ELEMENTARY PARTICLES IN A NEW QUANTUM SCHEME. 2

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We continue to develop the theory of fundamental (Lagrangian) particles (beginning in [1]) based on the realistic model of relativistic bi-Hamiltonian dynamics. Two representations of the Heisenberg algebra connected with the hadron and lepton era are considered. Lepton and barolepton fields are found. The explicit form of particle fields and numerical values of their masses are obtained.

1. Two Representations of Algebra $h_{16}^{(*)}$

In the case of the algebra $h_{16}^{(*)}$, the coordinates on the Lagrangian plane $L \subset h_{16}^{(*)}$ are denoted by $\varphi_{\alpha k}$, $\overline{\varphi}_{\alpha k}$. The representation of this algebra (see (2) in [1]) given by the operators

$$\Phi^H = \begin{pmatrix} \Lambda\, \partial / \partial\, \overline{\varphi}_{\alpha k} \\ \varphi_{\alpha k} \end{pmatrix}, \qquad \overline{\Phi}^{\,H} = \left(\overline{\varphi}_{\alpha k},\ -\ \Lambda \frac{\partial}{\partial\, \varphi_{\alpha k}} \right) \quad (1)$$

is called the H-representation (or hadronic). In this representation, the dynamical variables of a system (quadratic Hamiltonians $\frac{1}{\Lambda}\overline{\Phi}\,\Phi$, $\frac{1}{\Lambda}\Phi\,\Phi$, $\frac{1}{\Lambda}\overline{\Phi}\,\overline{\Phi}$) form the Lie algebra $d = \{d_-, d_0, d_+\}$ graduated by powers of Λ in which

$$d_+ :\quad \Lambda\, \partial_{\alpha k}\, \partial_{\beta m}, \quad \Lambda\, \overline{\partial}_{\alpha k}\, \overline{\partial}_{\beta m}, \quad \Lambda\, \partial_{\alpha k}\, \overline{\partial}_{\beta m};$$

$$d_- :\quad \frac{1}{\Lambda}\varphi_{\alpha k}\,\varphi_{\beta m}, \quad \frac{1}{\Lambda}\overline{\varphi}_{\alpha k}\,\overline{\varphi}_{\beta m}, \quad \frac{1}{\Lambda}\overline{\varphi}_{\alpha k}\,\varphi_{\beta m};$$

$$d_0 :\quad \varphi_{\alpha k}\,\partial_{\beta m}, \quad \overline{\varphi}_{\alpha k}\,\overline{\partial}_{\beta m}, \quad \overline{\varphi}_{\alpha k}\,\partial_{\beta m}, \quad \varphi_{\alpha k}\,\overline{\partial}_{\beta m}$$

(here $\partial_{\alpha k} = \frac{\partial}{\partial\,\varphi_{\alpha k}}$, $\overline{\partial}_{\beta m} = \frac{\partial}{\partial\,\overline{\varphi}_{\beta m}}$). Obviously we have

$$[d_0, d_0] \subset d_0, \qquad [d_\pm, d_0] \subset d_\pm,$$

$$[d_+, d_+] = [d_-, d_-] = 0, \qquad [d_+, d_-] \subset d_0.$$

Another representation, the squeezed one (or long-drawn one, if $\Lambda > 1$), is given by the operators

$$\Phi^L = \begin{pmatrix} \partial / \partial\, \overline{\varphi}'_{\alpha k} \\ \Lambda\, \varphi'_{\alpha k} \end{pmatrix}, \qquad \overline{\Phi}^L = \left(\Lambda\, \overline{\varphi}'_{\alpha k},\ -\ \frac{\partial}{\partial\, \varphi'_{\alpha k}} \right), \quad (2)$$

where $\varphi'_{\alpha k}$ is connected with $\varphi_{\alpha k}$ by a (generally speaking) purely isotopic transformation V, $\varphi'_{\alpha k} = V_{km}\,\varphi_{\alpha m}$, $(V \in \mathrm{SL}_i(2, \mathbf{C}))$, is called the L-representation (or leptonic one).

4-momentum variables in the H-representation are written in the form ${}^H p_\mu = \frac{1}{\Lambda}\,\overline{\varphi}\,\overset{+}{\sigma}_\mu\,\varphi = \frac{1}{\Lambda}\,\pi_\mu$, ${}^H\dot{p}_\mu = -\,\Lambda\,\partial\,\overline{\sigma}_\mu\,\overline{\partial}$, and, in the L-representation, they are ${}^L p_\mu = \Lambda\,\overline{\varphi}\,\overset{+}{\sigma}_\mu\,\varphi$, ${}^L\dot{p}_\mu = -\,\frac{1}{\Lambda}\,\partial\,\overline{\sigma}_\mu\,\overline{\partial}$.

Obviously, at $\Lambda \neq 1$, the mass spectrum formula for hadrons remains the same as that given by formula (36) in [1], leaving also the definition of 4-momenta P_μ and Q_μ as well as the formula for particle fields and transition amplitudes unchanged. However, by X_μ and Y_μ, now it is necessary to understand

$$X_\mu = \frac{1}{2}\left(\frac{1}{\Lambda}x_\mu + \Lambda\,\dot{x}_\mu\right), \qquad Y_\mu = \frac{1}{2}\left(\frac{1}{\Lambda}x_\mu - \Lambda\,\dot{x}_\mu\right). \quad (3)$$

In [2], it was shown that Λ can impossibly be an arbitrary number; Λ turns out to be connected with the dimensionality dim d of the algebra of dynamical variables d by the formula $\Lambda = \sqrt{\dim d}$. In the model $h_{16}^{(*)}$, the dimensionality dim d = 136.

2. Hadron and Lepton Era

1) In the present theory, the creation of particles occurs in strict order: at first, hadrons are generated, and

then, with decrease in the density of quanta f, leptons will be generated.

The creation of hadrons (hadron era) is described by the H-representation of algebra (1). In this representation, a 4-momentum of quanta $\dot{f}$ is large ${}^{H}\dot{p}_{\mu} = -\Lambda\,\partial\,\bar{\sigma}_{\mu}\,\bar{\partial}$ (factor Λ), the state $\dot{f}_{z}$ is of the form (18) in [1], where $z_k = z\begin{pmatrix}\varepsilon\\1\end{pmatrix}$ (here, $\varepsilon = z_1/z_2$, $z = z_2$), and a 4-momentum of quanta f is small ${}^{H}p_{\mu} = \frac{1}{\Lambda}\,\bar{\varphi}\,\overset{+}{\sigma}_{\mu}\,\varphi$ (factor $1/\Lambda$). Therefore, the distribution function w_f (15) in [1] is rather essential. As a consequence of the irreversible quantum transition $f \to \dot{f}$ (this aspect being emphasized!), both neutral and charged hadrons are created. It is important to notice that ${}^{H}\dot{p}_{\mu}$ and ${}^{H}p_{\mu}$ are neutral operators in the sense that they commute with the operator of electromagnetic charge Q : $[{}^{H}\dot{p}_{\mu}, Q] = [{}^{H}p_{\mu}, Q] = 0$. However, states $\dot{f}_{z}$ (at $\varepsilon \neq 0$) are characterized by an uncertain charge Q not equal to zero, since $Q\dot{f}_{z} \neq 0$ (that is why charged hadrons occur). Due to the 100 per cent fermion-antifermion asymmetry (see [3]), only fermions will be created (the antifermion creation will happen subsequently as a result of switching on the interactions).

