NON-UNITARY QUANTUM THEORY (MATHEMATICAL FOUNDATIONS). 2

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Extended Fock representations of Heisenberg algebras interlaced with their non-Fock representations are built. The main result of Part 2 is the proof of the existence of additional variables in the non-unitary scheme (see Part 1), which are Lorentzian scalars. A new physical entity, called physical vacuum or ether, is described by them [1]. It is shown that the fermion-antifermion symmetry is invalid on the space of additional variables. Other principal consequences of the theory are discussed.

5. Extended Fock Representation of the Algebra h_4. Additional Variables

1) The non-Fock representation $T_1(h_4)$ is, in any sence, an extension of the Fock representation $T_0(h_4)$ for the same algebra, i.e., $T_0(h_4) \subset T_1(h_4)$.

Statement 1. The representation $T_1(h_4)$ is interlaced with the extended Fock representation of the algebra h_4 which, as the Fock representatoion, is given by the operators $a^1_\alpha = \partial/\partial z_\alpha$, $a^2_\alpha = z_\alpha\,(\alpha = 1, 2)$, but is realized in the space $\mathbf{F} = \mathcal{F}_F \otimes \mathcal{F}_0$, where $\mathcal{F}_F$ is the space of the Fock representation $T_0(h_4)$ formed by functions of two complex variables z_1, z_2, and $\mathcal{F}_0$ is a space of functions depending on an additional variable $z \doteq z_2$ (in this case, $\partial/\partial z \doteq \partial/\partial z_2$) having non-standard transformational properties.

The existence of additional variables in $T_1(h_4)$ is connected with existence of two various units on the space $\mathbf{F}^{(\lambda)}_\zeta$ (see Part 1). This becomes obviously if $\mathbf{F}^{(\lambda)}_\zeta$ is written as a columm

$$\mathbf{F}^{(\lambda)}_\zeta = \begin{pmatrix} \vdots \\ \mathcal{F}_{\lambda+\frac{1}{2}} \\ \mathcal{F}_\lambda \\ \mathcal{F}_{\lambda-\frac{1}{2}} \\ \vdots \end{pmatrix}. \qquad (1)$$

Then one of units which we denote by **1** is represented by a unity diagonal matrix infinite in all sides

$$\mathbf{1} = \begin{pmatrix} \ddots & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & 1 & \\ & & & & \ddots \end{pmatrix} \qquad (2)$$

(a warning: this unit should not be mixed up with the unit of the basic field of scalars, which enters the right part of commutation relations (Eq. (22), Part 1) and is denoted by $\mathbf{1}^0$). In (2), the unit **1** as an operator on the subspace $\mathcal{F}_\lambda$ will be invariant under transformations from the group SU (2) [2]:

$$T_\lambda(u)\cdot 1 \cdot T^{-1}_\lambda(u) = 1, \quad u = \begin{pmatrix} \alpha & \beta \\ -\bar\beta & \bar\alpha \end{pmatrix} \in \mathrm{SU}(2), \qquad (3)$$

where the operators $\vec{L}^{(\lambda)}$ (Eq. (31), Part 1) are generators of $T_\lambda(u)$. Thus, the unit **1** is an SU (2)-scalar (it is also a scalar under transformations of $U(2)$ and $U^c(2)$).

Another unit is the second component of the spinor a^1_α (see formulae (21), (25), Part 1). If the space $\mathbf{F}^{(\lambda)}_\zeta$ is written down as (1), this unit is represented by a matrix in which units are displaced (in comparison with the matrix (2)) by one step in the right top corner

$$a_2^1 = \begin{pmatrix} \ddots & \ddots & & & & \\ & 0 & 1 & \mathbf{0} & & \\ & & 0 & 1 & & \\ & \mathbf{0} & & 0 & 1 & \\ & & & & \ddots & \ddots \end{pmatrix}. \tag{4}$$

The matrix can be considered as diagonal: we see, in the case the concept of diagonal is rather conditional (there is no first element in (1)). So both matrices **1** and a_2^1 are actually indistinguishable. However, in the case of a_2^1, the unit 1 in (4) as a operator on $\mathcal{F}_\lambda$ is transformed absolutely under another law, namely, [2]:

$$T_{\lambda-\frac{1}{2}}(u)\cdot 1\cdot T_\lambda^{-1}(u) = -\,\overline{\beta}\frac{d}{d\zeta} + \overline{\alpha}\cdot 1 \tag{5}$$

following from (Eq. (26), Part 1). An equality of quantities having different transformational properties denotes as[1]

$$a_1^2 \doteq \mathbf{1}. \tag{6}$$

If now to pass from the ζ-realization (Eq. (25), Part 1) to the Fock z-realization with the help of the transformation [2]

$$f(\zeta) = \int K(\zeta;\bar z_1,\bar z_2)\, f(z_1,z_2)\, d\mu(z_1,z_2) = \hat K f(z_1,z_2), \tag{7}$$

where $d\mu(z_1,z_2)$ is the Gauss measure on $\mathbf{C}^2$ (see Eq. (8), Part 1)), and (compare the structure of K with the structure of the space $\mathbf{F}_\zeta$ (Eq. (25), Part 1))

$$K(\zeta;\bar z_1,\bar z_2) \equiv \bigoplus_{p\in\mathbf{Z}} \bar z_2^{\,2\lambda+p} \exp \zeta\frac{\bar z_1}{\bar z_2}. \tag{8}$$

