^3 k_2}{2\omega_2} \frac{I}{m^4}, \tag{B.1}$$

where ω_1 (ω_2) is the energy of a photon with 4-momentum k_1 (k_2). PD records the total energy of both photons $\omega_1 + \omega_2 = (1-x)E$. The quantity I reads

$$I = \frac{2Dx_2 - x\eta_1\eta_2 - (1-x_2) D \eta_1 - 2x\eta_1 + 4x}{D^2 \eta_1^2} -$$

$$- \frac{2x(1-x_1)}{x_2 (1-x) D \eta_1} - \frac{2x(1-x_2)}{x_2 (1-x) D \eta_2} + \frac{2(1-x_1)(1-x_2)}{x_1 x_2 \eta_1 \eta_2} +$$

$$+ \frac{2D(D + x(\eta_1 + \eta_2)) + 8x - 2x(\eta_1 + \eta_2)}{D^2 \eta_1 \eta_2} +$$

$$+ ((x_2 - x_1) D + (1-x)((1-x_2)\eta_1 -$$

$$- (1-x_1)\eta_2) + 2(2x + x_1 x_2))/(D \eta_1 \eta_2 x_1) +$$

$$+ ((x_1 - x_2) D + (1-x)((1-x_1)\eta_2 -$$

$$- (1-x_2)\eta_1) + 2(2x + x_1 x_2))/(D \eta_1 \eta_2 x_1) - \frac{2x(1-x_2)}{x_1 (1-x) D \eta_2} +$$

$$+ \frac{2Dx_1 - x\eta_1\eta_2 - (1-x_1) D \eta_2 - 2x\eta_2 + 4x}{D^2 \eta_2^2} +$$

$$+ \frac{2x(2(1-x_2) - x_2 \eta_2)}{D \eta_2^2 x_1} - \frac{2x(1-x_1)}{x_1 (1-x) D \eta_1} +$$

$$+ \frac{2x(2(1-x_1) - x_1 \eta_1)}{D \eta_1^2 x_2}. \tag{B.2}$$

Here, we use the same notation as in [19] with the substitution $p_2 \to -p_2$, $\Delta \to D$.

In the considered case, it is convenient to write the angular phase space of photons in the following form:

$$\int d\Omega_1 \int d\Omega_2 = \pi^2 \left(\frac{m^4}{E^4} \right) \int\limits_0^{z_0} dz_1 \int\limits_0^{z_0} dz_2 \int\limits_0^{2\pi} \frac{d\varphi}{2\pi}, \tag{B.3}$$

where $z_{1,2} = \frac{E^2 \theta_{1,2}^2}{m^2}$, $\theta_{1,2} < \theta_0$. The necessary angular integrals are

$$\frac{1}{\eta_1 \eta_2} \to \frac{1}{x_1 x_2} \ln^2 z_0,$$

$$\frac{1}{D\eta_1} \to - \frac{1}{x_1 x_2 (1-x_1)} \left(\frac{1}{2} \ln^2 z_0 + \ln z_0 \ln \frac{x_2 (1-x_1)^2}{x_1 x} + J \right),$$

$$\frac{1}{D\eta_1^2} \to - \frac{1}{x_1^2 x_2 (1-x_1)} \ln z_0 + \left(\ln \frac{x_2 (1-x_1)^2}{x_1 x} - \right.$$

$$\left. - \frac{(1-x)(1-x_1)}{x_2} \ln \frac{(1-x_1)(1-x)}{x_1 x} \right),$$

$$\frac{1}{D^2 \eta_1} \to \frac{1}{x_1 x_2^2} \ln \frac{(1-x_1)(1-x)}{x_1 x},$$

$$\frac{1}{D^2\,\eta_1^2}\to\frac{1}{x_1^2\,x_2^2}\left(1-\frac{x_1\,x}{x_2}\ln\frac{(1-x_1)(1-x)}{x_1\,x}\right),$$

$$\frac{1}{D\,\eta_1\,\eta_2}\to\frac{1}{x_1^2\,x_2^2}\left(\ln\frac{(1-x_1)(1-x_2)}{x}+\right.$$

$$\left.+\,x_1\ln\frac{x_1\,x}{(1-x)(1-x_1)}+x_2\ln\frac{x_2\,x}{(1-x)(1-x_2)}\right),$$

$$\frac{1}{D^2\eta_2\eta_1}\to\frac{1}{3x_1^2\,x_2^2}\left(-\frac{(3-2x_2)x_2}{2x_1}\ln\frac{x_2\,x}{(1-x)(1-x_2)}-\right.$$

$$\left.-\frac{(3-2x_1)\,x_1}{2x_2}\ln\frac{x_1\,x}{(1-x)(1-x_1)}+\frac{1}{2x_1\,x_2}\ln\frac{x}{(1-x_1)(1-x_2)}-1\right),$$

$$\frac{\eta_2}{D^2\,\eta_1}\to\frac{1}{x_1\,x_2}\,\frac{1}{(1-x_2)^2}\left(\frac{1}{2}\ln^2 z_0+\right.$$

$$+\left(\left(\ln\frac{x_2\,(1-x_1)^2}{x_1\,x}+\frac{x_1\,x_2-x}{x}\right)\ln z_0+J\right)+$$