Clearly, in the hadron era, a large positive charge, not compensated by anything arises (before the transition, the charge of quanta f was equal to zero, see above: in the model $\dot{h}_{16}^{(*)}$, the coherent fields, see formula (13) in [1], are neutral leptons with zero mass). In fact, the probabilities of creation of various charged components of the baryon octet (main hadron component) satisfy the condition (they result from (40) in [1])

$$W_{p^{+}} \gg W_{\Sigma^{\pm}} \gg W_{\Xi^{-}} \qquad (4)$$

so that negative Ξ^{-}-baryons cannot compensate the positive charge of protons. Hence, the process creating only hadrons would be accompanied by a violation of the electric charge conservation law (for mesons, this problem is not present: the charge ρ^{+} is compensated by the charge ρ^{-}, and the charge K^{*+} is by the charge K^{*-}). To compensate the charge of protons, only opposite charged leptons can occur due to the following.

2) After the creation of hadrons (the transition $N_{+} \to N_{-}$), the density of quanta f sharply decreases, but the density of quanta $\dot{f}$ on the lower half of the light cone ($\rho \in N_{-}$) increases. Hence, now basically there are transitions at the vertex of the cone: $N_{+} \to \{0\}$ (lepton era).

This era is characterized by the density function $w_f = 1$ (as $\rho_{\mu} = 0$ that is equivalent to the parameter $z = 0$) and is described by the (long-drawn) L-representation (2) of the Heisenberg algebra. In this representation, the 4-momentum ${}^{L}p_{\mu} = \Lambda\,\bar{\varphi}'\,\overset{+}{\sigma}_{\mu}\,\varphi'$ is large (factor Λ), and ${}^{L}\dot{p}_{\mu} = -\frac{1}{\Lambda}\,\partial'\,\bar{\sigma}_{\mu}\,\bar{\partial}'$ is small (factor $\frac{1}{\Lambda}$). Picturely speaking, the yoke $({}^{H}\dot{p}, {}^{H}p)$ can overturn. As a result, it can take another position $({}^{L}\dot{p}, {}^{L}p)$ such that when p rises, $\dot{p}$ drops to slide in $\{0\}$ (at the vertex of the cone), an additional purely isotopic rotation $\varphi_k \to V_{km}\varphi_m$ occurs (the process of transition is described by Lorentz-invariant amplitudes, and it occurs in the space F_0 of additional variables, Lorentzian scalars $\varphi \doteq \varphi_2$, $\bar{\varphi} \doteq \bar{\varphi}_2$ where Lorentz transformations are trivial).

As a consequence, a 4-momentum $\frac{1}{\Lambda}\,\bar{\varphi}\,\overset{+}{\sigma}_{\mu}\,\varphi$ transits into a 4-momentum $\Lambda\,\pi'_{\mu}$:

$$\frac{1}{\Lambda}\,\bar{\varphi}\,\overset{+}{\sigma}_{\mu}\,\varphi \to \Lambda\,\pi'_{\mu} = \Lambda\,\bar{\varphi}'\,\overset{+}{\sigma}_{\mu}\,\varphi' = \Lambda\,\bar{\varphi}\,\overset{+}{\sigma}_{\mu}\,V^{+}\,V\,\varphi =$$

$$= \frac{1}{\Lambda}\,\bar{\varphi}\,\overset{+}{\sigma}_{\mu}\,\tilde{V}^{+}\,\tilde{V}\,\varphi,$$

where $\varphi'_{\alpha k} = V_{km}\,\varphi_{\alpha m}$, $V \in \mathrm{SL}_i(2, \mathbf{C})$ $(\det V = 1)$, and $\tilde{V} = \Lambda\,V \in \mathrm{GL}_i(2, \mathbf{C})$ $(\det \tilde{V} = \Lambda^2)$. Since it may be written as $V^{+}V = \overset{+}{\tau}_m A_m$, where A_m is a time-like 4-isovector ($A^2 = 1$, since $\det V = 1$), it may be written as $\pi'_{\mu} = \pi^{A}_{\mu}$, where $\pi^{A}_{\mu} = \bar{\varphi}\,\overset{+}{\sigma}_{\mu}\,\overset{+}{A}\,\varphi$, and $\overset{+}{A} = \overset{+}{\tau}_m A_m$. Being distinct from π_{μ}, the 4-momentum π^{A}_{μ} carries an electrical charge, since it does not commute with the charge operator Q : $[\pi^{A}_{\mu}, Q] \neq 0$. However, the state $\dot{f}_0 = 1/Z$ corresponding to the vertex of the cone is neutral: $Q\,\dot{f}_0 = 0$ (that is why finishing transitions will be at the vertex of the cone and that is why an isotopic rotation V becomes possible: at $z \neq 0$, the transformation is not admissible. In this case, only dilatations $\varphi \to \Lambda\,\varphi$ are allowed).

With the transformed 4-momentum, the lepton fields are written in the form (compare with (13) in [1]):

$$\psi^{\Sigma}(X + Y) = \frac{1}{(2\pi)^{3/2}}\,\frac{1}{Z}\int e^{i\,\Lambda\,\pi'(X + Y)} \times$$

$$\times\ \theta(\pi_0)\,\delta(\pi^2)\,d^4\pi\,O^{\Sigma}(\pi), \qquad (5)$$

where

$$O^{\Sigma}(\pi) = \int d^4 \nu\, O^{\Sigma}(\varphi).$$

In (5), it is convenient to go over from the variables φ to variables $\varphi' = V\varphi$. Since, for such transformations, the measure $d\mu_f$ does not change, we come to the formula

$$\psi^{\Sigma}(X+Y) = \frac{1}{(2\pi)^{3/2}} \frac{1}{Z} \int e^{i\Lambda\pi(X+Y)} \times$$

$$\times\ \theta(\pi_0)\,\delta(\pi^2)\, d^4\pi\, \tilde{O}^{\Sigma}(\pi),$$

where

$$\tilde{O}^{\Sigma}(\pi) = \int d^4\nu\, O^{\Sigma}(V^{-1}\varphi). \tag{6}$$

As always, all skeletons with non-zero hypercharge and isospin will result in zero. However, now also charged fields occur.

For an example of the lowest states of fermions, the skeletons are in fact of the form

$$O^{\Sigma}(\varphi) = \overline{\varphi}\,\tau_n\,\varphi_\alpha,$$

and the integral

$$O^{\Sigma}(\pi) = \int d^4\nu\,\overline{\varphi}\,\tau_n\,\varphi_\alpha = \delta_{n0}\,\varphi_\alpha(\pi)$$

is only different from zero at $n = 0$, i.e., for $\overline{\varphi}\,\varphi_\alpha$. This neutral lepton field is a neutrino. Next, as $O^{\Sigma}(V^{-1}\varphi) = \overline{\varphi}\, V^{+^{-1}}\,\tau_n\, V^{-1}\varphi_\alpha$, and $V^{+^{-1}}\tau_n V^{-1} = L_{nn'}(V^{-1})\,\tau_{n'}$, we have

$$\tilde{O}^{\Sigma}(\pi) = L_{nn'}(V^{-1}) \int d^4\nu\,\overline{\varphi}\,\tau_{n'}\,\varphi_\alpha = L_{n0}(V^{-1})\,\varphi_\alpha(\pi),$$

where $L_{n0} = \frac{1}{2}\,\mathrm{Sp}\, V^{+^{-1}}\tau_n V^{-1}$. Fields with L_{00} and L_{30} are neutral, and fields with $L_{+0} = L_{10} + iL_{20} \sim \varepsilon$ and with $L_{-0} = L_{10} - iL_{20} \sim \overline{\varepsilon}$ are charged. As can be seen, charged leptons occur.