(The operator $\hat K$ in (7) possesses the property

$$\hat K a_\alpha^a(z) = a_\alpha^a(\zeta)\hat K \tag{9}$$

and refers to as interlacing. Here, $a_\alpha^a(z)$ are defined by Eq. (27), Part 1, and $a_\alpha^a(\zeta)$ are by Eq. (25), Part 1, by virtue of the diagram (here, in accordance with (7), we have written down $\hat K$ as $\hat K = K\circ d\mu$ and have taken into account that $\bar z_\alpha d\mu = -\frac{\partial}{\partial z_\alpha} d\mu$, and T is the mapping of a cotangent space in a tangent space)

$$\begin{array}{ccccccc}
a_1^1(\zeta) & \xrightarrow{T} & a_1^2(\zeta) & & & & \\
\uparrow K & & \uparrow K & & & & \\
\bar z_1 & \xrightarrow{T} & \partial/\partial\bar z_1 & & & & \\
\uparrow d\mu & & \uparrow d\mu & & & & \\
\partial/\partial z_1 & \xleftarrow{T} & z_1 & & & & \\
a_2^2(\zeta) & \xleftarrow{T} & a_2^1(\zeta) & \doteq & \mathbf{1} & \rightarrow & ? \\
\uparrow K & & \uparrow K & & & & \uparrow K \\
\partial/\partial\bar z_2 & \xleftarrow{T} & \bar z_2 & \doteq & \bar z & \rightarrow & \partial/\partial\bar z \\
\uparrow d\mu & & \uparrow d\mu & & \uparrow d\mu & & \uparrow d\mu \\
z_2 & \xrightarrow{T} & \partial/\partial z_2 & \doteq & \partial/\partial z & \xleftarrow{T} & z
\end{array} \tag{10}$$

equality (6) will cause existence of the variable $z \doteq z_2$ being by an SU(2) $-$ scalar as well as **1**.

Statement 2. If $f(\bar z_2)$ is a function, so $f(\bar z_2)K = CK$, where K is defined by (8), and $C = f(1)$ [2][2].

It follows from this that K has a non-trivial kernel: Ker $K = \{f(\bar z_2) \mid f(1) = 0\}$, therefore, $\hat K$ has no inverse (so, indeed, $\hat K$ is an interlacing operator).

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[1] An analogous situation takes place in the case of the Dirac matrices $\beta \doteq \gamma_4$ where β enters the definition of Dirac conjugate bispinor $\bar\psi = \psi^+\beta$ and is transformed under the formula $S^+\beta S = \beta$, while γ_4 being by the fourth component of a vector γ_μ is transformed under the law $S^{-1}\gamma_4 S = L_{4\mu}\gamma_\mu$. The operators

$$\begin{pmatrix} \ddots & & & & \mathbf{0} \\ & 0 & & & \\ & & 1 & & \\ \mathbf{0} & & & 0 & \\ & & & & \ddots \end{pmatrix}$$

are marked, too, by the ambiguity specified above. This means that, in the non-unitary theory, there are neither projectors nor any quantum logic (compare with [3]).

[2] In particular, the series $\sum_{n=-\infty}^{\infty} z^n$ summable to the Dirac $\delta(1-z)$-function has the same property. Note that Euler puts this series equal to zero by mistake, which now is well clear.

Statement 3. The factor space $\mathbf{F}/\operatorname{Ker} K$ is isomorphic to the space $\mathbf{F}_\zeta^{(\lambda)}$. Hence, we have $\mathcal{F}_F \subset \mathbf{F}_\zeta^{(\lambda)} \subset \mathbf{F}$.

Consequence 1. The non-standard oscillator is equivalent to a double standard oscillator described by variables z_2, $\partial/\partial z_2$ and $z \doteq z_2$, $\partial/\partial z \doteq \partial/\partial z_2$.

This statement is almost a trivial consequence of the Zermelo theorem (or the axiom of choice): it follows from an opportunity of complete ordering of the set $\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, which is written down as $\mathbf{Z} = \{0, 1, 2, \ldots; -1, -2, \ldots\} = \{\mathbf{Z}_+, \overline{\mathbf{Z}}_+\}$ where $\mathbf{Z}_+ = \{0, 1, 2, \ldots\}$ and $\overline{\mathbf{Z}}_+ = -\{\mathbf{Z}_+ \setminus 0\}$. Here, the states of the standard oscillator are marked by the set $\mathbf{Z}_+$ and the states of the second oscillator by the set $\overline{\mathbf{Z}}_+$ (states with negative occupation numbers), which describe the states of the physical vacuum (ether). However, the fact that the second oscillator has trivial transformational properties is a non-trivial result at all.

It is worth noting that the physical vacuum (as a ground state of the World) was postulated by Heisenberg [4].

Thus, it is possible to write down that $T_1(h_4) \sim T_0(h_4) + T_0(h_2)$, and the space of the extended Fock representation is $\mathbf{F} = \overline{U[z_1, z_2]\,\mathcal{F}_0}^{\,\tau}$ where the space of additional variables $\mathcal{F}_0$ plays a role of a cyclic subspace (as is known, the space of the Fock representation is generated by a unique cyclic vector, i.e., by the mathematical vacuum 1, therefore, $\mathcal{F}_F = \overline{U[z_1, z_2]\cdot 1}^{\,\tau}$).

In general, we have $T_k(h_{2n}) \sim T_0(h_{2n}) + T_0(h_{2k})$, where h_{2k} is a subalgebra in h_{2n}, so that $T_k(h_{2n})$ contains k additional variables.

It is interesting to notice that the existence of an additional variable, the $SU^c(2)$-scalar $z \doteq z_2$, was felt for a long time. It is enough to look at the formulae which set representations of the group SU(2) in the class of functions depending on the projective variable z_1/z_2 (for example, see [5]). The pure geometrical proof of existence of an SU(2)-scalar is given in [2]. However, here we are not stopped on the proof.

