POLARIZATION EFFECTS IN SYNCHROTRON RADIATION IN ULTRA-QUANTUM APPROXIMATION

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UDC 539.1.01
‰ 2001

The effect of electronic self-polarization in synchrotron radiation in the approximation of weakly excited state of an electron and a strong magnetic field is investigated. An essential difference of the process probabilities with spin reorientation of an electron along and opposite the field is denoted, which indicates a more expressed effect of self-polarization of electrons in comparison with the quasi-classical approximation.

Despite the fact that the sufficiently much literature was devoted to the investigation of quantum-electrodynamic processes with photons and electrons, this direction continues up to the present days to set the problems which don't lose actuality both for experimental and theoretical research. The general relativistic theory of synchrotron radiation was constructed and investigated, and the common form of expressions describing this process was obtained in [1 – 5]. In particular, the problem was solved in [4, 5] with application of the operator method by analyzing the properties of the mass operator of an electron. However, it is necessary to mark that this process was described more in detail in the quasi-classical ultra-relativistic approximation. An effect of electron self-polarization researched in [2, 6 – 8] in such an approximation indicates that synchrotron radiation brings to the directional process of orientation of the electron spin relative to the external magnetic field. The authors of these works showed that, as a result of synchrotron radiation, 92% of electrons acquire the spin oriented against the magnetic field. In the quasi-classical approximation, the process of synchrotron radiation under a sufficiently small value of the magnetic field proceeds with electrons in strongly excited states with quasi-continuous energy levels n determinated to within $n + dn$. The investigation of polarization effects of this process with nonrelativistic electrons in weakly excited states was not paid so far enough attention [9, 10]. The reason for this is that the description of synchrotron radiation of such electrons requires using the ultra-quantum approximation that implies a relatively large value of the magnetic field comparable with the critical Schwinger magnetic field $H_0 = \dfrac{mc^3}{e\hbar} = 4.41 \cdot 10^{13}$Gs. After the discovery of strongly magnetized rotating neutron stars (pulsars) where the value of the magnetic field is compared with H_0, the problem of analysis of quantum-electrodynamic processes with photons and weakly excited electrons in a strong magnetic field has acquired actuality and great importance to astrophysics.

The purpose of present paper is a research of polarization effects of synchrotron radiation in the approximation where electrons are in weakly excited states in a strong magnetic field (ultra-quantum approximation).

Below, we use the relativistic system of units where $\hbar = c = 1$.

We take the following expression for the electron wavefunction in a uniform constant magnetic field H [11, 12]:

$$\Psi = \frac{A}{\sqrt{S}} \exp\left(-i\varepsilon_n t + ip_y r_y + ip_z r_z\right) \times$$

$$\times \left[i\sqrt{2neH}\, U_n(\xi) + (m + \mu\tilde{m}) U_{n-1}(\xi)\gamma^1\right] u_n, \qquad (1)$$

where ε_n is the energy of an electron in a magnetic field; p_y, p_z are the momenta of an electron along the y and z axes, respectively; S is the normalization area in the yz plane; $\xi = \sqrt{eH}(x + p_y/eH)$ is the displaced coordinate along a quantized direction x; e is the electron charge; $U_n(\xi) = \dfrac{1}{\sqrt{\sqrt{\pi}\, 2^n n!}} \times \exp(-\xi^2/2) H_n(\xi)$ is the Hermite function of order n; μ is the electron polarization which assumes the values ± 1; γ^1 is a Dirac matrix in the standart representation; $\tilde{m}^2 = m^2 + 2neH$; A is the normalization coefficient and u_n is the constant bispinor, their explicit forms are shown in [12].