3) It remains to find a special kind of the transformation V. Since the given transformation is only performed with one purpose, namely to compensate any charge, neutral fields (nonzero before the transformation) remain so after transformation. In particular, this concerns a field with the skeleton $\overline{\varphi}\,\tau_3\,\varphi_\alpha$ which was equal to zero before the transformation. After the transformation, it is $\sim L_{30}$. Therefore, it must be $L_{30} = 0$. This means that

$$\mathrm{Sp}\, V^{+^{-1}}\tau_3 V^{-1} = 0. \tag{7}$$

If hadrons begin to appear after the transformation $\varphi \to \tilde{V}\varphi$, all of them should be neutral. This means that it must be

$$\overline{z}\,\tilde{V} = \overline{z}^{\,0},$$

where $z^0 \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is a neutral isospinor. From the condition

$$\overline{z}\,\tilde{V}\,\tilde{V}^{+} z = \overline{z}\,z = |z|^2 (1 + |\varepsilon|^2) \tag{8}$$

(since neither energy nor temperature of quanta $\dot{f}$ should be changed for the transformation $\tilde{V}$), it follows that $z^0 = z\sqrt{1+|\varepsilon|^2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. From equations (7), (8), we find

$$V = \begin{pmatrix} \dfrac{\Lambda}{\sqrt{1+|\varepsilon|^2}} & \dfrac{\varepsilon}{\Lambda\sqrt{1+|\varepsilon|^2}}\left(1 + \dfrac{\sqrt{\Lambda^4 - 1}}{|\varepsilon|}\right) \\ -\dfrac{\Lambda\,\overline{\varepsilon}}{\sqrt{1+|\varepsilon|^2}} & \dfrac{1}{\Lambda\sqrt{1+|\varepsilon|^2}}\left(1 - |\varepsilon|\sqrt{\Lambda^4 - 1}\right) \end{pmatrix}. \tag{9}$$

At last, the most important condition should be thought over. If the transformation V is performed again, it may not lead to a new position of the yoke $(\dot{p}, p)$ or to a new generation (kind) of particles. Actually, the system has to return to the H-representation. With reference to V, this means that the twice applied transformation V must be proportional to a unit transformation: $V^2 = \alpha$. Hence, we obtain $\det V = \pm\,\alpha$. As $\det V = 1$, $\alpha = \pm 1$. For matrices in the form (9), only the condition $\alpha = -1$ is acceptable, therefore,

$$V^2 = -1. \tag{10}$$

It follows from (10) that, in (9),

$$|\varepsilon| = \sqrt{\frac{\Lambda^2 + 1}{\Lambda^2 - 1}}, \tag{11}$$

and, consequently, the matrix V must be of the form

$$V = \begin{pmatrix} \sqrt{\dfrac{\Lambda^2 - 1}{2}} & \sqrt{\dfrac{\Lambda^2 + 1}{2}}\, e^{i\chi} \\ -\sqrt{\dfrac{\Lambda^2 + 1}{2}}\, e^{-i\chi} & -\sqrt{\dfrac{\Lambda^2 - 1}{2}} \end{pmatrix} =$$

$$= \sqrt{\frac{\Lambda^2 - 1}{2}} \begin{pmatrix} 1 & \varepsilon \\ -\overline{\varepsilon} & -1 \end{pmatrix}, \tag{12}$$

where $\chi = \arg \varepsilon$. Since we have $\Lambda^2 = 136$ in the model $h_{16}^{(*)}$, so $|\varepsilon| = \sqrt{137/135}$, and the matrix V takes the form

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{135} & \sqrt{137}\, e^{i\chi} \\ -\sqrt{137}\, e^{-i\chi} & -\sqrt{135} \end{pmatrix}.$$

It is interesting to note that the *a priori* probabilities of creation of a proton W_p and a neutron W_n are not equal (see (39) in [1])

$$\frac{W_p}{W_n} = |\varepsilon|^2 = \frac{137}{135} = 1.015. \tag{13}$$

Thus, the transition $f \to \dot{f}$ will generate slightly more protons than neutrons.

From the formula $V^+ V = \overset{+}{\tau}_m A_m$, it is possible to find components of the isovector A_m:

$$A_0 = \Lambda^2, \quad A_3 = 0, \quad A_1 = \sqrt{\Lambda^4 - 1}\cos\chi,$$

$$A_2 = \sqrt{\Lambda^4 - 1}\sin\chi, \tag{14}$$

and, from the formula $\frac{1}{2}\,\mathrm{Sp}\, V^{+^{-1}} \tau_n V^{-1} = L_{n0}$, the values L_{n0} are

$$L_{00} = \Lambda^2, \quad L_{30} = 0, \quad L_{+0} = -\sqrt{\Lambda^4 - 1}\, e^{-i\chi},$$

$$L_{-0} = -\sqrt{\Lambda^4 - 1}\, e^{i\chi}. \tag{15}$$

4) So, hadrons arise from the transition ${}^H p \to {}^H \dot{p}$, and leptons arise from the transition ${}^L p \to {}^L \dot{p}$. There are two more transition possibilities: ${}^H p \to {}^L \dot{p}$ and ${}^L p \to {}^H \dot{p}$.

In the first case, both 4-momenta ${}^H p$ and ${}^L \dot{p}$ are small (zero as a matter of fact). In this case, the system goes over from the vertex of the upper half of the light cone into the vertex of the lower half of light cone. This is a vacuum transition. Its amplitude is equal to the factor $1/Z$.

In the second case, both 4-momenta are large: ${}^L p = \Lambda \bar{\varphi} \overset{+}{\sigma}_\mu \varphi$, ${}^H \dot{p} = -\Lambda\, \partial\, \bar{\sigma}_\mu \bar{\partial}$ (factor Λ, matrix $V = 1$). This is the limiting case where the parameter $\mu^2 \to \infty$, see [4]. In this limit, being only meaningful for fermions ($F = 1$), Eq. (29) in [1] passes into the equation (it is necessary to pay attention on the factor Λ^2)

$$\left(\Box_X - \Lambda^2 F_{\Sigma}^{0}\right) O^{\Sigma}(X;Y) = \mathfrak{J}^{\Sigma}(X;Y),$$

and formula (36) in [1] gives

$$M_{T\Sigma}^2 = -\Lambda^2\left[(N+5)^2 + 7 - 4i(i+1)\right]. \tag{16}$$

As can be seen, in our space-time (coordinates X_μ), the objects arising in this case are characterized by an imaginary mass. In addition, they have no wave function, since solutions of the equations

$$\left(\Box_X - M_{T\Sigma}^2\right)\psi^{\Sigma}(X;Y) = 0$$

do not represent any fields. Such objects do not exist as particles. However, in the second space (coordinates Y_μ), these objects represent usual particles, since $\psi^{\Sigma}(X;Y)$ as a function of Y satisfy the Klein–Gordon equation,

$$\left(\Box_Y + M_{T\Sigma}^2\right)\psi^{\Sigma}(X;Y) = 0,$$

and are characterized by the real masses

$$|M_{T\Sigma}^2| = \Lambda^2\left[(N+5)^2 + 7 - 4i(i+1)\right].$$

But the second (internal) space is inaccessible for observations. In our space, the objects in question can show themselves only as virtons, being objects which are described by a distribution function or propagator (for the lack of any wave function), compare with [5]. To go out (because of their large mass for an instant) from the second space in our space, they certainly have to return into the second space where they exist as usual particles. In that way, these objects stick together different points of the discontinuum to transform it in a continuum [6].

