ABOUT THE END OF THE ELECTRON SPECTRUM IN FIVE-LEPTON μ^+ DECAY

N. P. MERENKOV, O. N. SHEKHOVTSOVA

Kharkiv Institute of Physics and Technology
(*1, Academichna Str., Kharkiv 61108, Ukraine*)

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The spectrum of very fast electrons in five-lepton decay $\mu^+ \to e^- e^+ e^+ \nu\bar{\nu}$, that is the main background decay at the study of the muonium-antimuonium conversion in vacuum, is considered. An essential decrease of the spectral distribution is demonstrated when the energy of one positron in this decay is small. Some arguments for such a decrease for arbitrary positron energies are given.

1. The search of deviation from the Standard Model (SM) and the probe physics beyond it are the main goals of the current and future experiments in elementary particle physics. In this connection, the study of the spontaneous conversion of muonium ($\mu^+ e^-$) into antimuonium ($\mu^- e^+$) is of great interest nowadays because the additive lepton family (generation) number would be violated by two units in this process. There are different theoretical models beyond SM where such a violation is allowed [1 – 5]. In recent experiments [6, 7], the observation of muonium atom in vacuum has been used to investigate the conversion process. The conversion event would manifest itself by the registration of a fast electron that appears due to the standard decay of μ^- from antimuonium atom

$$e^- \mu^+ \to e^+ \mu^- \to e^+ + e^- + \nu + \bar{\nu}. \quad (1)$$

The main electrodynamic background for such events in vacuum arises due to the five-lepton decay of μ^+ in muonium

$$\mu^+ (p) \to e^- (p_3) + e^+ (p_1) + e^+ (p_2) + \nu\bar{\nu}(q). \quad (2)$$

It is well known that, in the case where the energies of both positrons in decay (2) are small enough, the energy distribution of the electrons is strongly suppressed at the end of their spectrum. The form of the spectrum under these conditions is [8, 9]

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = \frac{\alpha^2}{\pi^2}(1-y)^2 F(L, l), \; L = \ln\frac{M^2}{m^2},$$

$$l = \ln(1-y), \quad \Gamma_0 = \frac{G^2 M^5}{192\pi^3}, \quad 1-y \ll 1,$$

$$x_1 + x_2 < 1 - y,$$

$$y = \frac{2\varepsilon_-}{M}, \quad x_{1,2} = \frac{2\varepsilon_{1,2}}{M}, \quad (3)$$

where M (m) is the muon (electron) mass, ε_-, $\varepsilon_{1,2}$ are the energies of the electron and positrons, respectively, and $F(L, l)$ is the known function (see below). The additional smallness $(1-y)^2$ arises because of the trivial phase space factor of positron energy fractions: $\Delta x_1 \Delta x_2 \approx (1-y)^2$.

Just this additional smallness of the probability of the background decay (2) makes the selection events near the end-point of the electron spectrum in process (1) very attractive to observe the muonium-antimuonium conversion.

In [6], it was suggested that, in the general case where the positron energy fractions x_1 and x_2 can be arbitrary form (3) of the differential width breaks down and the small factor $(1-y)^2$ disappears. In the present work, we want to explain that this factor remains independent of x_1 and x_2. The physical reason for this assertion is a decrease of the angular phase space of the positron with increase in energy. Namely, if the electron in decay (2) carries away the energy fraction y such that $(1-y) \ll 1$ and the positron with 4-momentum $p_1 (p_2)$ has the energy fraction $x_1(x_2) \gg (1-y)$, then it flies just in the opposite direction respect to the energetic electron one and $\Delta c_1 \Delta c_2 \sim (1-y)$, where $c_{1,2} = \cos \mathbf{p}_{1,2}\mathbf{p}_3$. We calculate analytically the function $F(L, l)$ in the case where the energy of one positron is smaller than $(1-y)$ and the energy of the other is arbitrary. While calculating, we neglect terms of the order of $(1-y)$ in this function.