2) In the suggested theory, the additional variables play a role of hidden parameters which try to enter for a long time and unsuccessfully into quantum theory (their introduction is forbidden by the well-known von Neumann theorem [3]). Nevertheless, an introduction of additional variables proves to be possible. However, it is connected with a radical reorganization of the Heisenberg – Schrödinger quantum theory: it turns from the unitary theory into a non-unitary one. Moreover, the sence of our hidden variables is absolutly other: they are not additional variables of usual quantum objects (as they were understood by de Broglie [6]), they are variables of the absolutely other physical object – the dynamical system (physical vacuum or ether) generating usual quantum objects (elementary particles). And it will be shown further.

If the formula for spaces $\mathbf{F}_\zeta^{(\lambda)} = \hat{K}(\mathcal{F}_F \otimes \mathcal{F}_0)$ interlaced by the operator $\hat{K}$ is written for representations of the group SU(2), which is realized in these spaces, we obtain the relation (compare with (9))

$$\left(\bigoplus_{p\in\mathbf{Z}} D^+\left(\lambda + \frac{p}{2}\right)\right)\hat{K} = \hat{K}\left(\bigoplus_{n\in\mathbf{Z}_+} D\left(\frac{n}{2}\right)\otimes D_h(0)\right), \tag{11}$$

where $D\left(\frac{n}{2}\right)$ are finite-dimensional representations realized in the Fock space $\mathcal{F}_F$ and $D_h(0)$ is the non-trivial unity representation realized in the space of additional variables $\mathcal{F}_0$. It is necessary to pay attention that relation (11) holds at any λ with the exception of integer or half-integer numbers.

As is proved in [2], the semispinor representation $D^+(\lambda)\,(\lambda \neq n/2)$ of the algebra su(2) is irreducible algebraically and topologically. Hence, it is not equivalent to the well-known finite-dimensional representations of this algebra. However, the infinite system of semispinor representations is equivalent to an infinite system of spinor representations (see formula (11)). This fact is of fundamental importance for physics: formula (11) describes how the complete symmetry of SU(2) and $SU^c(2)$ (inherent to finite-dimensional objects – spinors) arises from the spontaneously broken symmetry inherent to semispinors – inifinite-dimensional objects (see Section 4).

However, in spite of the symmetry of group SU(2) being restored, the complete symmetry of the group $Sp(2, \mathbf{R})$ in the space of the extended Fock representation remains still broken. The breaking is caused by a topologization of representation space (more exactly, by the topology of the dual pair of spaces (Φ', Φ)): a representation of the subgroup $e^{i\vec{\theta}(\vec{\Gamma} + i\vec{N})}$ is realized only in the space of generalized functions of the exponential type Φ', while a representation of another subgroup $e^{i\vec{\theta}(\vec{\Gamma} - i\vec{N})}$ is in Φ. It is interesting to notice that if to reject the nontrivial unity representation $D_h(0)$, we come to the unitary theory. This implies that, from the point of view of the non-unitary theory, the unitary theory is not complete. The latter gives a description of quantum objects only, while the non-unitary theory gives a joint description of quantum and subquantum objects (a subquantum object is understood to be the physical vacuum related to the space $\mathcal{F}_0$).

3) It is interesting also to notice that the splitting of the phase transformation $U(1) = e^{i\alpha L_0}$ is connected with additional variables. Indeed, on the one hand, it should be $L_0 z = 0$, as z is an SU(2)-scalar, and, hence, $\vec{L} z = 0$ and $\vec{L}^2 z = L_0 (L_0 + 1) z = 0$. The operator L_0 is denoted by $L_0^{(l)}$. On the other hand, z cannot be an $(\vec{N}, \vec{\Gamma})$-scalar, as conditions $\vec{N} z = \vec{\Gamma} z = 0$, $\Gamma_0 z = \frac{1}{2} z$ are not conformed with values of the Casimir operators $\vec{L}^2 - \vec{N}^2 = -\frac{3}{4}$, $\vec{\Gamma}^2 - \Gamma_0^2 = \frac{1}{2}$. z must behave under these transformations as z_2, i.e. (as $z \doteq z_2$), it should be $L_0 z = \frac{1}{2} z$. The operator L_0 is denoted by $L_0^{(i)}$. Thus, we have $L_0^{(l)} z = 0$ and $L_0^{(i)} z = \frac{1}{2} z$. In this connection, it should be to put that $\vec{L}^2 = L_0^{(l)} (L_0^{(l)} + 1)$, $\vec{N}^2 = L_0^{(l)} (L_0^{(l)} + 1) + \frac{3}{4}$ and $\Gamma_0 = L_0^{(i)} + \frac{1}{2}$, $\vec{\Gamma}^2 = (L_0^{(i)} + \frac{1}{2})^2 + \frac{1}{2} = L_0^{(i)} (L_0^{(i)} + 1) + \frac{3}{4}$. The contradiction specified above is completely removed.

It is resonable to attach the phase transformation $e^{i\alpha L_0^{(l)}}$ to $e^{i \vec{L}\vec{\theta}}$, expanding, thus, the group SU(2) up to U(2), and to add the transformation $e^{-i\alpha_0 \Gamma_0}$ to the transformaition $e^{i\vec{\alpha}\vec{\Gamma}}$ and to get the transformation $e^{i\alpha_\mu \Gamma_\mu}$. Note that an analogous splitting of phase transformation was postulated by Heisenberg [4] in his theory of a ground state of the World.