APPENDIX 1

Here, we find the explicit amplitudes $O^{\Sigma}(P,Q)$ determined by formula (25) in [1]:

$$O^{\Sigma}(P,Q) = \int d^4\nu\, O^{\Sigma}(\varphi_k, \bar{\varphi}_k)\, e^{-\frac{\pi\rho}{2\mu^2} - 2i\,\mathrm{Im}\,\bar{z}_k \varphi_k} =$$

$$= \exp\left(-\frac{-P^2}{4\mu^2}\right) O^{\Sigma}\left(-\frac{\partial}{\partial\bar{z}}, \frac{\partial}{\partial z}\right) I, \tag{17}$$

where $O^{\Sigma}(\varphi)$ are the skeletons of particles, and

$$I(\pi,\rho) = \int d^4\nu \exp\left(\bar{\varphi}_{\alpha k} z_{\alpha k} - \bar{z}_{\alpha k}\varphi_{\alpha k}\right) \tag{18}$$

such that $z_{\alpha k} = \delta_{\alpha 2} z_k$, and the measure $d^4\nu$ is defined by formula (9) in [1].

At first, it is useful to consider the integral appearing in the model $h_8^{(*)}$ (see [4]):

$$J = \int_0^{2\pi} \frac{d\omega}{2\pi} e^{\bar{\varphi}_\alpha z_\alpha - \bar{z}_\alpha \varphi_\alpha}, \quad z_\alpha = z\,\delta_{\alpha 2}, \tag{19}$$

where $\omega = \arg \varphi_2$. It follows from the definitions $\pi_\mu = \bar{\varphi}\overset{+}{\sigma}_\mu \varphi$, $\rho_\mu = \bar{z}\,\bar{\sigma}_\mu\, z$ $(\pi^2 = \rho^2 = 0)$ that $\pi\rho = 2\,|\,\bar{z}_\alpha \varphi_\alpha\,|^2$. As $2\pi\rho = -P^2$, $\bar{z}_\alpha \varphi_\alpha = \frac{1}{2}\sqrt{-P^2}\, e^{i\chi}$. Therefore,

$$J = \frac{1}{2\pi}\int_0^{2\pi} e^{i\sqrt{-P^2}\sin\chi}\, d\omega.$$

Since

$$\varphi_\alpha = e^{i\omega}\begin{pmatrix} e^{i\psi}\,|\,\varphi_1| \\ |\,\varphi_2| \end{pmatrix}, \quad z_\alpha = e^{i\nu}\begin{pmatrix} e^{i\alpha}\,|\,z_1| \\ |\,z_2| \end{pmatrix},$$

we obtain

$$\bar{\varphi}_\alpha z_\alpha - \bar{z}_\alpha \varphi_\alpha = -2i\,[\sin(\omega - \nu + \psi - \alpha)\,|\,\varphi_1 z_1| +$$

$$+ \sin(\omega - \nu)\,|\,\varphi_2 z_2|\,],$$

$$\bar{\varphi}_\alpha z_\alpha + \bar{z}_\alpha \varphi_\alpha = 2\,[\cos(\omega - \nu + \psi - \alpha)\,|\,\varphi_1 z_1| +$$

$$+ \cos(\omega - \nu)\,|\,\varphi_2 z_2|\,].$$

It follows from this that

$$d\omega \frac{d}{d\omega}\sqrt{-P^2}\sin\chi = \sqrt{-P^2}\cos\chi \frac{d\chi}{d\omega}\, d\omega =$$

$$= \sqrt{-P^2}\cos\chi\, d\omega,$$

i.e., $d\chi = d\omega$ and, consequently,

$$J = \frac{1}{2\pi}\int_0^{2\pi} e^{i\sqrt{-P^2}\sin\chi}\, d\chi = J_0(\sqrt{-P^2}),$$

where J_0 is the Bessel function.

Integrating now (17) with respect to ω_1 and ω_2, we obtain

$$I = \int d^4\nu\, e^{\bar{\varphi}_{\alpha k} z_{\alpha k} - \bar{z}_{\alpha k}\varphi_{\alpha k}} = \int d^2\nu\, J_0(\sqrt{-P_1^2})\, J_0(\sqrt{-P_2^2}),$$

where $-P_2^2 = 2\rho^{(2)}\Pi$, $-P_1^2 = 2\rho^{(1)}(\pi - \Pi)$, and

$$d^2\nu = \frac{2}{\pi}\frac{\theta(\Pi_0)\,\theta(\pi_0 - \Pi_0)}{\theta(\pi_0)}\,\delta(\Pi^2)\,\delta(2\pi\Pi - \pi^2)\, d^4\Pi.$$

Expanding the function J_0 in a series, we have

$$I = \sum_{n,m=0}^{\infty} \frac{(-1)^n}{2^n (n!)^2}\frac{(-1)^m}{2^m (m!)^2}\int d^2\nu\,(\rho^{(2)}\Pi)^n\,(\rho^{(1)}(\pi - \Pi))^m.$$

As

$$(\rho^{(1)}(\pi - \Pi))^m = \sum_{l=0}^{m} (\rho^{(1)}\pi)^{m-l}\,(\rho^{(1)}\Pi)^l\,(-1)^l C_l^m,$$

(C_l^m is the number of combinations),

$$I = \sum_{n,m,l} \frac{(-1)^{n+m+l}}{2^n (n!)^2\, 2^m (m!)^2}\, C_l^m\,(\rho^{(1)}\pi)^{m-l}\int d^2\nu\,\times$$

$$\times\,(\rho^{(1)}\Pi)^l\,(\rho^{(2)}\Pi)^n.$$

We need to calculate integrals in the form $(\pi^2 = 0)$:

$$\int d^2\nu\,\overbrace{\Pi_\alpha \ldots \Pi_\omega}^{s} = \frac{1}{\pi}\int \overbrace{\Pi_\alpha \ldots \Pi_\omega}^{s}\,\frac{\theta(\Pi_0)\,\theta(\pi_0 - \Pi_0)}{\theta(\pi_0)}\,\times$$

$$\times\,\delta(\Pi^2)\,\delta(\pi\Pi)\, d^4\Pi.$$

It is clear that the integration result must be of the form $A(s)\,\overbrace{\pi_\alpha \ldots \pi_\omega}^{s}$ such that it remains only to find $A(s)$. For this purpose, we pass to a system in which $\pi_\mu = (0, 0, \pi_3, \pi_0)$ such that $\pi_3 = \pm\,\pi_0$. In it, we have

$$\frac{1}{\pi}\int \frac{\theta(\Pi_0)\,\theta(\pi_0 - \Pi_0)}{\theta(\pi_0)}\,\delta(\Pi_1^2 + \Pi_2^2 + \Pi_3^2 - \Pi_0^2)\,\delta(\pi_0\,\times$$

$$\times\,(\Pi_3 - \Pi_0))\,\Pi_\alpha \ldots \Pi_\omega\, d\Pi_1\, d\Pi_2\, d\Pi_3\, d\Pi_0 =$$

$$= \frac{1}{\pi}\int \delta(\Pi_1^2 + \Pi_2^2)\, d\Pi_1\, d\Pi_2 \int_0^{\pi_0} \frac{d\Pi_0}{\pi_0}\,\Pi_\alpha \ldots \Pi_\omega.$$

As

$$\frac{1}{\pi}\int \delta(\Pi_1^2 + \Pi_2^2)\, d\Pi_1\, d\Pi_2 = \frac{1}{\pi}\int_0^{2\pi} d\theta \int_0^{\infty} \delta(r^2)\, r\, dr = 1,$$

and the integral $\frac{1}{\pi_0}\int_0^{\pi_0} d\Pi_0\,\overbrace{\Pi_\alpha \ldots \Pi_\omega}^{s}$ is only nonzero for $\alpha, \ldots, \omega$ equal to 3 or 0, and $\Pi_3 = \Pi_0$,

$$\frac{1}{\pi_0}\int_0^{\pi_0} d\Pi_0\,\overbrace{\Pi_\alpha \ldots \Pi_\omega}^{s} = \frac{1}{\pi_0}\int_0^{\pi_0} d\Pi_0\,\Pi_0^s = \frac{\pi_0^s}{s+1}.$$