$$+\frac{1}{2x_1\,x_2\,x\,(1-x_1)}\left(2\,(x-x_1)\ln\frac{(1-x_1)(1-x)}{x_1\,x}+\right.$$

$$\left.+\frac{3x-x_1\,x_2}{1-x_1}L_2-(1-x_2)\,L_1\right). \qquad \text{(B.4)}$$

In this notation, the symbol $\to$ means the corresponding angular interaction and the quantities J, L_1, and L_2 are determined as follows:

$$J=\int\limits_0^1\frac{dz}{z}\ln\frac{x_2(1-x_1)H(z)+z x_1 x_2\,(x-x_1x_2)+x_2^2(1-x_2)^2}{2x_2^2\,(1-x_1)^2\left(1+\dfrac{x_2\,z}{x_1\,x}\right)},$$

$$L_1=\ln\frac{(x_1\,(1-x_2)\,H(1)+x_1x_2\,(x-x_1x_2)+x_1^2\,(1-x_2)^2)}{(2\,x_1\,x_2\,x)},$$

$$L_2=\ln\frac{(x_2\,(1-x_1)\,H(1)+x_1x_2\,(x-x_1x_2)+x_2^2\,(1-x_1)^2)}{(2\,x_2^2\,(1-x_1)^2)},$$

$$H(z)=\sqrt{x_2^2\,(1-x_1)^2+2x_1x_2(x-x_1x_2)\,z+x_1^2(1-x_2)^2\,z^2}. \qquad \text{(B.5)}$$

To derive the differential cross-section, respect to the fraction of the energy deposited in PD by both collinear photons, one needs to integrate also over the energy fraction of one photon (for example over x_1) provided that $x_2+x_1=1-x$, $\Delta<x_1<1-x$. In order to calculate the corresponding contribution into the quantity $\tilde{K}$ (see Eq. (7)), we need to select in (B.4) the terms which do not contain any power of $\ln z_0$. Therefore, the following integrals are requierd:

$$\int\frac{dx_1}{1-x_1}\ln\frac{x_2\,(1-x_1)^2}{x_1\,x}=-\frac{1}{2}\ln^2 x, \qquad \text{(B.6)}$$

$$\int dx_1\left\{1;\frac{1}{1-x_1};\frac{1}{x_2};\frac{1}{x_2^2}\right\}\ln\frac{(1-x_1)(1-x)}{x_1\,x}=$$

$$=\left\{-\ln x;\frac{1}{2}\ln^2 x+\mathrm{Li}_2\,(1-x);\right.$$

$$\left.\frac{\pi^2}{6}+\frac{1}{2}\ln^2 x+\mathrm{Li}_2\,(1-x);\ \frac{1}{(1-x)\,x}\left(1+\ln\frac{x\,(1-x)}{\Delta}\right)\right\},$$

$$\int dx_1\left\{\frac{1}{x_1};\frac{1}{x_1^2}\right\}\ln\frac{(1-x_1)(1-x_2)}{x}=$$

$$=\left\{\frac{1}{2}\ln^2 x;\frac{1+x}{x}\ln x+\frac{1-x}{x}\left(1+\ln\frac{1-x}{\Delta}\right)\right\},$$

$$\int dx_1\left\{\frac{1}{x_2};\frac{1}{x_1}\right\}\ln\frac{(1-x_1)^2}{x}=\left\{\ln x\,\ln\frac{x\,(1-x)}{\Delta}+\right.$$

$$\left.+\,2\,\mathrm{Li}_2\,(1-x);\ \ -\ln x\,\ln\frac{1-x}{\Delta}-2\,\mathrm{Li}_2\,(1-x)\right\}.$$

The straightforward calculation, using (B.2), (B.4) and (B.6), leads to the following contribution into the quantity $\tilde{K}$ in the case in question:

$$\tilde{K}^{\,h}=-\frac{8}{3}+\frac{4}{3}\left(3-\frac{1}{1-x}\right)+\frac{4x}{1-x}\ln\frac{(1-x)}{\Delta}+$$

$$+\left(-\frac{17}{3}+\frac{28}{3(1-x)}-\frac{8}{3(1-x)^2}\ln x+\left(\frac{9}{2}-\frac{7}{6}x-\frac{8}{1-x}+\right.\right.$$

$$\left.+\frac{16}{3(1-x)^2}-\frac{4}{3(1-x)^3}\right)\ln^2 x+\left(1-\frac{7x}{3}\right)\mathrm{Li}_2\,(1-x)+$$

$$+\int\limits_0^{1-x}\left(\frac{x^2+(1-z)^4}{z(1-z-x)(1-z)^2}J+\right.$$

$$\left.+\frac{z\,(1-z-x)-3x}{2(1-z)^2}L_2+\frac{x+z}{2\,(1-z)}L_1\right)dz. \qquad \text{(B.7)}$$

In order to compensate the auxiliary parameter Δ in RC, it is necessary to take into consideration the hard photon radiation (with energy more than ΔE) by a positron. The corresponding quantity can be easily obtained from Eq. (21) of [8] and its contribution to $\tilde{K}$ that contains $\ln\Delta$ is equal to $\frac{4x}{1-x}\ln\frac{1}{\Delta}$. Thus, the final result for $\tilde{K}$, that is the sum of the last term, (A.4) and (B.7), has the form of (11) and does not depend on Δ.

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