6. Non-Fock Representation $T_2(h_8^{(*)})$ of the Aalgebra $h_8^{(*)}$

1) The algebra h_4, appeared for the first time in Majorana's work [7] in connection with the known equations that now bear his name (see also [8]), is unsufficient from the physical point of view for many reasons. First, though the representation theory of this algebra allows one to build relativistic (but Majorana's) 4-spinors under the formula $\psi_\alpha = \langle \dot{f}, \phi_\alpha f \rangle$ where $\phi_\alpha = \begin{pmatrix} a_\alpha^1 \\ a_\alpha^2 \end{pmatrix}$, however, it does not allow one to obtain fields $\psi(X)$ on the space-time $\mathbf{A}_{3,1}$ without postulating the existence of the space $\mathbf{A}_{3,1}$. Certainly, it is a serious lack. And the point is that the group of automorphisms Sp (2, **R**) of the algebra h_4, locally isomorphic to the de Sitter group SO(3, 2) which acts in the Majorana fiber $S_4 \ni \psi$, contains neither the Poincare group P nor the translation group with which only it would be possible to connect (not to postulate) the existence of the space $\mathbf{A}_{3,1}$ (nevertheless, Dirac particles are necessary in particle theory rather than Majorana ones. Secondly, a representation of the subalgebra sl(2, **C**) (generated in $\mathbf{F}_\zeta^{(\lambda)}$ by the operators $\vec{L}, \vec{N}$, see Section 3, Part 1), as well as a representation of the algebra sp (1, **C**) in the space F_F (see Section 2, Part 1) is a cycle of two representations $[\frac{1}{2}, 0]^+ \oplus [0, \frac{1}{2}]^+$ which do not form a complete system. The decycling results in an infinite complete system of real semispinor representations of the group $SU^c(2) \approx SL(2, \mathbf{C})$ of the following kind (an SU^c-analog of Eq. (30), Part 1 [9])[3]

$$\bigoplus_{p \in \mathbf{Z}} \bigoplus_{q \in \mathbf{Z}} \left(\lambda + \frac{p}{2}, \lambda' + \frac{q}{2} \right)^+ \tag{12}$$

(real representations $(\lambda, \lambda')^+ = (\lambda, 0)^+ + \overline{(\bar{\lambda}', 0)^+}$, where $(\bar{\lambda}', 0)^+ = (0, \lambda')^+$ is an antianalytic semispinor re-presentation, are investigated in [10, 11]). The space of representation (12) is written down analogously (an analog of Eq. (25), Part 1)

$$\mathbf{F}_{\zeta, \bar{\zeta}}^{(\lambda, \lambda')} = \bigoplus_{p \in \mathbf{Z}} \bigoplus_{q \in \mathbf{Z}} F_{\lambda + \frac{p}{2}, \lambda' + \frac{q}{2}}. \tag{13}$$

Now together with the operators φ^α, ${}^{(p)}\bar{\varphi}_\alpha$ (Eq. (25), Part 1) having properties

$$\varphi^\alpha : F_{\lambda + \frac{p}{2}, \lambda' + \frac{q}{2}} \to F_{\lambda + \frac{p-1}{2}, \lambda' + \frac{q}{2}};$$

$${}^{(p)}\bar{\varphi}_\alpha : F_{\lambda + \frac{p}{2}, \lambda' + \frac{q}{2}} \to F_{\lambda + \frac{p+1}{2}, \lambda' + \frac{q}{2}},$$

it is necessary to consider the complex conjugate operators $\varphi^{*\dot{\alpha}}$, ${}^{(q)}\bar{\varphi}^*_{\dot{\alpha}}$ (spinors with dotted indices) having the properties

$$\varphi^{*\dot{\alpha}} : F_{\lambda + \frac{p}{2}, \lambda' + \frac{q}{2}} \to F_{\lambda + \frac{p}{2}, \lambda' + \frac{q-1}{2}};$$

$${}^{(q)}\bar{\varphi}^*_{\dot{\alpha}} : F_{\lambda + \frac{p}{2}, \lambda' + \frac{q}{2}} \to F_{\lambda + \frac{p}{2}, \lambda' + \frac{q+1}{2}},$$

where * stands for complex conjugation. By analogy with Eq. (21), Part 1, we introduce the operators a_α^a and a_α^{a*}. Their restrictions on the subspaces

[3] The appearence of a double sum in (12) is a simple consequence of that $su^c(2)$ is a Lie algebra of the rank 2 in which a representation are characterized by a pair of independent numbers (λ, λ') satisfying only one condition $\lambda - \lambda' \neq \frac{n}{2}$.

$\mathcal{F}_{\lambda+\frac{p}{2},\,\lambda'+\frac{q}{2}} = \mathcal{F}_{\lambda+\frac{p}{2}} \times \overline{\mathcal{F}_{\bar{\lambda}'+\frac{q}{2}}}$ are the operators $\varphi, \bar{\varphi}, \varphi^*, \bar{\varphi}^*$ entered just now, and we consider the operators

$$\phi = \begin{pmatrix} -a_\alpha^{2*} \\ a_\alpha^1 \end{pmatrix}, \quad \bar{\phi} = (a_\alpha^{1*}, a_\alpha^2). \qquad (14)$$