In any system, the result is written as $\frac{1}{s+1}\overbrace{\pi_\alpha \ldots \pi_\omega}^{s}$. Therefore, we have

$$A(s) = \frac{1}{s+1}.$$

In view of the equality

$$\int d^2\nu\,(\rho^{(1)}\Pi)^l(\rho^{(2)}\Pi)^n = \frac{1}{l+n+1}\,(\rho^{(1)}\pi)^l\,(\rho^{(2)}\pi)^n,$$

we obtain

$$I = \sum_{n,m,l} \frac{(-1)^{n+m+l}}{2^n (n!)^2\, 2^m (m!)^2}\, C_l^m\,\frac{(\pi\rho^{(2)})^n (\pi\rho^{(1)})^m}{l+n+1}.$$

As

$$\sum_{l=0}^{m} \frac{(-1)^l}{l+n+1} C_l^m = \frac{m!\,n!}{(m+n+1)!},$$

$$I = \sum_{n,m=0}^{\infty} \frac{(-\pi\rho^{(1)}/2)^m}{m!} \frac{(-\pi\rho^{(2)}/2)^n}{n!\,(m+n+1)!} =$$

$$= \sum_{n=0}^{\infty} \frac{(-\pi\rho^{(1)}/2)^m}{m!} J_{m+1}(\sqrt{2\pi\rho^{(2)}}) \left(\frac{2}{\sqrt{2\pi\rho^{(2)}}}\right)^{m+1},$$

where J_{m+1} is the Bessel function. To take advantage of the formula [7]

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \left(\frac{t(z+t/2)}{z}\right)^m J_{m+1}(z) = \frac{z}{z+t} J_1(z+t)$$

in which it is necessary to put $z = \sqrt{2\pi\rho^{(2)}}$, $\pi\rho^{(1)} = t(\sqrt{2\pi\rho^{(2)}} + t/2)$, i.e. $t = \sqrt{2\pi\rho} - \sqrt{2\pi\rho^{(2)}}$, where $\rho = \rho^{(1)} + \rho^{(2)}$, we find the final expression

$$I(\pi, \rho) = \frac{2}{\sqrt{2\pi\rho}} J_1(\sqrt{2\pi\rho}) = \frac{2}{\sqrt{-P^2}} J_1(\sqrt{-P^2}),$$

$$-P^2 = 2\pi\rho.$$

At $\rho_\mu = 0$ (i.e., $z = 0$), it follows from this that

$$\int d^4\nu = 1.$$

It is not difficult to show that definition (17) results in a factorized expression for the amplitudes $O^\Sigma(P, Q)$ or $O^\Sigma(\pi, \rho)$:

$$O^\Sigma(\pi, \rho) = O_\Sigma^{(l)}(\pi, \rho)\, O_\Sigma^{(i)}(z)\, N_\Sigma(X), \quad X = \sqrt{2\pi\rho}, \tag{20}$$

where $O_\Sigma^{(l)}$ and $O_\Sigma^{(i)}$ are the Lorentzian and isotopic factors.

Let us find, at first, all three factors in the model $h_8^{(*)}$. In this model, L-skeletons $O^\Sigma(\varphi)$ are written in the form

$$O^\Sigma(\varphi) = \overbrace{\varphi_\alpha \ldots \varphi_\chi}^{a}\, \overbrace{\bar\varphi_{\dot\alpha} \ldots \bar\varphi_{\dot\omega}}^{b}.$$

Spinor indices equal to 2 answer additional variables. For definiteness (due to the complete fermion-antifermion asymmetry), we hold that $a \geq b$. Denoting $a + b = N$ and $a - b = Y$, we rewrite $O^\Sigma(\varphi)$ in the form

$$O^\Sigma(\varphi) = \overbrace{\varphi_\delta \ldots \varphi_\chi}^{Y}\, \overbrace{\varphi_\alpha \bar\varphi_{\dot\alpha} \ldots \varphi_\omega \bar\varphi_{\dot\omega}}^{(N-Y)/2}.$$

As $\pi_\mu = \overset{+}{\bar\varphi}\sigma_\mu\varphi$, $\varphi_\alpha\bar\varphi_{\dot\alpha} = \frac{1}{2}\bar\pi_{\alpha\dot\alpha}$, where $\bar\pi = \pi_\mu \overset{+}{\sigma}_\mu$. Therefore, by definition of the amplitudes $O^\Sigma(\pi, \rho)$, we have

$$O^\Sigma(\pi, \rho) = e^{-\frac{X^2}{4\mu^2}} O^\Sigma\left(-\frac{\partial}{\partial\bar z^\alpha}, \frac{\partial}{\partial z^{\dot\alpha}}\right) J_0(X) =$$

$$= e^{-\frac{X^2}{4\mu^2}} \overbrace{\frac{1}{2}\bar\pi_{\alpha\dot\alpha} \ldots \frac{1}{2}\bar\pi_{\omega\dot\omega}}^{(N-Y)/2} \left(-\frac{\partial}{\partial\bar z^\delta}\right) \ldots \left(-\frac{\partial}{\partial\bar z^\chi}\right) J_0(X).$$

By writing $\frac{\partial}{\partial\bar z^\alpha} = (\bar\pi z)_\alpha \frac{1}{X}\frac{\partial}{\partial X}$ and using the formula [7]

$$\left(\frac{1}{X}\frac{\partial}{\partial X}\right)^m \frac{1}{X^\nu} J_\nu(X) = (-1)^m \frac{1}{X^{\nu+m}} J_{\nu+m}(X),$$

we obtain (here, $(\bar\pi z)_\delta = \bar\pi_{\delta 2} z$)

$$O^\Sigma(\pi, \rho) = e^{-\frac{X^2}{4\mu^2}} \overbrace{(\bar\pi z)_\delta \ldots (\bar\pi z)_\chi}^{Y} \overbrace{\frac{1}{2}\bar\pi_{\alpha\dot\alpha} \ldots \frac{1}{2}\bar\pi_{\omega\dot\omega}}^{(N-Y)/2} \times$$

$$\times \frac{1}{X^Y} J_Y(X). \tag{21}$$

Here, $Y \geq 0$. In the case of $b \geq a$ $(Y \leq 0)$, this formula can be written as

$$O^\Sigma(\pi, \rho) = e^{-\frac{X^2}{4\mu^2}} \overbrace{(\bar z\bar\pi)_{\dot\delta} \ldots (\bar z\bar\pi)_{\dot\chi}}^{|Y|} \overbrace{\frac{1}{2}\bar\pi_{\alpha\dot\alpha} \ldots \frac{1}{2}\bar\pi_{\omega\dot\omega}}^{(N-Y)/2} \times$$

$$\times \frac{(-1)^Y}{X^{|Y|}} J_{|Y|}(X).$$

To distinguish additional variables in $O^\Sigma(\varphi)$, it is necessary to write

$$O^\Sigma(\varphi) = \varphi^{a_s}\bar\varphi^{b_s} \overbrace{\varphi_\alpha \ldots \varphi_\omega}^{a'}\, \overbrace{\bar\varphi_{\dot\alpha} \ldots \bar\varphi_{\dot\chi}}^{b'},$$

where $a' = a - a_s$, $b' = b - b_s$. Here, $\varphi \doteq \varphi_2$, $\varphi \doteq \bar\varphi_2$ are Lorentzian scalars. From $O^\Sigma(\varphi)$, it is possible to construct the SL(2, **C**)-multiplet, realizing a finite-dimensional representation $(s, \dot s) = (a'/2, b'/2)$ of the group $SL_l(2, \mathbf{C})$. A canonical basis of such representations is written in the form

$$O^\Sigma(\varphi) = \varphi_2^a \bar\varphi_2^b\, O_{j, j_3}^{(s_0, s_1)}(\zeta), \tag{22}$$

where (see [8])

$$O_{j,j_3}^{(s_0,s_1)}(\zeta) = (-1)^{j-s_0} \sqrt{\frac{(j+s_0)!\,(j-s_0)!}{(j+j_3)!\,(j-j_3)!}} \times$$

$$\times \zeta^{s_0+j_3}(1+|\zeta|^2)^{s_1-s_0-1} P_{j-s_0}^{(s_0-j_3, s_0+j_3)}\left(\frac{|\zeta|^2-1}{|\zeta|^2+1}\right). \tag{23}$$