The constant magnetic field H is directed along the z axis. Wavefunction (1) corresponds to a gauge of the external field where the 4-vector potential is defined as

$$A_0 = 0, \quad \mathbf{A} = (0, Hx, 0). \qquad (2)$$

The energy spectrum of an electron takes the form:

$$\varepsilon_n = \sqrt{m^2 + p_z^2 + 2neH} = \sqrt{\tilde{m}^2 + p_z^2}, \qquad (3)$$

n is the principal quantum number (Landau level number), $n = 0, 1, 2,$` . The differential probability of the process per unit time, in which the summation is performed over photon polarizations at various polarizations of the electron both in the initial and final states looks:

$$dW^{\mu\mu'}_{nn'} = \left| -ie\int_{-\infty}^{\infty} d^4x\,\overline{\psi}'(\zeta')\gamma_\nu A^\nu\psi(\xi)\right|^2 \frac{1}{T}dN =$$

$$= \frac{\alpha m\omega}{\varepsilon_n(\varepsilon'_{n'} + \omega u^2)}\left[\sum_{i=1}^{5}\Phi_i\right]du, \tag{4}$$

where T is a large time interval; $u = \cos\theta$ is the cosine of the angle between the directions of the magnetic field and the motion of the final photon; α is the fine-structure constant; dN is the number of final states which equals:

$$dN = \frac{SVd^3kd^2p}{(2\pi)^5}. \tag{5}$$

$$d^2p = dp_y dp_z\,;\; d^3k = \omega^2 d\omega d\varphi du.$$

The components under the summation symbol in formula (4) have the form:

$$\begin{cases} d\Phi_1 = -\dfrac{\mu\mu' nn'(eH)^2}{(\tilde{m} + \mu m + \&)(\tilde{m}' + \mu' m + \&')}J^2(n', n)\,, \\ d\Phi_2 = -\dfrac{1}{4}\mu\mu'\lambda\lambda'(\tilde{m}' + \mu' m)(\tilde{m} + \mu m)\times \\ \qquad \times J^2(n'-1, n-1)\,, \\ d\Phi_3 = \dfrac{n'\lambda(\tilde{m} + \mu m)(eH)}{2\tilde{m}\tilde{m}'(\tilde{m}' + \mu' m + \&')}(\varepsilon_n\varepsilon'_{n'} - p_z p'_z u + \mu\mu'\tilde{m}\tilde{m}')\times \\ \qquad \times J^2(n', n-1)\,, \\ d\Phi_4 = \dfrac{n\lambda'(\tilde{m}' + \mu' m)(eH)}{2\tilde{m}\tilde{m}'(\tilde{m} + \mu m + \&)}(\varepsilon_n\varepsilon'_{n'} - p_z p'_z u + \mu\mu'\tilde{m}\tilde{m}')\times \\ \qquad \times J^2(n'-1, n)\,, \\ d\Phi_5 = -\sqrt{nn'}\lambda\lambda'(eH)J(n', n)J(n'-1, n-1)\,. \end{cases} \tag{6}$$

In Eqs.(4), (6), and further, by using symbol ($'$), we mark the final state of an electron (the state after the radiation process).

The probability of synchrotron radiation averaged over the polarizations of the initial electron and summed over the polarizations of the final electron has the form [13]:

$$\frac{dW_{nn'}}{du} = [-m^2J^2(n', n) + (pp' - m^2)\times$$

$$\times (J^2(n', n-1) + J^2(n'-1, n)) -$$

$$- 4m^2h\sqrt{n'n}\,J(n'-1, n-1)J(n', n) -$$

$$- m^2J^2(n'-1, n-1)]\,\frac{\alpha\omega}{\varepsilon_n(\varepsilon_n - \omega(1-u^2))}, \tag{7}$$

where $pp' = \varepsilon_n\varepsilon'_{n'} - p_z p'_z$.

