PARAMETRIC ION CYCLOTRON AND ION-SOUND TURBULENCE

K. N. STEPANOV

National Science Center 'Kharkiv Institute of Physics and Technology“
(1, Academichna Str., Kharkiv 61108, Ukraine)

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The results of analytic investigations and computer simulations using the macroparticle technique of two parametric instabilities of plasma, i.e., parametric ion cyclotron instability in the electric pump field of fast magnetosonic wave with the frequency of order of the ion cyclotron one and the kinetic ion-sound instability in the electric field of helicon, are presented. The experiments in which these instabilities may play an important role, namely the experiments on RF heating and plasma transport studying on the URAGAN-3M torsatron made by E.D.Volkov et al. in KhIPT and the experiments on plasma production in helicon sources made by F.F.Chen et al. in UCLA, USA, are discussed.

Academician N.N.Bogolyubov made the outstanding contribution to the development of plasma physics. Starting from the first principles, he derived the kinetic equation with self-consistent fields and obtained the collision integral for a system with Coulomb interaction. His idea about a reduced description in the presence of the hierarchy of relaxation times was widely used in plasma physics, specifically, at KhIPT when constructing the nonlocal theory of interaction between waves and particles in the nonuniformly magnetized plasma and the nonlinear theory of kinetic parametric instabilities.

Studying the interaction of electromagnetic fields with plasma is one of the important problems of plasma physics that is of fundamental importance and has many applications. Such is, e.g., the problem of plasma heating up to fusion temperatures in magnetic traps (tokamaks, stellarators, etc.) with the help of electromagnetic waves. In this case, electromagnetic waves are absorbed by resonant particles of plasma for which the following condition holds:

$$\omega(\mathbf{k}) = n\omega_{c\alpha} + k_{||}\upsilon_{||} \quad (n = 0, \pm 1, \pm 2, \ldots) \qquad (1)$$

where $\omega(\mathbf{k})$ is the wave frequency, $\mathbf{k}$ is the wave vector, $k_{||}$ is its projection on the direction of the external magnetic field $\mathbf{B}_0$, $\omega_{c\alpha} = e_\alpha B_0 / m_\alpha c$ is the cyclotron frequency of α-species particles with the change e_α and mass m_α, $\upsilon_{||}$ is the particle velocity along $\mathbf{B}_0$. Condition (1) was pointed out in the paper by A.I.Akhiezer and L.E.Pargamanik [1] that was the first work on the kinetic theory of oscillations of the magnetized electron plasma. Electron cyclotron damping of electromagnetic waves was found by A.G.Sitenko and the reporter [2]. Cyclotron and Cherenkov damping of various branches of electromagnetic waves in plasma were later studied in detail in a number of papers (cf. monographs [3, 4]).

For thermal particles, the Doppler shift of the frequency $k_{||}\upsilon_{||}$ is small compared with $n\omega_{c\alpha}$ and the cyclotron resonance condition in the traps where the magnetic field is nonuniform holds within a comparatively narrow layer $\upsilon \approx \upsilon_{res}$ where the wave frequency is close to $n\omega_{c\alpha}$, $\omega > n\omega_{c\alpha}(\mathbf{r}_{res})$.

Passing through the cyclotron resonance zone, a particle acquires a push from the wave electric field increasing or decreasing its energy depending on the wave-particle phase. On the average, travelling along the magnetic line of force through the cyclotron resonance zones, the particle would acquire the energy from the RF field when the wave-particle phase is random during the passage through resonance zones and the particle distribution is Maxwellian-like.