Here, $s_0 = s - \dot s = \frac{a' - b'}{2} = \frac{1}{2}F$, $s_1 = s + \dot s + 1 = \frac{1}{2}(a' + b') + 1 = \frac{1}{2}D + 1$, $s_0 \leq j \leq s_1 - 1$, $-j \leq j_3 \leq j$, $\zeta = \varphi_1/\varphi_2$, and $P_n^{(\alpha,\beta)}$ are Jacobi polynomials. The functions $O_{j,j_3}^{(s_0,s_1)}(\zeta)$ are normalized by the condition

$$\frac{i}{2\pi}\int \overline{O_{j',j'_3}^{(s_0,s_1)}(\zeta)}\, O_{j,j_3}^{(s_0,s_1)}(\zeta)\, d\zeta \wedge d\bar\zeta = \frac{(j-s_0)!}{2j+1}\delta_{jj'}\delta_{j_3 j'_s}$$

(pay attention that the representation $(s_0, -s_1)$ is conjugate to (s_0, s_1)). Functions (23) can developed further so $(Y_s = Y - F)$

$$O^{\Sigma}(\varphi) = (-1)^{j-s_0}\sqrt{\frac{(j+s_0)!(j-s_0)!}{(j+j_3)!(j-j_3)!}}\times$$

$$\times\ \varphi_1^{s_0+j_3}\varphi_2^{Y_3+s_0-j_3}(\overline{\varphi}_2\varphi_2)^{b_s}(\overline{\varphi}\varphi)^{s_1-s_0-1}\times$$

$$P_{j-s_0}^{(s_0-j_3,\,s_0+j_3)}\left(\frac{\overline{\varphi}\sigma_3\varphi}{\overline{\varphi}\varphi}\right).$$

At the same time, the amplitude $O^{\Sigma}(\pi, \rho)$ is written as

$$O^{\Sigma}(\pi,\rho) = (-1)^{j-s_0}\sqrt{\frac{(j+s_0)!(j-s_0)!}{(j+j_3)!(j-j_3)!}}\times$$

$$\times\left(\frac{\pi_0-\pi_s}{2}\right)^{b_s}\pi_0^{s_1-s_0-1}P_{j-s_0}^{(s_0-j_3,\,s_0+j_3)}\left(\frac{\pi_3}{\pi_0}\right)\times$$

$$\times\ (\overline{\pi}z)_2^{Y_s+s_0-j_3}e^{-\frac{X^2}{4\mu^2}}\frac{1}{X^Y}J_Y(X),$$

having obviously a factorized structure.

Now we go over to the model $h_{16}^{(*)}$. In the system where $\varphi_{1k} = 0$ (or in the space F_0), the skeletons of particles are expressed in terms of additional variables $\varphi_k \doteq \varphi_{2k}$, $\overline{\varphi}_k \doteq \overline{\varphi}_{2k}$ (isotopic variables now play a spin role),

$$O^{\Sigma}(\varphi) = \underbrace{\varphi_\alpha \ldots \varphi_f}_{A}\,\overbrace{\overline{\varphi}_g \ldots \overline{\varphi}_z}^{B},$$

in the way analogous to the model $h_8^{(*)}$. Representation (22) is now rewritten in the form

$$O^{\Sigma}(\varphi) = \varphi_2^A\,\overline{\varphi}_2^B\,O_{i,i_3}^{(i_0,i_1)}(\zeta),\qquad \zeta = \frac{\varphi_1}{\varphi_2},$$

where $i_0 = \frac{1}{2}(A-B) = \frac{1}{2}Y$, $i_1 = \frac{1}{2}(A+B)+1 = \frac{1}{2}N+1$, $\frac{1}{2}Y \le i \le \frac{1}{2}N$, $-i \le i_3 \le i$ are weights of a representation of the isotopic group $SL_i(2, \mathbf{C})$. In the same way as earlier, we have

$$O^{\Sigma}(\varphi) = (-1)^{i-i_0}\sqrt{\frac{(i+i_0)!(i-i_0)!}{(i+i_3)!(i-i_3)!}}\times$$

$$\times\ \varphi_1^{i_0+i_3}\varphi_2^{i_0-i_3}(\overline{\varphi}\varphi)^{i_1-i_0-1}P_{i-i_0}^{(i_0-i_3,\,i_0+i_3)}\left(\frac{\overline{\varphi}\tau_3\varphi}{\overline{\varphi}\varphi}\right).$$

Expanding Jacobi polynomials in a series

$$P_n^{(\alpha,\beta)}(x) = \frac{1}{2^n}\sum_{m=0}^{n}C_m^{n+\alpha}C_{n-m}^{n+\beta}(x-1)^{n-m}(x+1)^m,$$

we rewrite the previous formula:

$$O^{\Sigma}(\varphi) = \omega_{i,i_3}^{(i_1,i_0)}\varphi_1^{i_0+i_3}\varphi_2^{i_0-i_3}(\overline{\varphi}\varphi)^{i_1-i_0-1}\times$$

$$\times\sum_{m=0}^{i-i_0}(-1)^m C_m^{i-i_3}C_{i-i_0-m}^{i+i_3}(\overline{\varphi}_1\varphi_1)^m(\overline{\varphi}_2\varphi_2)^{i-i_0-m},$$

where $\omega_{i,i_3}^{(i_1,i_0)} = \sqrt{\frac{(i+i_0)!(i-i_0)!}{(i+i_3)!(i-i_3)!}}$. While calculating $O^{\Sigma}(\pi, \rho)$, it is necessary to put $\varphi_k = -\frac{\partial}{\partial\overline{z}_k}$ such that $-\frac{\partial}{\partial\overline{z}_k} = = \frac{\pi_0-\pi_3}{2}z_k D$, where $D = -\frac{2}{X}\frac{d}{dX}$ and $X = \sqrt{2\pi\rho} = = \sqrt{2(\pi_0-\pi_3)\overline{z}_k z_k}$.