There are two conservation laws for the projection of the momenta p_y, p_z on the y, z axes and the conservation law for the energy of the process considered:

$$p_y = p'_y + k_y\,,\; p_z = p'_z + k_z\,,\; \varepsilon_n = \varepsilon'_{n'} + \omega. \tag{8}$$

The expression for the photon frequency in synchrotron radiation follows from the conservation laws (8):

$$\omega = \frac{(\varepsilon_n - p_z u) - \sqrt{(\varepsilon_n - p_z u)^2 - 2(n - n')(eH)(1-u^2)}}{1-u^2}. \tag{9}$$

Integration over the x coordinate (quantized direction) in (4) brings to the appearance of special functions $J = J(n, m)$ (Laguerre functions) [2], the form of which is:

$$J(n, m) = \exp(-\eta/2)\times$$

$$\times\begin{cases} \eta^{(m-n)/2}\sqrt{\dfrac{m!}{n!}}\,\dfrac{1}{(m-n)!}\times \\ \qquad \times F(-n, m-n+1, \eta)\,, \quad m > n\,, \\ (-1)^{m-n}\eta^{(n-m)/2}\sqrt{\dfrac{n!}{m!}}\,\dfrac{1}{(n-m)!}\times \\ \qquad \times F(-m, n-m+1, \eta), \quad n > m. \end{cases} \tag{10}$$

Here, m and n are nonnegative integer numbers, and F is the confluent hypergeometric function which is a Laguerre polynomial accurate within a coefficient.

The parameter η in (10) is determined as

$$\eta = \frac{k_x^2 + k_y^2}{2eH} = \frac{\omega^2(1-u^2)}{2eH}. \tag{11}$$

It equals to the square of the product of the transversal projection of a photon wave vector to the direction of the magnetic field $k_\perp$ by a specific transverse size of the electron wavefunction R (Landau radius): $\eta = (k_\perp R)^2$.

We consider the radiation process in the strong external magnetic field with weakly excited electron states. Such an approximation implies the experimental distinguishing of Landau levels. It corresponds to the condition:

$$\Delta n = 1, \qquad (12)$$

where Δn is the number of levels in the final states (5).

Nevertheless, we consider the value of the magnetic field to be much less than the critical Schwinger magnetic field $H_0 = 4.41 \cdot 10^{13}$:

$$h = \frac{H}{H_0} << 1. \qquad (13)$$

Hereinafter, we will suppose that $p_z = 0$. This condition doesn't reduce the generality of the process under study because the transition to the reference system where $p_z \neq 0$ is always possible. In addition, the value of the magnetic field doesn't change. Granting the proposed simplification and making the series expansion in the small parameter h, we will rewrite the presented above expressions (3), (9), (11) in the approximation of synchrotron radiation by an electron in the weakly excited state (12).

The electron energy spectrum is:

$$\varepsilon_n = m\sqrt{1 + 2nh} = m + mnh - \frac{1}{2}mn^2h^2 + O(h^3). \qquad (14)$$

The frequency of photon radiation

$$\omega = (n - n')mh - \frac{1}{2}(n - n')m\,[(n + n') +$$

$$+ (n - n')u^2]h^2 + O(h^3). \qquad (15)$$

It is obvious from (14) and (15) that, in the considered approximation, the electron energy takes nonrelativistic values:

$$\varepsilon \sim m,\ \omega \sim hm << m. \qquad (16)$$

Eq.(11) becomes

$$\eta = \frac{\omega^2(1 - u^2)}{2hm^2} = \frac{1}{2}(n - n')^2(1 - u^2)h -$$

$$- \frac{1}{2}(n - n')^2(1 - u^2)\,[(n + n') +$$

$$+ (n - n')u^2]h^2 + O(h^3). \qquad (17)$$

Since h is a small quantity, the parameter η in the ultra-quantum approximation

$$\eta << 1. \qquad (18)$$

The explicit forms of the probabilites per unit time of the considered synchrotron radiation in approximation (12), (16), (18) under arbitrary polarizations of an electron μ before and μ' after radiation are:

$$\frac{dW^{++}}{du} = \frac{\alpha m}{\hbar}\,\frac{(n - n')^{2(n - n') + 1}(n - 1)!}{2^{(n - n')}(n' - 1)!((n - n')!)^2}\times$$

$$\times (1 - u^2)^{(n - n' - 1)}(1 + u^2)h^{(n - n' + 1)},$$

$$\frac{dW^{--}}{du} = \frac{\alpha m}{\hbar}\,\frac{(n - n')^{2(n - n') + 1}n!}{2^{(n - n')}n'!((n - n')!)^2}\times$$