Binary Coulomb collisions in high temperature plasma are rare, the collision frequency ν_{coll} is small and the direct effect of collisions on wave damping is negligible. The mechanism of wave absorption under cyclotron resonance is essentially collisionless. However, collisions still play a fundamental role in the cyclotron heating of plasma in magnetic traps. Were they completely absent, the variation of the distribution function of resonant particles under the action of RF field would lead to the cessation of the interaction between waves and particles and to the quenching of plasma heating. This does not occur because resonant particles leaving the cyclotron resonance zone travel a large distance along the magnetic line of force before they enter the cyclotron resonance zone again. During this time interval owing to binary collisions, the particles acquire a random change of velocity along the magnetic field $\delta\upsilon_{||}$ found from the relation $\overline{\delta\upsilon_{||}^2} \sim \nu_{coll}\, t\, \upsilon_{T\alpha}^2$. The wave-particle phase will change by a quantity which considerably exceeds unity even for fusion-like traps. Therefore, one can regard the wave-particle phase in this case as a random quantity uniformly distributed within the interval $(0, 2\pi)$. This enables one to apply the quasilinear theory for waves with dynamic phases without the assumption on the random distribution of phases within the wave packet generated by the antenna. This fact was pointed out by T.Stix (1975). Such a theory taking into account the decorrelation collisional effects and the nonuniformity of the magnetic field along the line of force, i.e., the nonlocal theory of cyclotron and Cherenkov absorption of electromagnetic waves and quasilinear variation of the distribution function of resonant particles, has been constructed (cf. review [5] by S.V.Kasilov, A.I.Pyatak and K.N.Stepanov). The theory has been applied to the problems of wave absorption, plasma heating, and current drive in tokamaks, stellarators, and open traps [5]. Collisions between resonant and nonresonant particles lead to the heating of the latter. Collisions also make Maxwellian the bulk ion distribution function.

At present, one efficiently applies fast magnetosonic waves (FMSW) for plasma heating in the ion cyclotron range of frequencies. When the wave frequency approaches the cyclotron frequency for bulk ions, the FMSW polarization becomes unfavourable with its electric field vector rotating opposite to ion rotation in the magnetic field $\mathbf{B}_0$ and its damping becomes very small. The variation of the FMSW polarization may be reduced if one applies the cyclotron resonance for minority ions (this method of plasma heating was proposed by A.V.Longinov at KhIPT and by J.Adam and A.Samain in France (1971) [6]). At KhIPT, other scenarios of plasma heating with FMSW were proposed that are promising for large toroidal traps including those for a fusion reactor (cf. review [6]).

At the JET tokamak, minority heating using He^3 ions has led to fusion temperatures of electrons and ions $T_e \sim T_i \sim 10$ keV for deuterium and deuterium-tritium plasmas with the densities $n_0 = 10^{13} \div 10^{14}\,\text{cm}^{-3}$ [7]. Under such heating, He^3 ions diffuse in the velocity space under the action of the FMSW RF field in the quasilinear regime attaning MeV energies and imparting their energy to electrons and bulk ions due to binary Coulomb collisions. Therefore, one can regard the problem of heating plasma in traps with good thermal insulation as fundamentally solved.

Nonlinear effects, in particular, the onset of parametric instabilities under the interaction of RF fields with plasma, play an important role in many laboratory experiments as well as in space [8]. Even for moderate values of the pumping field amplitude at relatively low temperatures or plasma densities, these phenomena arise also in fusion devices. For example, under plasma production by RF techniques at $\omega_0 \gtrsim \omega_{ci}$ in stellarators and tokamaks, one obtains plasma with low temperature. On heating plasma in small-size stellarators with small energy lifetime, one applies strong alternate electric fields to attain high temperatures. In these cases, there may arise parametric instabilities on electrostatic ion cyclotron waves (ion Bernstein modes) [9 – 11]. Simulation of such instabilities has been performed by V.V.Olshansky et al. [12 – 15] by the macroparticle technique. The first part of the report will give a brief review of the results [12 – 15].

Parametric phenomena may also play an important role in sustaining the discharge in so-called helicon plasma sources as was shown by A.I.Akhiezer, V.S.Mikhailenko, and the author [21]. This problem is dealt with in the second part of the report.