Now we arrive at

$$\varphi_1^{i_0+i_3+m}\varphi_2^{i-i_3-m}\rightarrow\left(\frac{\pi_0-\pi_3}{2}\right)^{i+i_0}\times$$

$$\times\ z_1^{i_0+i_3+m}z_2^{i-i_3-m}D^{i+i_0}.$$

Therefore as $\overline{\varphi}_k\varphi_k = \frac{1}{2}(\pi_0-\pi_3)$, we have

$$O^{\Sigma}(\pi,\rho) = e^{-\frac{X^2}{4\mu^2}}\omega_{i,i_3}^{(i_1,i_0)}(i+i_3)!(i-i_3)!\times$$

$$\times\left(\frac{\pi_0-\pi_3}{2}\right)^{i_1+i_0-1}\sum_{m=0}^{i-i_0}(-1)^m\frac{(\partial/\partial z_1)^m}{m!}\times$$

$$\times\ \frac{(\partial/\partial z_2)^{i-i_0-m}}{(i-i_0-m)!}\frac{z_1^{i_0+i_3+m}}{(i_0+i_3+m)!}\frac{z_2^{i-i_3-m}}{(i-i_3-m)!}D^{i+i_0}I.$$

Denote $\upsilon_k = \frac{\pi_0-\pi_3}{2}|z_k|^2 D$. The validity of the following formula is verified by the method of induction:

$$\frac{1}{m!}\left(\frac{\partial}{\partial z_m}\right)^m z_k^{\alpha+m}f(X) = z_k^{\alpha} : L_m^{\alpha}(\upsilon_k) : f(X).$$

Here, $f(X)$ is any function depending only on z_k by means of X, and L_m^{α} are Laguerre polynomials. A normal product sign: : suggests that the degree $(\upsilon_k)^n$ is understood to be $\left(\frac{\pi_0-\pi_3}{2}|z_k|^2\right)^n D^n$. Therefore, $O^{\Sigma}(\pi, \rho)$ may be written as

$$O^{\Sigma}(\pi,\rho) = e^{-\frac{X^2}{4\mu^2}}\omega_{i,i_3}^{(i_1,i_0)}(i+i_3)!(i-i_3)!\times$$

$$\times\left(\frac{\pi_0-\pi_3}{2}\right)^{i_1+i_0-1}z_1^{i_0+i_3}z_2^{i_0-i_3}\sum_{m=0}^{i-i_0}(-1)^m\times$$

$$\times\ \frac{L_m^{i_0+i_3}(\upsilon_1)L_{i-i_0-m}^{i_0-i_3}(\upsilon_2)}{(i_0+i_3+m)!(i-i_3-m)!}D^{i+i_0}I(X).$$

We cannot sum up here the series if υ_1 and υ_2 are arbitrary. But this is unnecessary. Since the parameters z_1 and z_2 of the theory are large (see [1]), it suffices to know the result at $\upsilon_1, \upsilon_2 \to \infty$. As $L_n^{\alpha}(x) \xrightarrow[x\to\infty]{} (-x)^n/n!$, for large υ_1 and υ_2, we have

$$\sum_{m=0}^{i-i_0} (-1)^m \frac{L_m^{i_0+i_3}(\upsilon_1)\, L_{i-i_0-m}^{i_0-i_3}(\upsilon_2)}{(i_0+i_3+m)!\,(i-i_3-m)!} \to$$

$$\to (-\upsilon_2)^{i-i_0} \sum_{m=0}^{i-i_0} \left(-\frac{\upsilon_1}{\upsilon_2}\right)^m [m!\,(i_0+i_3+m)!\,(i-i_0-$$

$$-m)!\,(i-i_3-m)!]^{-1} = \frac{(-\upsilon_2)^{i-i_0}}{(i-i_0)!\,(i-i_3)!\,(i_0+i_3)!}\times$$

$$\times\, {}_2F_1(-i+i_0, -i+i_3; i_0+i_3+1; -\upsilon_1/\upsilon_2) =$$

$$= \frac{(\upsilon_1+\upsilon_2)^{i-i_0}}{(i-i_3)!\,(i+i_3)!} P_{i-i_0}^{(i_0-i_3,\, i_0+i_3)}\left(\frac{\upsilon_1-\upsilon_2}{\upsilon_1+\upsilon_2}\right).$$

Here, ${}_2F_1$ is the hypergeometric function, and $P_n^{(\alpha,\beta)}$ are Jacobi polynomials. As $\upsilon_1+\upsilon_2 = \frac{\pi_0-\pi_3}{2}\bar{z}\, z\, D$, and $\frac{\upsilon_1-\upsilon_2}{\upsilon_1+\upsilon_2} = \frac{\bar{z}\,\tau_3\, z}{\bar{z}\, z}$, we obtain finally:

$$O^{\Sigma}(\pi,\rho) = e^{-\frac{X^2}{4\mu^2}}\, \omega_{i,i_3}^{(i_1,i_0)} \left(\frac{\pi_0-\pi_3}{2}\right)^{i_1+i_0-1} \times$$

$$\times\, z_1^{i_0+i_3} z_2^{i_0-i_3} (\bar{z}\, z)^{i-i_0} P_{i-i_0}^{(i_0-i_3,\, i_0+i_3)}\left(\frac{\bar{z}\,\tau_3\, z}{\bar{z}\, z}\right) D^{2i} I(X).$$

Thus, we have come again to Jacobi polynomials from which we proceeded. As

$$D^{2i} I = \left(-\frac{2}{X}\frac{d}{dX}\right)^{2i} \frac{2}{X} J_1(X) = \left(\frac{2}{X}\right)^{2i+1} J_{2i+1}(X),$$

see [7], by keeping in mind that the equality $\frac{\pi_0-\pi_3}{2} = \frac{X^2}{4\bar{z}\, z}$ holds on the spase F_0, we have

$$O^{\Sigma}(\pi,\rho) = e^{-\frac{X^2}{4\mu^2}}\, \omega_{i,i_3}^{(i_1,i_0)} \frac{z_1^{i_0+i_3} z_2^{i_0-i_3}}{(\bar{z}\, z)^{i_1+i_0-1}} \times$$

$$\times\, P_{i-i_0}^{(i_0-i_3,\, i_0+i_3)}\left(\frac{\bar{z}\,\tau_3\, z}{\bar{z}\, z}\right)\left(\frac{X}{2}\right)^{2i_1-3} J_{2i+1}(X).$$

This expression takes place when a representation of the algebra $h_{16}^{(*)}$ is considered on the subspace F_0, i.e., when $\varphi_{1k} = 0$. In this case, the additional variables φ_{2k} and spinors $\varphi_{\alpha k}$ have no distinctions. It is not difficult to pass to the case where a representation of $h_{16}^{(*)}$ is considered on the whole space $\mathbf{F} = F_{\mathrm{F}} \otimes F_0$. It is only necessary that not all $\frac{\pi_0-\pi_3}{2}$ be equal to $\frac{X^2}{4\bar{z}z}$. Namely, $O^{\Sigma}(\pi,\rho)$ is written as

$$O^{\Sigma}(\pi,\rho) = \left(\frac{\pi_0-\pi_3}{2}\right)^F \omega_{i,i_3}^{(i_1,i_0)} \frac{z_1^{i_0+i_3} z_2^{i_0-i_3}}{(\bar{z}\, z)^{(i_1+i_0-F-1)}} \times$$

$$\times\, P_{i-i_0}^{(i_0-i_3,\, i_0+i_3)}\left(\frac{\bar{z}\,\tau_3\, z}{\bar{z}\, z}\right)\left(\frac{X}{2}\right)^{2i_1-2F-3} J_{2i+1}(X)\, e^{-\frac{X^2}{4\mu^2}}, \quad (24)$$

where F is the fermion charge of the amplitude. The rest $\frac{\pi_0-\pi_3}{2}$ is the second component of the spinor $\varphi_\alpha(\pi) = \frac{1}{2}\begin{pmatrix}\pi_1 + i\pi_2\\ \pi_0-\pi_3\end{pmatrix} = \frac{1}{2}\bar{\pi}_{\alpha 2}$ (the first component disappears when passing to F_0 and occurs in the return transition from F_0 to $\mathbf{F}$). Therefore, on $\mathbf{F}$, the Lorentzian factor $\left(\frac{\pi_0-\pi_3}{2}\right)^F$ in (20) is written in the form

$$O_l^{\Sigma} = \frac{1}{2^F}\overbrace{\bar{\pi}_{\alpha 2} \ldots \bar{\pi}_{\omega 2}}^{F} = \varphi_\alpha(\pi) \ldots \varphi_\omega(\pi). \quad (25)$$