$$\times (1 - u^2)^{(n - n' - 1)}(1 + u^2)h^{(n - n' + 1)},$$

$$\frac{dW^{+-}}{du} = \frac{\alpha m}{\hbar}\,\frac{(n - n')^{2(n - n') + 3}(n - 1)!}{2^{(n - n' + 1)}n'!((n - n')!)^2}\times$$

$$\times (1 - u^2)^{(n - n' - 1)}(1 + u^2)h^{(n - n' + 2)},$$

$$\frac{dW^{-+}}{du} = \frac{\alpha m}{\hbar}\,\frac{(n - n')^{2(n - n') + 3}n!}{2^{(n - n' + 3)}(n' - 1)!((n - n' + 1)!)^2}\times$$

$$\times (1 - u^2)^{(n - n' - 1)}[1 + (n - n')^2u^6 -$$

$$- (3(n - n')^2 + 2(n - n'))u^4 + (4(n - n')^2 +$$

$$+ 6(n - n') + 1)u^2]\,h^{(n - n' + 4)}. \qquad (19)$$

The analysis of the obtained expressions (19) shows that the probability of the process essentially depends on initial and final electron polarizations. The processes without reorientation of the electron spin are the most probable (the power of the small parameter h is the smallest) while the processes with a change of polarization (spin flip transition) are less probable (the power of the small parameter h is relatively higher) and in addition they have a different power of parameter h. Also it is necessary to note that the most probable process among all the possible ones is the process of photon radiation with the electron transition

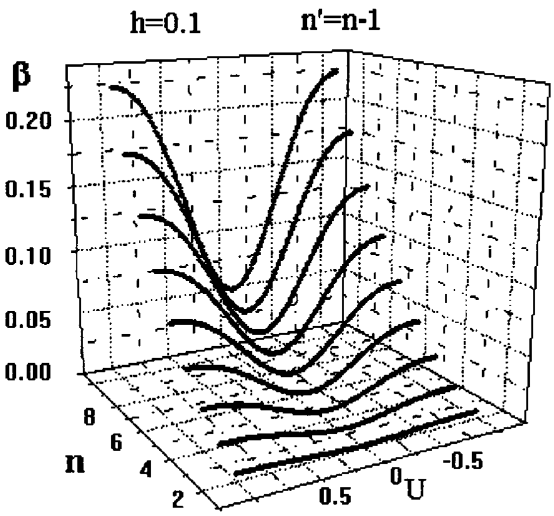

Fig.1. Parameter β versus both angle of photon radiation and initial electron energy level n

to the adjacent energy level $\nu = (n - n') = 1$. Every next increase of ν results in an additional power of the parameter h in Eqs.(19) for differential probability. For the most probable transition $\nu = 1$, Eqs.(19) take the form:

$$\frac{dW^{++}}{du} = \frac{\alpha m}{\hbar} \frac{1}{2}(n - 1)(1 + u^2)h^2,$$

$$\frac{dW^{--}}{du} = \frac{\alpha m}{\hbar} \frac{1}{2} n(1 + u^2)h^2,$$

$$\frac{dW^{+-}}{du} = \frac{\alpha m}{\hbar} \frac{1}{4}(1 + u^2)h^3,$$

$$\frac{dW^{-+}}{du} = \frac{\alpha m}{\hbar} \frac{1}{64} n(n - 1)(1 + 11u^2 - 5u^4 + u^6)h^5. \tag{20}$$

Further, polarization effects connected with reorientation of the electron spin will be considered. We introduce the parameter

$$\beta = \frac{dW^{-+}/du}{dW^{+-}/du} = \frac{1}{16} n(n - 1)\,[1 + 11u^2 -$$

$$- 5u^4 + u^6]\,\frac{1}{(1 + u^2)} h^2 \tag{21}$$

characterizing the difference of probabilities with spin reorientation.