2. In our calculations, we do not take into account the identity of positrons, because the corresponding effects consist no more than five percent in a general case (in accordance with Monte Carlo calculations [10]) and trend to decrease in the considered here case where $(1-y) \ll 1$ [9]. Besides, we use the relativistic approximation and neglect the electron mass always where it is possible. We start from the differential width of the five-lepton decay (2) in the following form:

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = \frac{\alpha^2}{4\pi^2} R x_1 x_2\, dx_1\, dx_2\, dc_1 \frac{d\Omega_2}{2\pi}, \tag{4}$$

where the quantity R can be written in the used approximation as a contraction of two tensors [9]

$$R = [a_1 \tilde{g}_{\mu\nu} + 2a_2 \tilde{p}_{2\mu} \tilde{p}_{2\nu} + 2a_3 \tilde{p}_\mu \tilde{p}_\nu +$$

$$+ 2a_4 (\tilde{p}\tilde{p}_2)_{\mu\nu}][(p_1 p_3)_{\mu\nu} - \frac{k^2}{2} g_{\mu\nu}],$$

$$k^2 = (p_1 + p_3)^2,\ \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2},$$

$$\tilde{p}_\mu = p_\mu - \frac{pk}{k^2} k_\mu,\quad (ab)_{\mu\nu} = a_\mu b_\nu + a_\nu b_\mu, \tag{5}$$

and quantities a_i on the right side of Eq. (5) are given in [9, Appendix 1].

The constraints on the possible angles and energy fractions of positrons can be obtained from the condition on the invariant mass of neutrinos: this last has to be positive:

$$1 - y - x_1 - x_2 + \frac{x_1 y}{2}(1 - c_1) +$$

$$+ \frac{x_2 y}{2}(1 - c_2) + \frac{x_1 x_2}{2}(1 - c_{12}) > 0, \tag{6}$$

where $c_{12} = \cos \widehat{\mathbf{p}_1 \mathbf{p}_2}$.

To simplify calculations, we divide the phase space of positron energy fractions into four kinematical regions, where restrictions on the positron energy fractions, positron angles, and form of R on the right side of Eq. (3) are different. In the first region, the condition

$$1 - x_1 - x_2 - y \geq 0$$

is satisfied. One can see from inequality (6) that, in this case, all angles for positrons are permitted. The quantity R is very simple in this region

$$R_1 = \frac{M^4}{2}\left(\frac{1}{k^2 B} - \frac{1}{B^2}\right),\ B = (p_1 + p_2 + p_3)^2. \tag{7}$$

Within the chosen accuracy, we can use $k^2 + 2p_2 p_3$ for B in R_1 because the quantity $p_1 p_2$ is always negligible in this region. Then the integration R_1 over the phase space of positrons gives the well-known result obtained in [8] (see also [9])

$$F_1(L, l) = \frac{1}{8}L^2 + L\left(-1 + \frac{1}{2}l\right) + \frac{1}{2}l^2 - 2l +$$

$$+ 3 - \frac{\pi^2}{12} + O(1-y). \tag{8}$$

In the second region, the energy fractions of positrons satisfies inequalities

$$1 - y - x_2 < x_1 < 1 - y,\quad 0 < x_2 < 1 - y. \tag{9}$$

Because of the smallness of both x_1 and x_2 in this region, we can omit the last term in condition (6) and obtain the constraints on c_1 and c_2 in the form

$$-1 < c_1 < 1 + \frac{2(1 - x_1 - x_2 - y)}{x_1 y},$$

$$-1 < c_2 < 1, \tag{10}$$

and

$$1 < c_2 < 1 + \frac{2(1 - x_1 - x_2 - y)}{x_2 y} + \frac{x_1}{x_2}(1 - c_1),$$

$$1 > c_1 > 1 + \frac{2(1 - x_1 - x_2 - y)}{x_1 y}. \tag{11}$$

Within the chosen accuracy, the expression for R in the second region coincides with R_1. The angular integration of the separate terms in R_1 over regions (10) and (11) gives

$$\int_{(2)} dc_1\, dc_2 \frac{M^4}{k^2 B} = \frac{4}{x_1 x_2}\left[-L \ln \frac{x_1 + x_2 + y - 1}{y x_2} - \right.$$

$$- \mathrm{Li}_2\left(-\frac{x_1}{x_2}\right) - \frac{1}{2}\ln^2 \frac{x_1(x_1 + x_2 + y - 1)}{y} +$$

$$\left. + 2\ln x_1 \ln x_2 \right],$$

$$\int_{(2)} dc_1\, dc_2 \frac{M^4}{B^2} = -\frac{4}{x_1 x_2}\left[\ln \frac{(x_1 + x_2)}{x_1 x_2} + \right.$$

$$+ \ln (x_1 + x_2 + y - 1)\Big], \qquad (12)$$

where the form B on the left side of relations (12) is the same as in the first kinematical region.