They satisfy the commutation relations (that follow from commutation relations for a and a^*)

$$[\phi_\alpha, \bar{\phi}_\beta] = \delta_{\alpha\beta}, \quad [\phi_\alpha, \phi_\beta] = [\bar{\phi}_\alpha, \bar{\phi}_\beta] = 0, \qquad (15)$$

which define the Heisenberg algebra with involution $h_8^{(*)} = h_4 + h_4^*$. Its representation in the space $\mathbf{F}^{(\lambda,\lambda')}$ (13) is denoted by $T_2(h_8^{(*)})$ and refers to as non-Fock. A representation (connected to it) of the algebra $\mathrm{sl}(2, \mathbf{C})$ (12) is quite obviously given by the operators $I_{\mu\nu} = \bar{\phi}\,\Sigma_{\mu\nu}\,\phi$, where $\Sigma_{\mu\nu} = \frac{1}{4i}[\gamma_\mu, \gamma_\nu]$ and γ_μ are Dirac matrices. Also it is obvious that the representation of $\mathrm{sl}(2, \mathbf{C})$ is extended up to a representation of $\mathrm{gl}(2, \mathbf{C})$ with the help of the operators $A = \bar{\phi}\,\phi$ and $B = \bar{\phi}\,\gamma_5\,\phi$. Proceeding from the commutation relations (15), we come to the relations

$$[I_{\mu\nu}, I_{\rho\sigma}] = \delta_{\nu\rho} I_{\mu\sigma} + \delta_{\mu\sigma} I_{\nu\rho} - \delta_{\mu\rho} I_{\nu\sigma} - \delta_{\nu\sigma} I_{\mu\rho},$$

$$[I_{\mu\nu}, A] = [I_{\mu\nu}, B] = [A, B] = 0.$$

So the relativization of spin (Formula (12)) results in consideration of the Heisenberg algebra with involution $h_8^{(*)}$ which we was already obtained earlier in [1]. There, the variables $\bar{\phi}$ were Dirac conjugate to ϕ, i.e., they were connected with ϕ by the formula $\bar{\phi} = \phi^+\,\beta$, where $\beta \doteq \gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is the Dirac matrix, and $+$ is a conjugation with respect to a sesquilinear form which it will be necessary now to set on $\mathbf{F}^{(\lambda,\lambda')}$. For the operators a_α^a, the condition for Dirac conjugation means that $(a_\alpha^1)^+ = a_\alpha^{1*}$, and $a_\alpha^{2+} = \bar{a}_\alpha^{\,2*}$ or (compare with Eq. (9), Part 1)

$$\langle \dot{f}, a_\alpha^1 f\rangle = \langle a_\alpha^{1*}\dot{f}, f\rangle, \quad \langle \dot{f}, a_\alpha^2 f\rangle = -\langle a_\alpha^{2*}\dot{f}, f\rangle, \qquad (16)$$

where $\langle .,.\rangle$ is the above-mentioned form on $f \in \mathbf{F}^{(\lambda,\lambda')}$, and $\dot{f} \in \dot{\mathbf{F}}^{(\lambda,\lambda')}$ is the space dual to $\mathbf{F}^{(\lambda,\lambda')}$ with respect to the form $\langle .,.\rangle$ determined in [10]. It is written down as

$$\langle \dot{f}, g\rangle = \sum_{p,q=-\infty}^{\infty} \langle f^{(-\bar{\lambda}'-1-\frac{q}{2},\,-\bar{\lambda}-1-\frac{p}{2})}, g^{(\lambda+\frac{p}{2},\,\lambda'+\frac{q}{2})}\rangle, \qquad (17)$$

where $g^{(\lambda+\frac{p}{2},\,\lambda'+\frac{q}{2})}$ is a projection of g onto the subspace $\mathcal{F}_{\lambda+\frac{p}{2},\,\lambda'+\frac{q}{2}}$, and $f^{(-\bar{\lambda}'-1-\frac{q}{2},\,-\bar{\lambda}-1-\frac{p}{2})} = \dot{f}^{(\lambda+\frac{p}{2},\,\lambda'+\frac{q}{2})}$ is a projection of $\dot{f}$ onto the subspace $\mathcal{F}_{-\bar{\lambda}'-1-\frac{q}{2},\,-\bar{\lambda}-1-\frac{p}{2}} = \dot{\mathcal{F}}_{\lambda+\frac{p}{2},\,\lambda'+\frac{q}{2}}$, in which the representation $(-\bar{\lambda}'-1-\frac{q}{2}, -\bar{\lambda}-1-\frac{p}{2})^+$ dual to $(\lambda+\frac{p}{2}, \lambda'+\frac{q}{2})^+$ (as is shown in [9]) is realized (the pair $(-\bar{\lambda}'-1, -\bar{\lambda}-1)$ is conjugate to the pair of numbers (λ, λ')), and $\langle .,.\rangle$ is given by the integral

$$\langle \dot{f}^{(\lambda,\lambda')}, g^{(\lambda,\lambda')}\rangle = \int \overline{\dot{f}^{(\lambda,\lambda')}(\zeta, \bar{\zeta})}\, Ig^{(\lambda,\lambda')}(\zeta, \bar{\zeta})\, d\mu(\zeta), \qquad (18)$$

where $d\mu = \frac{i}{4\pi} d\zeta \wedge d\bar{\zeta}$ is an $\mathrm{SL}(2, \mathbf{C})$-invariant measure on $\mathbf{C}$ and $Ig(\zeta, \bar{\zeta}) = g(-\zeta, -\bar{\zeta})$ [2].

The Hermitian property of the operators $p_\mu = i\bar{\phi}\,\gamma_\mu P_+\,\phi$ and $\dot{p}_\mu = -i\bar{\phi}\,\gamma_\mu P_-\,\phi$ (they expand the algebra $\mathrm{gl}(2, \mathbf{C})$ up to the algebra $\mathrm{u}(2, 2)$) is connected with the condition for Dirac conjugation of the variables ϕ and $\bar{\phi}$ in the metric $\langle .,.\rangle$, i.e., $\langle p_\mu \dot{f}, g\rangle = \langle \dot{f}, p_\mu g\rangle$ (and similarly for $\dot{p}_\mu$). However, to throw the exponents e^{ixp} (or $e^{i\dot{x}\dot{p}}$) from left to right (or from right to left) in $\langle .,.\rangle$ is not possible because the symmetry of the group $\mathrm{Sp}^*(4, \mathbf{C})$ is spontaneously broken (see [2], and also Section 4).