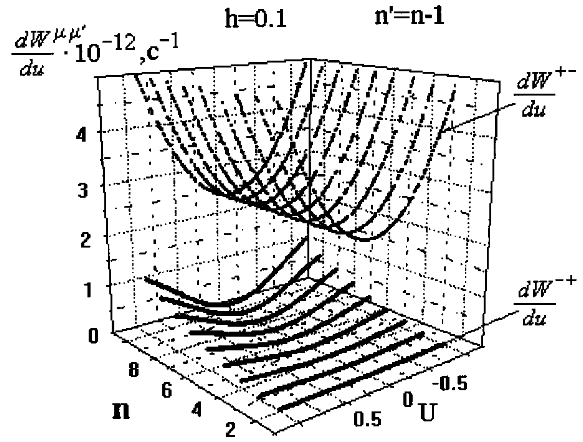

Fig.2. Differential probabilities dW^{+-}/du and dW^{-+}/du per unit time versus both angle of photon radiation and initial electron energy level n

The radiation degree of electron polarization is connected with the parameter β in the following way:

$$\zeta = \frac{dW^{+-}/du - dW^{-+}/du}{dW^{+-}/du + dW^{-+}/du} \approx 1 - 2\beta. \tag{22}$$

The parameter β versus both an angle of photon radiation and an initial electron energy level n is given in Fig.1. On the graph, a rise of the parameter β for more excited electron states is seen. Also one can see that photons are radiated in preference under an angle to the direction of electron motion. In particular, at $h = 0.1$, $n = 2$, $u = 0$, the parameter $\beta = 0.00125$ and the radiation degree of polarization $\zeta = 0.9975$.

It is necessary to mark that, in the quasi-classical approximation in the process of synchrotron radiation with spin reorientation, there are about ten electrons with spin -1 (spin-down state) accounted for one electron with spin $+1$ (spin-up state) in the final state. As a result of synchrotron radiation, only 92% of electrons have the spin oriented against the direction of the magnetic field [2]. Here, the effect of self-polarization doesn't depend on the value of the magnetic field. In the ultra-quantum approximation in the considered process with spin reorientation, there are nearly 1000 electrons with spin -1 accounted for one electron with spin $+1$ in the final state at $h = 0.1$. Also we note that the probabilities ratio β depends on the value of the magnetic field and is proportional to h^2.

Fig.2 shows the differential probabilities dW^{+-}/du and dW^{-+}/du per unit time versus both the angle of photon radiation and initial electron energy level n. It is seen from the graph that electrons with the initial spin directed along the magnetic field participate predominantly in the process of photon radiation with spin reorientation. The probability of the photon radiation for electrons with the opposite spin relative to the direction of the magnetic field is less and it increases comparatively slightly with the initial energy level.

We must note that, in the considered approximation (12), (16), (18) unlike the quasi-classical case [2], the effect of self-polarization essentially depends on the value of the magnetic field H and radiation angle u.

The essential difference in the process probabilities for electrons with spin reorientation (22) can be explained by analyzing the expression for the distribution of the probability density with a definite energy level:

$$j^0 = |\Psi|^2 = 2eA\varepsilon(\varepsilon - \mu\tilde{m})(2nem^2hU_n^2 +$$

$$+ (m + \mu\tilde{m})^2U_{n-1}^2). \tag{23}$$

For two electrons which are on the adjacent energy levels $n_1 = 1$ and $n_2 = 2$ but with the contrary values of spins, the following expressions in approximation (12), (16), (18) take place:

$$j^0 = 2\xi^2 + \left(\frac{1}{2} - \xi^2\right)h + O(h^2), \quad n_1 = 1, \mu = -1,$$

$$j^0 = 2\xi^2 + \left(\frac{1}{2} - 4\xi^2 + 2\xi^4\right)h + O(h^2), \quad n_2 = 2, \mu = 1. \tag{24}$$