The further integration over energy fractions of the positrons defines the contribution of the second kinematical region (see inequalities (9) – (11)) into the function $F(L, l)$

$$F_2(L, l) = \frac{1}{4}L + \frac{1}{2}l + \ln 2 - 2 + \frac{\pi^2}{12}. \qquad (13)$$

The restrictions on energy fractions in the third region are as follows:

$$0 < x_1 < 1 - y, \quad 1 - y < x_2 < 1.$$

Note that the condition on the energies of the visible particles in decay (2) gives the quantity $2 - y - x_1$ for the upper limit of x_2, which differs from 1 by the value of order $1 - y$. The accounting of that difference leads to the contribution of order $1 - y$ to $F(L, l)$ which is beyond our accuracy.

Let us analyze condition (6) in the third kinematic region. If x_2 are near $(1 - y)$, one can neglect, as before, the last term in (6) and obtain the same constraints for c_2 as in the first inequality in (11). The extension of them to large values of $x_2 >> (1 - y)$ means that the corresponding values of c_2 are near -1 with $\Delta c_2 \sim (1 - y)$. The physical content of this circumstance is very transparent: in an event with the large-energy electron $(y \approx 1)$, the large-energy positron $(x_2 >> 1 - y)$ must fly in the opposite direction with respect to the direction of the electron 3-momentum. On the contrary, the conservation of 3-momentum in decay (2) would be violated.

Therefore, we can correct the above restriction taking into account the last term in (6) at $c_2 = -1$. In this way, we derive the constraints on c_2 and c_1 in the third region as

$$-1 < c_2 < -1 + \frac{2(1 - y - x_1)(1 - x_2)}{x_2 y} +$$

$$+ \frac{x_1(y - x_2)}{x_2 y}(1 - c_1), \quad -1 < c_1 < 1. \qquad (14)$$

The quantity R in the third region reads

$$R_3 = \frac{M^4}{B^2}\left(-\frac{1}{2} + \frac{x_2}{2} + 2x_2^2\right) + \frac{M^2}{k^2}(-1 + 2x_2 + 6x_2^2) +$$

$$+ \frac{a_{23}}{k^2}\left(-\frac{1}{2} - 7x_2\right) + \frac{3a_{23}^2}{M^2k^2} + \frac{M^4}{k^2B}\left(\frac{1}{2} - x_2^2 - 2x_2^3\right),$$

$$a_{23} = 2p_2 p_3. \qquad (15)$$

In accordance with our prescription, in this region, we have to take the term $2(p_1 p_2)$, that enters B, at $c_2 = -1$. Such a procedure leads to

$$\frac{M^2}{B} = \frac{2}{x_2}\left(1 - c_2 + 2x_1 + \frac{x_1}{x_2}(y - x_2)(1 - c_1)\right)^{-1}$$

on the right side of Eq. (15).

The list of the necessary angular integrals in R_3 reads

$$\int\limits_{(3)} dc_1\, dc_2 \frac{M^4}{B^2} = \frac{4}{x_2 x_1}\left(\frac{x_1}{x_1 + x_2 + y - 1} - \ln\frac{x_1 + x_2}{x_2}\right),$$

$$\int\limits_{(3)} dc_1\, dc_2 \frac{M^2}{k^2} = \int\limits_{(3)} \frac{dc_1\, dc_2}{x_2}\frac{a_{23}}{k^2} = \int\limits_{(3)} \frac{dc_1\, dc_2}{x_2^2}\frac{a_{23}^2}{k^2M^2} =$$

$$= \frac{4(1 - x_2)}{x_2 x_1}(x_1 + (1 - y - x_1)(L + 2\ln x_1)),$$

$$\int\limits_{(3)} dc_1 dc_2 \frac{M^4}{k^2B} = \frac{4}{x_1 x_2}\left[-\mathrm{Li}_2\left(-\frac{x_1}{x_2}\right) + (L + 2\ln x_1)\times\right.$$

$$\left.\times\left(-x_2\ln\frac{x_1 + x_2}{x_2} + \ln\frac{x_2(y + x_1)}{x_1 + x_2 + y - 1}\right)\right]. \qquad (16)$$

While writing these integrals, we neglect all terms which lead to terms of order $(1 - y)^3$ in the electron spectrum.