1. Simulating Ion Cyclotron Instability of Plasma

Let a plasma be located in the electric field $\mathbf{E}_{0\perp}^{\sim}$ of oscillations with frequency of order of the ion cyclotron frequency, $\omega_0 \sim \omega_{ci}$. Electrons and ions of plasma oscillate in this field across the magnetic field with the velocity $u_\perp \sim c\,(E_{0\perp}^{\sim} / B_0) < \upsilon_{Ti}$. In this case, it is possible to generate short wavelength oscillations with the following frequency and growth rate (Kitsenko A.B. et al. [9, 10]):

$$\omega^{(n)}(k) = n\omega_{ci}\left(1 + \frac{1}{\sqrt{2\pi}\; k\,\rho_i}\right), \tag{2}$$

$$\gamma(\mathbf{k}) \sim \omega_{ci} / k\rho_i \sim \omega_{ci}\,(u_\perp / \upsilon_{Ti}). \tag{3}$$

These oscillations are excited by resonant electrons moving along $\mathbf{B}_0$ with the velocity equal to the phase velocity of beats formed between the pumping wave and unstable oscillations

$$\upsilon_{||} = (\omega(k) - p\,\omega_0) / k_{||} \sim \upsilon_{Te} \quad (p = 1, 2, \ldots). \tag{4}$$

The oscillations may also grow coherently under the parametric resonance conditions $\omega^{(n)} \pm \omega^{(m)} = N\omega_0$ (N is an integer).

One may expect that saturation of these instabilities may be due to nonlinear scattering of unstable oscillations by ions [16] (i.e., nonlinear spreading cyclotron resonances at $\Delta\omega_{NL} \sim \gamma$). This condition furnishes the following estimate for the energy density of oscillations:

$$\frac{W}{n_0 T_i} \sim \left(\frac{u_\perp}{\upsilon_{Ti}}\right)^4. \tag{5}$$

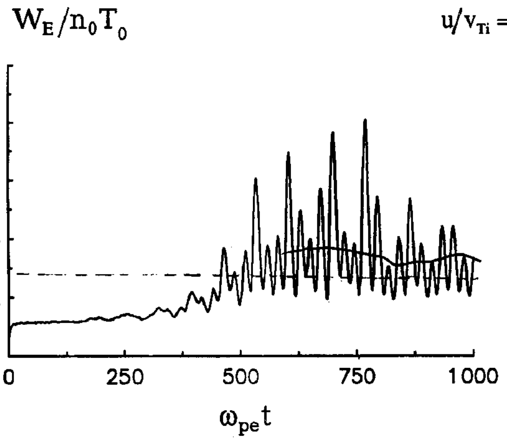

Fig. 1. Time dependence of the electric field energy density W_E ($\omega_0 = 0.8\,\omega_{ci}$, $u/\upsilon_{Ti}(0) = 0.5$). Broken line corresponds to formula (5)

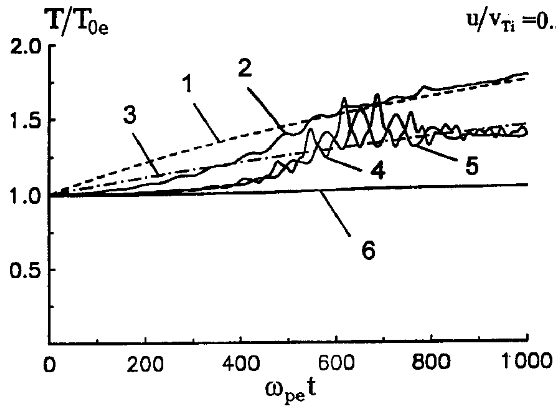

Fig. 2. Time dependences of the temperature of particles. The curves correspond to: *1, 3* are for theoretical formula (6) for electrons and ions, respectively; *2* and *6* are for the longitudinal temperature of electrons and ions, respectively; *4, 5* are for the transverse temperatures of ions

In this case, there occurs the heating of electrons along $\mathbf{B}_0$ and the transverse temperature of ions increases with a rate that may be estimated according to the quasilinear theory generalized in [17] for finite displacements of particles oscillating in the pumping electric field:

$$\frac{dT_{e||}}{dt} \sim \frac{dT_{i\perp}}{dt} \sim \gamma\left(\frac{u_\perp}{\upsilon_{Ti}}\right)^4 \sim \omega_{Ti}\left(\frac{u_\perp}{\upsilon_{Ti}}\right)^5. \qquad (6)$$

One sees from these formulas that the turbulence level and the rate of turbulent heating fall fast with $u_\perp$ decreasing. Therefore, the turbulent heating arises at sufficiently strong pumping electric fields.