As one can see, the expressions for j^0 at zeroth order of the small parameter h coincide in both cases. The similar situation holds also in the following cases with more excited states of an electron. Thereby, the following equality is true:

$$|\Psi_n^-|^2 = |\Psi_{n+1}^+|^2, \; n = 0, 1, 2, \ldots \; . \tag{25}$$

By applying the additional quantum number

$$N = n - \frac{1}{2} - \frac{\mu}{2}, \; n = 0, 1, 2, \ldots \; , \tag{26}$$

one can rewrite (25) in terms of N:

$$|\Psi_N^-|^2 = |\Psi_N^+|^2. \tag{27}$$

One sees from (27) that the quantum number N characterizes the spatial distribution of electron probability density while the principal quantum number n describes the electron energy spectrum.

There are two cases of the synchrotron radiation process with spin reorientation in the indices of the principal quantum number n and quantum number N schematically depicted in Fig.3. Fragments (a and

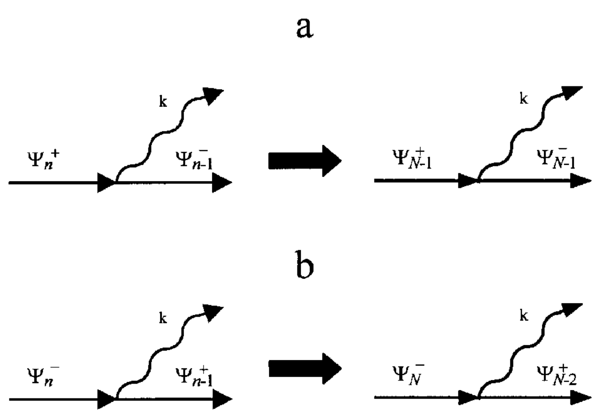

Fig.3. Feynman diagrams of the process of synchrotron radiation with spin reorientation of electron

b) correspond to the considered process with electron spin reorientation against and along the direction of the magnetic field. In Fig.3, we see that, in the first case (fragment a), the distribution of electron probability density doesn't change during radiation. While, in the second case (fragment b), one can observe a change of the distribution of electron probability density and the reduction of the quantum number N by 2. As was marked supra, the transition onto a lower Landau level brings to the appearance of an additional power of h in the equations for the process probabilities (19), (20). This fact explains the difference in probabilities by two orders of the small parameter h in (19), (20).

Integrating over the angle u of photon radiation, the total probabilities of synchrotron radiation at various electron polarizations can be put into the form:

$$W^{++} = \frac{\alpha m}{\hbar}\,\frac{4}{3}(n-1)h^2, \; n = 2, 3, 4, \ldots \; ,$$

$$W^{--} = \frac{\alpha m}{\hbar}\,\frac{4}{3}nh^2, \; n = 1, 2, 3, \ldots \; ,$$

$$W^{+-} = \frac{\alpha m}{\hbar}\,\frac{2}{3}h^3, \quad n = 1, 2, 3, \ldots \; ,$$

$$W^{-+} = \frac{\alpha m}{\hbar}\,\frac{5}{42}n(n-1)h^5, \; n = 2, 3, 4, \ldots \; . \tag{28}$$

The total probability of the process averaged over polarizations of an initial electron and summed over polarizations of a final one has the form:

$$W = \frac{1}{2}\sum_{\mu,\mu' = -1}^{1} W_{nn'}^{\mu\mu'} = \frac{2}{3}\,\frac{\alpha m}{\hbar}\,(2n-1)h^2 =$$

$$= \frac{2e^4H^2}{3m^3c^5\hbar}(2n-1). \quad (29)$$

It can be noted that the obtained expressions (28) for the probabilities of synchrotron radiation with the exception of W^{-+} at the least possible value of the principal quantum number n to within a coefficient ~ 1 coincide with analogous expressions well known in the quasi-classical approximation. In particular, the value $1/W^{+-}$ congruents with the polarization time and $1/W$ with the emission time of photons [9].

The authors are grateful to P.I. Fomin for his constant help in holding this work and also to S.P. Roshchupkin for useful discussions.

The work partly was supported by the State Fund of Fundamental Researches.