Using Eqs. (16) and integrating over positron energy fractions, we derive the contribution of the third kinematical region into the function $F(L, l)$ in the form

$$F_3(L, l) = -\frac{1}{3}L - \frac{1}{4}Ll - \frac{1}{2}l^2 - \frac{1}{6}l +$$

$$+ \frac{31}{24} - \frac{\pi^2}{24} - \ln 2. \qquad (17)$$

In the fourth kinematical region,

$$0 < x_2 < 1 - y, \quad 1 - y < x_1 < 1.$$

The restrictions on c_1 and c_2 in this region can be obtained from (14) by the simple substitution

$$x_1 \leftrightarrow x_2, \quad c_1 \leftrightarrow c_2$$

because of the obvious symmetry of the positron phase space in the third and fourth regions.

The expression for R in this region has the following form:

$$R_4 = \frac{M^4}{B^2}\left(-\frac{1}{2} + x_1\right)(1 + x_1) + \frac{M^4}{k^2B}\left(\frac{1}{2} - \right.$$

$$-\frac{x_1}{2}-x_1^2-\frac{3x_1}{1+x_1}\Bigg)+\frac{M^4}{k^4}\frac{x_1(1-2x_1)}{1+x_1}. \qquad (18)$$

The angular integrals in the considered case are

$$\int_{(4)} dc_1\,dc_2\,\frac{M^4}{B^2}=\frac{4}{x_1x_2}\left(\frac{x_2}{x_1+x_2+y-1}-\ln\frac{x_1+x_2}{x_1}\right),$$

$$\int_{(4)} dc_1\,dc_2\,\frac{M^4}{k^4}=\frac{4}{x_1^2}\left(-1+\ln\frac{x_1+x_2+y-1)}{x_1+y-1}\right),$$

$$\int_{(4)} dc_1\,dc_2\,\frac{M^4}{k^2B}=\frac{4}{x_1x_2}\left[\mathrm{Li}_2\left(\frac{x_2}{x_1+x_2+y-1}\right)+\right.$$

$$\left.+\mathrm{Li}_2\left(-\frac{x_2}{x_1}\right)-\mathrm{Li}_2\left(\frac{x_1x_2}{x_1+x_2+y-1}\right)-\mathrm{Li}_2(-x_2)\right]. \qquad (19)$$

Using the expression for R_4 and angular integrals (19), we perform the integration over positron energy fractions in the fourth region and derive

$$F_4(L,l)=3/4-\ln 2. \qquad (20)$$

Thus, the spectrum of very fast electrons in the five-lepton decay (2) (provided the energy fraction of one positron is smaller than $(1-y)$ and the energy fraction of other one is arbitrary) is defined as the sum of contributions of the considered above four kinematical regions and can be written in the following form:

$$\frac{d\Gamma}{\Gamma_0\,dy}=\frac{\alpha^2}{\pi^2}(1-y)^2\left[\frac{1}{8}L^2+\left(\frac{1}{4}l-\frac{13}{12}\right)L-\frac{5}{3}l-\right.$$

$$\left.-\ln 2-\frac{\pi^2}{24}+\frac{73}{24}\right]. \qquad (21)$$

Note that the terms containing L^2 and L in our final result coincide with those computed in [9], where the collinear and semicollinear kinematics of the five-lepton μ decay were investigated.

Here, we have considered the case where the energy of only one (from two) positron in decay (2) is small. But we are sure that the factor $(1-y)^2$ in the fast electron spectrum would appear at arbitrary positron energies because of an essential squeeze of the angular phase space of positrons along the direction opposite to the electron 3-momentum, if their energies become large enough. The effect of such a squeeze is seen from our analytic calculations. Therefore, we conclude that, under study of the muonium-antimuonium conversion in vacuum by the observation of a very fast electron near its maximum energy, the probability of the main background decay is always very small. For example, in accordance with our estimations, it equals about $1.24\cdot 10^{-7}\,\Gamma_0$ if $y=0.9$ and is essentially decreasing when the electron energy goes on to increase.

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