It is easy to see that a special (non-Wigner) representation of the Lie algebra of the Poincare group $\mathcal{P} = \mathrm{SL}(2, \mathbf{C}) \times) T_{3,1}$ (likewise for the group $\dot{\mathcal{P}} = \mathrm{SL}(2, \mathbf{C}) \times) \dot{T}_{3,1}$ represented in the dual space $\dot{\mathbf{F}}^{(\lambda,\lambda')}$) is realized on the space $\mathbf{F}^{(\lambda,\lambda')}$. First, it is necessary to notice that here all the generators of the group $\mathcal{P}$ (and they are $I_{\mu\nu}, p_\mu$) are vertical vectors (acting in a fiber, see [1, 2]). In the Wigner representation, $p_\mu = -i\frac{\partial}{\partial X_\mu}$ are horizontal vectors (they act in the base). Secondly, it is easy to see that, in $\mathbf{F}^{(\lambda,\lambda')}$, there is a flag from p_μ-invariant subspaces such that a solvable representation of the Lie algebra of the subgroup $T_{3,1}$ is realized in $\mathbf{F}^{(\lambda,\lambda')}$ (the Wigner approach where there is the base $\mathbf{A}_{3,1}$ makes use of irreducible representations of the group $T_{3,1}$).

2) The majority of statement for the non-Fock representation theory of the algebra $h_8^{(*)}$ copies statements of the algebra h_4. So, as for the algebra h_4, the non-Fock representation of the algebra $h_8^{(*)}$ is interlaced with its extended Fock representation such

that the relations

$$\phi(\zeta)\hat{K} = \hat{K}\phi(z), \quad \bar{\phi}(\zeta)\hat{K} = \hat{K}\bar{\phi}(z) \qquad (19)$$

are fulfilled, in which the interlacing operator $\hat{K}$ is defined by the formula

$$f(\zeta, \bar{\zeta}) = \hat{K} f(z, \bar{z}) =$$

$$\int K(\zeta, \bar{\zeta}; z, \bar{z})\, \overline{f(z, \bar{z})}\, d\mu(z).$$

Here,

$$K(\zeta, \bar{\zeta}; z, \bar{z}) =$$

$$= \Big(\bigoplus_{p \in \mathbf{Z}} z_2^{2\lambda + p} \exp(\zeta \frac{z_1}{z_2})\Big) \times \Big(\bigoplus_{q \in \mathbf{Z}} \bar{z}_2^{2\lambda' + q} \exp(\zeta \frac{\bar{z}_1}{\bar{z}_2})\Big)$$

and $d\mu(z) = \left(\frac{i}{4\pi}\right)^2 \prod_{\alpha = 1,2} dz_\alpha \wedge d\bar{z}_\alpha$ is an SL (2, **C**)-invariant measure on the Lagrangian plane $L \ni (z_\alpha, \bar{z}_\alpha)$. In (19), the operators $\phi(\zeta), \bar{\phi}(\zeta)$ are written down in ζ-realization, (Eq. (25), Part 1), and $\phi(z), \bar{\phi}(z)$ are in the z-realization, in which they have the same form, as in the Fock representation, i.e.,

$$\phi = \begin{pmatrix} \partial/\partial\bar{z}_\alpha \\ z_\alpha \end{pmatrix}, \quad \bar{\phi} = \left(\bar{z}_\alpha, -\frac{\partial}{\partial z_\alpha}\right), \quad \alpha = 1, 2. \qquad (20)$$

However, a representation space in the space

$$\mathbf{F} = \mathcal{F}_\mathrm{F} \otimes \mathcal{F}_0, \qquad (21)$$

where $\mathcal{F}_\mathrm{F}$ is the Fock representation space formed by functions of complex variables $z_\alpha, \bar{z}_\alpha$, and $\mathcal{F}_0$ is the space of functions depending on additional variables $z \doteq z_2$, $\bar{z} \doteq \bar{z}_2$ which are GL (2, **C**)-scalars (the same structure has the dual space $\dot{\mathbf{F}}$). An SL (2, **C**)-invariant sesquilinear form on **F**, connecting **F** and $\dot{\mathbf{F}}$ together in a dual pair of spaces, is defined by the formula [2]

$$\langle \dot{f}, g \rangle = \int \overline{\dot{f}(z)}\, g(z)\, d\mu(z). \qquad (22)$$

The variables ϕ and $\bar{\phi}$ (20) satisfy relations (16) in which now $\langle .,. \rangle$ is understood to be form (22).

An analog of formula (11) will be

$$\bigoplus_{p \in \mathbf{Z}} \bigoplus_{q \in \mathbf{Z}} \left(\lambda + \frac{p}{2}, \lambda' + \frac{q}{2}\right)^+ \hat{K} =$$

$$= \hat{K}\left(\left(\bigoplus_{n \in \mathbf{Z}_+} \bigoplus_{m \in \mathbf{Z}_+} \left(\frac{n}{2}, \frac{m}{2}\right)\right) \times (0, 0)_h\right), \qquad (23)$$

where $\left(\frac{n}{2}, \frac{m}{2}\right)$ are finite-dimensional (spinor) representations of the group SL (2, **C**) realized in $\mathcal{F}_\mathrm{F}$, and $(0, 0)_h$ is the non-trivial unity representation realized in $\mathcal{F}_0$. Here, real semispinor representations $\left(\lambda + \frac{p}{2}, \lambda' + \frac{q}{2}\right)^+$ (as well as analytic ones, see Section 4, Part 1) spontaneously break the symmetry of SL (2, **C**), lowering it up to the symmetry of its open subgroups, while finite-dimensional representations are characterized by the complete symmetry of SL (2, **C**). Nevertheless, despite the fact that the symmetry of GL (2, **C**) is complete in the extended Fock representation, $U(2, 2)$-symmetry remains broken by the topology of the dual pair of spaces (Φ', Φ) (compare with the case of the algebra h_4) that it will be seen from further considerations. In particular, the exponent e^{ipx} is connected with the space Φ', and $e^{i\dot{p}\dot{x}}$ with Φ.