In order to determine the characteristics of the ion cyclotron turbulence, it was simulated by the macroparticle technique [12 – 14]. The following plasma parameters were chosen: $m_e/m_i = 0.01$, $\omega_{pi}/\omega_{ci} = 2.5$. There were considered 2-*D* oscillations: $\mathbf{E} = -\nabla\varphi$, $\varphi = \varphi(x, z, t)$. The ion motion is three-dimensional, electrons are magnetized and they move only along the magnetic field, i.e., along the *z*-axis. The velocity $u_\perp$ was chosen to be less or order of ion thermal velocity. The initial temperatures of electrons and ions were equal, $T_i(0) = T_e(0)$. Fig. 1 shows the time dependence of the energy density of the electric field of unstable oscillations, and Fig. 2 depicts the longitudinal temperature of electrons $T_{e||}(t)$ and the transverse temperature of ions $T_{ix}(t)$ and $T_{iy}(t)$ (curves for $u_\perp/\upsilon_{Ti}(0) = 0.5$ and $\omega_0/\omega_{ci} = 0.8$). Fig. 3 shows the time dependence of the most unstable spatial harmonic $k_\perp\rho_i = 1$, $k_{||}\rho_i = 0.05$ (*a*) and the frequency spectrum of this harmonic (*b*) for $\omega_0/\omega_{ci} = 1.2$ and $u_\perp/\upsilon_{Ti}(0) = 0.9$. The broken lines show the values of these quantities according to estimates (5) and (6).

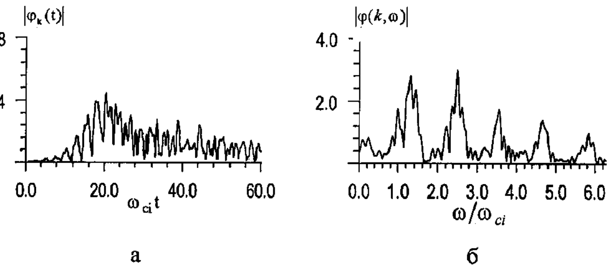

Fig. 3. Temporal behaviour of the amplitude of the most unstable mode (*a*); and frequency spectrum of this mode (*b*) ($\omega_0/\omega_{ci} = 1.2$, $u/\upsilon_{Ti}(0) = 0.9$, $k\,\rho_I = 1.2$, $k_{||}\,\rho_I = 0.05$)

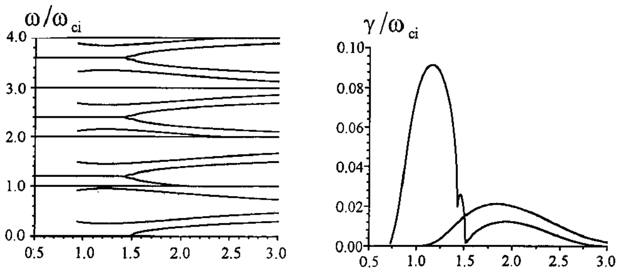

Fig. 4. Solution of the linear parametric dispersion equation for $\omega_0/\omega_{ci} = 1.2$, $u/\upsilon_{Ti}(0) = 0.9$, $k\,\rho_I = 1.2$, $k_{||}\,\rho_I = 0.05$. Dependence of the frequencies (*a*) and the growth rates (*b*) on the transverse wavenumber

The largest peaks in Fig. 3,*b* correspond to the solutions of the dispersion relation shown in Fig. 4 with (*a*) the frequency and (*b*) the growth rates. We see that, in the presence of the pumping field, the frequency of cyclotron oscillations is split into four frequencies, this situation being characteristic for the coherent parametric resonance (cf. [8] where these frequencies are found at $u_\perp/\upsilon_{Ti} \ll 1$). Peaks in Fig.