For the isotopic factor, we have

$$O_i^{\Sigma} = \omega_{i,i_3}^{(i_1,i_0)} \frac{z_1^{i_0+i_3} z_2^{i_0-i_3}}{(\bar{z}\, z)^{i_1+i_0-F-1}} P_{i-i_0}^{(i_0-i_3,\, i_0+i_3)}\left(\frac{\bar{z}\,\tau_3\, z}{\bar{z}\, z}\right), \quad (26)$$

and, for N_Σ, we obtain

$$N_\Sigma = \left(\frac{M_\Sigma}{2}\right)^{2i_1-2F-3} J_{2i+1}(M_\Sigma)\, e^{-\frac{M_\Sigma^2}{4\mu^2}} \quad (27)$$

(let us assume $X^2 = M_\Sigma^2$). As $\frac{\bar{z}\,\tau_3\, z}{\bar{z}\, z} = \frac{|\eta|^2-1}{|\eta|^2+1} = \frac{1}{\Lambda^2} = \frac{1}{136}$ it is possible to take $P_{i-i_0}^{(i_0-i_3,\, i_0+i_3)}(0)$ in (26).

Consider the L-spinor $\varphi_\alpha(\pi)$. It may be written as

$$\varphi_\alpha(\pi) = \frac{1}{2}\bar{\pi}_{\alpha 2} = \bar{\pi}\, a = \frac{\bar{P}+\bar{Q}}{2} a,$$

where $a = \frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix}$ is a constant spinor. Define now a R-spinor by the formula

$$\chi^{\dot{\alpha}}(P,Q) = \frac{\overset{+}{\rho}\,\bar{\pi}\, a}{X} = \frac{(\overset{+}{Q}-\overset{+}{P})(\overset{-}{P}+\overset{-}{Q})\, a}{4X}.$$

If we substitute $\psi = \begin{pmatrix}\chi^{\dot{\alpha}}(P,Q)\\ \varphi_\alpha(P,Q)\end{pmatrix}$ in (37) in [1] instead of $O^{\Sigma}(P,Q)$ and subsequently integrate over Q, previously having put $Y_\mu = 0$ (thus, linear powers of Q vanish), we come to a bispinor

Masses of 'bare“ fundamental hadrons (in GeV)

M_Σ	N: $i=\frac{1}{2}$, $Y=1$, $N=2n+1$		Λ: $i=0$, $Y=0$, $N=2n+2$		Σ: $i=1$, $Y=0$, $N=2n+2$		Δ: $i=\frac{3}{2}$, $Y=1$, $N=2n+3$	
n	Theor.	Exper.	Theor.	Exper.	Theor.	Exper.	Theor.	Exper.
0	1.14	0.94	1.15	1.12	1.22	1.19	1.34	1.23
1	1.27	~	1.31	1.33	1.36	~	1.44	–
2	1.39	1.39	1.43	1.40	1.47	1.38	1.55	1.55
3	1.49	1.47	1.53	1.52	1.57	1.58	1.64	1.65
4	1.60	1.65	1.63	1.60	1.67	1.67	1.73	1.69
5	1.69	1.67	1.72	1.69	1.76	1.76	1.91	1.90
6	1.78	1.78	1.81	1.82	1.84	1.84	1.89	1.89
7	1.86	1.81	1.89	1.87	1.92	1.92	1.97	1.95
8	1.94	1.99	1.97	2.01	1.99	2.00	2.04	~
9	2.02	2.00	2.05	2.08	2.06	2.03	2.11	~
10	2.09	2.10	2.12	2.11	2.13	2.10	2.18	2.16
M_Σ	Ξ: $i=\frac{1}{2}$, $Y=-1$, $N=2n+5$		ε: $i=0$, $Y=0$, $N=2n$		ρ: $i=1$, $Y=0$, $N=2n+2$		K^*: $i=\frac{1}{2}$, $Y=1$, $N=2n+3$	
n	Theor.	Exper.	Theor.	Exper.	Theor.	Exper.	Theor.	Exper.
0	1.39	1.31	0.73	0.70_{ε}	0.72	0.75_{ρ}	0.84	0.89_{K^*}
1	1.50	~	0.82	0.78_{ω}	0.85	~	0.95	~
2	1.60	1.53	0.93	0.98_{s^*}	0.96	0.98_{δ}	1.05	~
3	1.69	1.68	1.03	1.02_{Φ}	1.07	1.10_{A_1}	1.15	~
4	1.75	1.82	1.12	1.08_{η_N}	1.16	$1.17_{A_{1,s}}$	1.23	~
5	1.86	~	1.21	1.27_{f}	1.25	$1{,}25_{\rho'}$	1.32	1.28_{Q_1}
6	1.94	1.94	1.29	1.28_{D}	1.34	1.31_{A_2}	1.40	1.40_{Q_2}
7	2.02	2.03	1.37	1.30_{ε}	1.42	$1{,}41_{X}$	1.47	1.43_{K^*}
8	2.09	2.12	1.45	1.42_{E}	1.49	1.54_{F_1}	1.55	1.58_{L}
9	2.16	~	1.52	$1.51_{f'}$	1.57	$1.60_{\rho''}$	1.62	1.65_{K^*}
10	2.23	2.25	1.60	1.67_{ω}	1.66	1.66_{A_3}	1.68	1.70_{K_N}

$\psi = \begin{pmatrix} \chi \\ \varphi \end{pmatrix}$ equal to

$$\psi(P) = \frac{1}{2}\begin{pmatrix} M_\Sigma\, a \\ \bar{P}\, a \end{pmatrix}.$$

As can be seen, the wave function of a Dirac particle in the momentum representation is a Penrose twistor.

APPENDIX 2

$\mu^2 = 0.065$, $k^{-1} = 2 \cdot 10^{-14}$ cm, $khc = 1$ GeV. The experimental mass values are taken from [9].

We have to note that the masses of low-lying states (in particular, of nucleons) are appreciably renormalized by switching on interactions, the excistence of which are connected with the degeneration group of the state $\dot{f}_z$. It is necessary to keep in mind that quanta of degeneration fields (first of all, π, η, K-mesons, photon, graviton) are not fundamental particles, they are a new kind of elementary particles.

1. *Sannikov-Proskurjakov S.S.* // . – 2001. – **46,** ” 8. – . 775 – 783.
2. *Sannikov-Proskurjakov S.S* //Izvestiya Vysshikh Uchebnykh Zavedenii, seriya Fizika, N 10, 60 (1997) (translation in Soviet Physics Journal).
3. *Sannikov-Proskurjakov S.S*, *Cabbolet M.J.F.T.*// Ukrain. J. Phys. **45**, N 8, 895 (2000).
4. *Sannikov-Proskurjakov S.S* // Izvestiya Vysshikh Uchebnykh Zavedenii, seriya Fizika, N 11, 52 – 61 (1999) (translation in Russian Physics Journal); Ukr. Journ. Phys., **46**, N1, 5 – 13 (2001).
5. *Efimov G.V.* Problems of the Quantum Theory of Nonlocal Interactions (Nauka, Moscow, 1985).
6. *Sannikov S.S.* // Izvestiya Vysshikh Uchebnykh Zavedenii, seriya Fizika, N 2, 25 (1996) (translation in Russian Physics Journal).
7. *Gradshtein I.S., Ryzhik I.M.* Tables of Integrals, Series and Products: 4th ed. (Academic, New York, 1965).
8. *Sannikov S.S.* // J. Nucl. Phys. **4,** N 3, 587 (1996).
9. *Review* of Particle Properties // Phys. Lett. **B239**, 1990.

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