In the z-realization for the operators p_μ and $\dot{p}_\mu$, we have

$$p_\mu = \bar{z}\overset{+}{\sigma}_\mu z, \quad \dot{p}_\mu = -\frac{\partial}{\partial z}\bar{\sigma}_\mu\frac{\partial}{\partial \bar{z}}, \qquad (24)$$

where $\overset{\pm}{\sigma}_\mu = (\vec{\sigma}, \pm i)$ are the Pauli matriccs. The formulae of the extended Fock representation will play the extremely important role in further specific calculations.

7. Representation Structure of the Algebra $h_8^{(*)}$ on the Space $\mathcal{F}_0$ and the Fermion-Antifermion Asymmetry

It is meaningful to consider a restriction of the algebra $h_8^{(*)}$ from the space $\mathbf{F} = \mathcal{F}_\mathrm{F} \otimes \mathcal{F}_0$ on the subspace of additional variables $\mathcal{F}_0$ accompanied actually by restricting the algebra $h_8^{(*)}$ up to the algebra $h_4^{(*)} = h_2 \oplus h_2^*$. We denote generators of the algebra $h_4^{(*)}$ by $\phi, \bar{\phi}$ which are written down as

$$\phi = \begin{pmatrix} \partial/\partial\bar{z} \\ z \end{pmatrix}, \quad \bar{\phi} = \left(\bar{z}, -\frac{\partial}{\partial z}\right) \qquad (25)$$

and act on $\mathcal{F}_0$. The bilinear forms $\bar{\phi}_\alpha\phi_\beta$, $\phi_\alpha\phi_\beta$, $\bar{\phi}_\alpha\bar{\phi}_\beta$ will form the algebra $\mathrm{sp}^{(*)}(2, \mathbf{C})$. From them, real variables written down as $L_\mu = \bar{\phi}\sigma_\mu\phi$

($\mu = 0, 1, 2, 3$) will form the algebra u(1, 1). Explicitly, we have ($L_\pm = L_1 \pm L_2$)

$$L_+ = \bar{z}z, \quad L_- = -\frac{\partial}{\partial z}\frac{\partial}{\partial \bar{z}},$$

$$L_3 = -\frac{1}{2}\left(z\frac{\partial}{\partial z} + \bar{z}\frac{\partial}{\partial \bar{z}} + 1\right),$$

$$L_0 = -\frac{1}{2}\left(z\frac{\partial}{\partial z} - \bar{z}\frac{\partial}{\partial \bar{z}} + 1\right).$$

Note that L_+, L_-, L_0 are that remains after the restriction $h_8^{(*)} \supset h_4^{(*)}$ for the operators $p_\mu, \dot{p}_\mu, \frac{1}{2}A$, thus $p_\mu \to (0, 0, L_+, iL_+)$ and $\dot{p}_\mu \to (0, 0, L_-, -iL_-)$, i.e., the representation restriction from **F** to $\mathcal{F}_0$ is accompanied by restricting the group (vector) space $T_{3,1} \supset T_{1,1}$.

Further, it is easy to see that a representation of the algebra u(1, 1) in the class of functions of two complex variables z and $\bar{z}$ (it will be specified later) has the following structure:

$$\left(\bigoplus_{k=0}^{\infty} D^+\left(-\frac{k+1}{2}\right)\right) \oplus \left(\bigoplus_{k=1}^{\infty} D^+\left(\frac{k-1}{2}\right)\right). \qquad (26)$$

Here, $D^+\left(-\frac{k+1}{2}\right)$ is an irreducible representation of su(1, 1) with the weight $-\frac{k+1}{2}$ ($k = 0, 1, 2, \ldots$), which is realized in the class of entire functions in the form $z^k f(\bar{z}z)$, where $f(\bar{z}z)$ is an entire function of the variable $\bar{z}z$, i.e., of functions expanded in a Taylor series. Vectors of the subspace, on which the representations $D^+\left(-\frac{k+1}{2}\right)$ are realized, are characterized by positive values of the operator $F = -2L_0 - 1 = k \geq 0$ standing for the fermionic charge. In regard to the representations $D^+\left(\frac{k-1}{2}\right)$ with weights $\frac{k-1}{2}$ ($k = 1, 2, ..$) which are realized in the class of meromorphic functions expanded in a Laurent series, it is incompletely reducible and has the structure

$$D^+\left(\frac{k-1}{2}\right) = D\left(\frac{k-1}{2}\right) +)\, D'^+\left(-\frac{k+1}{2}\right), \qquad (27)$$