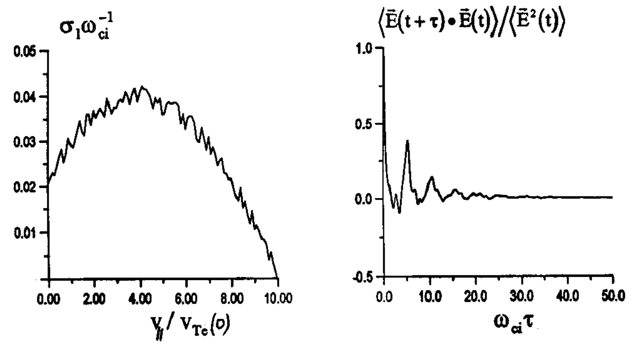

Fig. 5. Autocorrelation function of the electric field (*a*) and behaviour of the maximum Lyapunov exponential against initial parallel velocity of electrons (*b*)

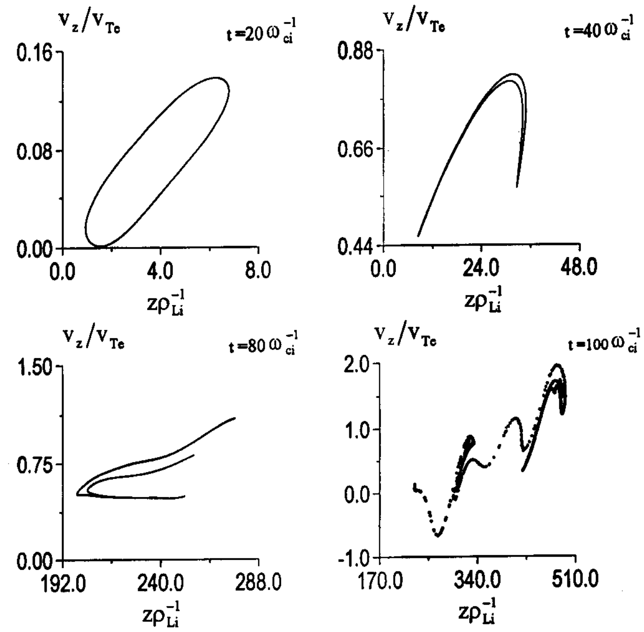

Fig. 6. Evolution of the phase volume shape versus time for electron motion along the magnetic field

4,*b* are strongly spread due to a nonlinear broadening of oscillations (frequency spectrum is obtained within the total time interval $\omega_{pe} t = 1000$ where the oscillations are already strongly nonlinear). The dependences of the autocorrelation function for the electric field and the maximum Lyapunov exponential on time (Fig. 5) indicate strong nonlinearity. Oscillations become independent after the time period $\Delta t \sim 1/\gamma$.

After the same time period, the dynamic chaos arises in the electron motion along the magnetic field under the action of the self-consistent electric field. Fig. 6 depicts the evolution of the conserved phase volume of electrons (z, υ_z): the circle with radius $z/\rho_i(0) = 1$ and $\upsilon_z / \upsilon_{Te}(0) = 1$ at $t = 0$ shifts with slight deformation after the time $20/\omega_{ci}$. After $t = 40/\omega_{ci}$, it transforms into a thin boomerang; after $t = 80/\omega_{ci}$, it is elongated into a thin doubly folded thread; and after $t = 100/\omega_{ci}$, it transforms into a thinner twisted thread (at this stage, the length of the thread is so large that the distance between particles on it becomes large, and the thread exhibits 'breaks").

Similar results have been obtained on simulating the parametric ion cyclotron instability of plasma with two ion species at $k_{||} = 0$ by the macroparticle technique [15]. In this case, the instability is driven by the relative motion of different ion species [11], electrons are regarded as magnetized and they do not affect the onset of the instability.

The simulation performed justifies estimates (5) and (6) that formed the ground for the explanation [18] of the mechanism of plasma heating and transport of particles and energy in RF experiments [19] on the Uragan-3M torsatron.

2. Ion-Sound Turbulence

Let plasma be in the FMSW (whistler or helicon) electric field with frequency within the range $\omega_{ci} \ll \omega_0 \ll |\omega_{ce}|$. Then, at $u > \upsilon_s$, $\upsilon_s = (T_e / m)^{1/2}$ being the ion-sound velocity, electrons moving along $\mathbf{B}_0$ with velocity equal to the beat velocity (4) excite short wavelength ($k_\perp \rho_e \gg 1$) ion-sound oscillations [20]. Here, $k_\perp$ is the transverse wavenumber, ρ_e is the Larmor radius of electrons with the thermal velocity. Nonlinear theory of such a kinetic parametric instability was developed in the paper by A.I.Akhiezer, V.S.Mikhailenko, and K.N.Stepanov [21]. The evolution of the spectral intensity $I = k^2 |\varphi_k|^2 / 4\pi$ of the ion sound is determined by the kinetic equation for waves $\partial I / \partial t = 2(\gamma - \hat{\gamma}_{NL}) I$, where $\hat{\gamma}_{NL}$ is the integral operator describing the nonlinear scattering of the ion sound by ions found by V.I.Petviashvili and B.B.Kadomtsev [22], and γ is the linear growth rate $\gamma \sim 0.05\,(|\omega_{ce}|\,\omega_{ci})^{1/2}$. In the saturation state, $(\gamma - \hat{\gamma}_{NL}) I = 0$. Using the expressions [22] for $\hat{\gamma}_{NL} I$, one obtains the following estimate for the energy density of ion-sound oscillations:

$$\frac{W}{n_0 T_e} \sim \frac{0.02}{\alpha} \frac{T_e}{T_i} \frac{\upsilon_s}{u_\perp} \frac{(|\omega_{ce}|\,\omega_{ci})^{1/2}}{\omega_s}, \qquad (7)$$

where $\omega_s = k\upsilon_s$ is the ion-sound frequency, α is the numerical factor depending on the angular distribution of the intensity ($\alpha \approx 1/8$). On the ground of the quasilinear theory [17] taking into account expression (7), one can obtain the following estimate of the rate of turbulent heating of electrons:

$$\frac{1}{\tau_n} = \frac{1}{T_{e||}} \frac{dT_{e||}}{dt} \sim 5 \cdot 10^{-4} \frac{|\omega_{ce}|\ \omega_{ci}}{\alpha\, \omega_s} \frac{T_e}{T_i} \left(\frac{\omega_0}{\omega_s} - 1 \right). \quad (8)$$

From the energy balance $\Gamma_0 W_0 = n_0 T_{e||}/\tau_n$, where $W_0 = |B^{\sim}|^2/8\pi$ is the energy density of a helicon, $\Gamma_0 = 2\nu_{\rm eff}(k_0 c/\omega_{pe})^2$ is its damping rate, k_0 is its wavenumber, one finds the effective frequency $\nu_{\rm eff}$ determining the absorption of a helicon as

$$\nu_{\rm eff} \sim \gamma \frac{W}{2W_0} \left(\frac{\omega_0}{\omega_s} - 1 \right) \frac{\omega_{pe}^2}{k_0^2 c^2}. \quad (9)$$

The detailed comparison of experimental data [23, 24] on the discharge sustained due to the excitation of a helicon with the theoretical values obtained has shown that, in helicon sources ($n_0 \sim 10^{13}$ cm^{-3}, $T_e \sim 4$ eV, $B_0 = 800$ G,the introduced power is 1 – 2 kW), there may be excited the parametric kinetic ion-sound instability in the vicinity of the antenna. Its development leads to the turbulent heating of electrons, intensive ionization of the neutral gas and discharge sustainment. The value of $\nu_{\rm eff}$ exceeds the frequency of binary collisions by an order of magnitude. Thus, the proposed mechanism of heating of electrons gives a possible explanation of the enigmatic efficiency of the operation of helicon sources.

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