where $D\left(\frac{k-1}{2}\right)$ is a finite-dimensional representation of su(1, 1), and $D'^+\left(-\frac{k+1}{2}\right)$ is its infinite-dimensional 'tail“ equivalent to an irreducible representation $D^+\left(-\frac{k+1}{2}\right)$, see [11]. The semidirect sum in (27) turns in the direction of the invariant subspace. If the 'tail“ $D'^+\left(-\frac{k+1}{2}\right)$ is realized in the class of entire functions in the form $\bar{z}^k f(\bar{z}z)$, the finite-dimensional representation is realized in the class of singular (in zero) functions in the form $\frac{1}{z^k}P(\bar{z}z)$, where $P(\bar{z}z)$ is a polynomial of degree $\leq k$ for $\bar{z}z$. Vectors, in which representations $D^+\left(\frac{k-1}{2}\right)$ are realized, are characterized by negative values of the fermionic charge $F \leq 0$. It is clear that singular functions are acceptable neither from the physical nor mathematical point of view, and therefore it is necessary to reject them. However, to reject only finite-dimensional blocks in (26) and to retain the invariant subspaces $D'^+\left(-\frac{k+1}{2}\right)$, it is impossible. First, $D\left(\frac{k-1}{2}\right)$ are non-invariant subspaces, and, consequently, the factorization on them is not allowable. Secondly, it is impossible to avoid blocks $D\left(\frac{k-1}{2}\right)$ (for the same reason), as only from them it is possible to get in $D'^+\left(-\frac{k+1}{2}\right)$ with the help of the raising operator L_+. There is only one opportunity to exclude the singular functions ‾ it is wholly to exclude (by means of factorization) all the representations $D^+\left(\frac{k-1}{2}\right)$, i.e., as a representation space of the algebra $h_4^{(*)}$, to consider the space

$$\mathcal{F}_0 = \bigoplus_{k=0}^{\infty} \mathcal{F}_0^{(k)}, \qquad (28)$$

where $\mathcal{F}_0^{(k)}$ is a class of functions in the form $z^k f(\bar{z}z)$. On this space, the fermionic charge takes only positive values. This means that the fermion-antifermion symmetry completely broken underlies the suggested theory. This conclusion is of fundamental importance for cosmology: at the moment of the creation of our Universe, the quantum transition $f \to \dot{f}$ will generate only fermions. Antifermions are produced only in result of switching on of interactions between particles.

8. Other Basic Consequences

The makings, occurring in the space-time, are considered in the kinetic theory which is classical in its nature. In the quantum theory, the makings are realized not in the space-time, but in the representation space, and, consequently, are essentially of a probability nature. If processes proceeding in the space-time are evolutionary and a cause-effect connection is typical of them, the quantum processes (in particular, jumps) follows another principle that one of purposefulness, in the case a causality is completely away. From the pure philosophical point of view, it is necessary to estimate the process of making (being of fundamental importance) the complete symmetry of the Lorentz group, related to (23), as well as another important process of making the four-dimensionality $\mathbf{R}_{3,1}$ from two-dimensionality $\mathbf{R}_{1,1}$, related to the extension of a representation of the algebra $h_8^{(*)}$ from space F_0 up to the space $\mathbf{F}$ (see the preceding Section). It is impossible to answer a question at which instant of time (time is not yet present as a category) the non-Fock representation of the algebra $h_8^{(*)}$ becomes the extended Fock representation and, hence, when the complete symmetry arises from the broken Lorentz symmetry. Also it is impossible to answer the question when a representation of $h_8^{(*)}$ extends from F_0 (connected with two-dimensionality) to $\mathbf{F}$ (connected with four-dimensionality). All these processes are essentially of a probability nature and do not obey the principle of causality. Thus, 'the present" can be determined by 'the future". This happens to be the case for such parameters of states of relativisrtic bi-Hamiltonian system as temperature T_f and $T_{\dot{f}}$ (they will appear in a further consideration), the numerical values of which are determined by conditions under which the process of quantum transition $f \to \dot{f}$ occurs. Before the transition, it is necessary to speak about them as about *a priori* quantities which, by the way, essentially determine the geometry of a space-time continuum, in which there is (before the transition $f \to \dot{f}$) an ensemble of states f [1]. In the suggested theory, the above-mentioned processes, as if they are pressed all together, exist and go 'simultaneously", determining that Leibnitz called the pre-established harmony (in this case, it is pre-established by the algebra $h_8^{(*)}$). Only after the creation of a space-time continuum, the processes develop as a chain of cause-effect events in it.

One more consequence of the non-unitary theory concerns a unequivalence of the Heisenberg picture and the Schrödinger one, see [1], about which Dirac [12] seriously throught in the framework of the usual unitary scheme, but which really is found out only within the framework of the non-unitary scheme using the dual pair of topological vector spaces. By the way, the pair of spaces $(\mathbf{F}', \mathbf{F})$ and the Gauss decomposition $B_+ B_-$, associated with it, of the group $\mathrm{Sp}^{(*)}(4, \mathbf{C})$ (such that $e^{ipx} \in B_+$, and $e^{i\dot{p}\dot{x}} \in B_-$; in [1], such a correspondence is called the polarization of a relativistic bi-Hamiltonian system) result not only in the T (and, hence, CP)-asymmetry of the non-unitary scheme (see earlier), but also in the P- and C-asymmetry. In addition to this, we notice that the T-symmetry is broken also by additional variables $z \doteq z_2$. Indeed, under the reflection of T, as is known, we have $z_2 \to i\,\bar{z}_1$. However, the additional variables $z' \doteq z_1$ are not present in the scheme so that the space F_0 is not invariant under the reflection of T.

To answer the frequently given question why our space has three dimensions (Kant, Poincare, Ehrenfest, Weyl) now it might be said so: three dimensions of space (and its appearance and also the mass spectrum of fundamental particles and their symmetry properties) are caused by the symmetry properties of the algebra $h_8^{(*)}$ and its representations underlaying the all Worldbuilding.

Our discussion would be incomplete if we do not note that the algebra $h_8^{(*)}$ represents a real form of the Penrose twistor algebra $h_8(\mathbf{C})$ [13] appeared on the pure geometrical way in the context of the cohomology theory. For the theory of elementary particles, the twistor program has resulted in rather limited results. However, if to proceed to the algebra $h_8^{(*)}$ from the point of view of functional analysis and the theory of representations, it is possible to achieve an essential progress in construction of the consecutive theory of elementary particles. These problems will be considered in next papers.

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